Computer algebra independent integration tests

`1_Algebraic_functions/1.1_Binomial_products/1.1.2Quadratic/1.1.2.3(a+bx^2)^p(c+dx^2)`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

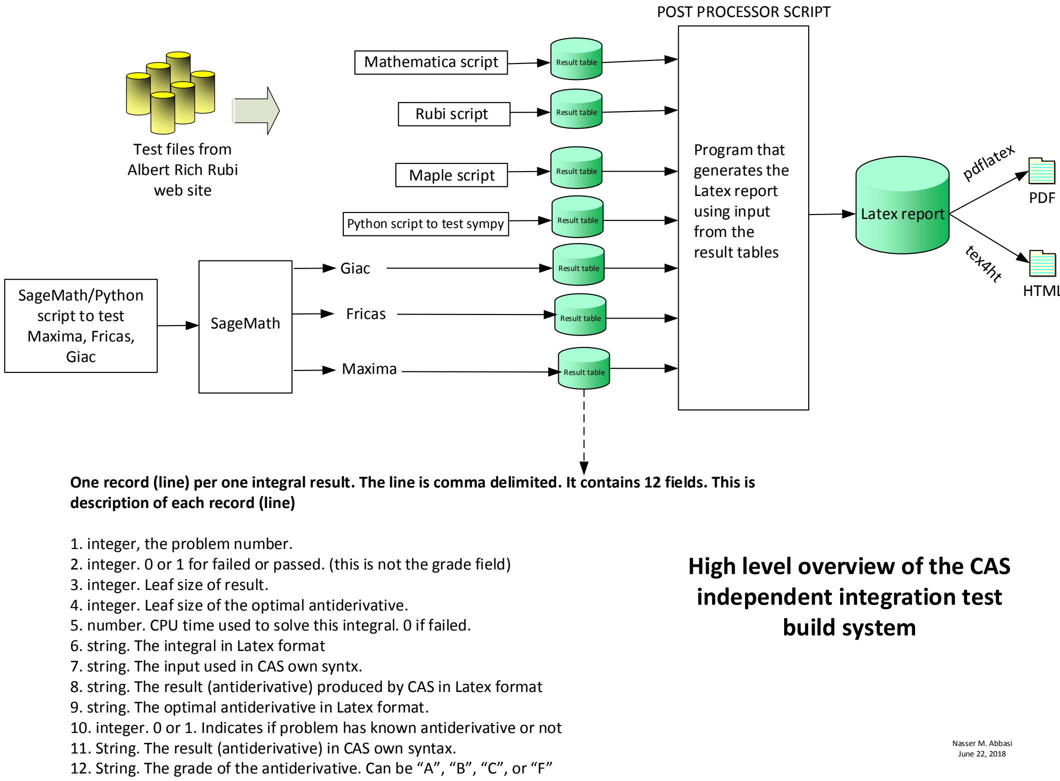
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (346)	% 0. (0)
Rubi in Sympy	% 86.42 (299)	% 13.58 (47)
Mathematica	% 100. (346)	% 0. (0)
Maple	% 70.23 (243)	% 29.77 (103)
Maxima	% 8.09 (28)	% 91.91 (318)
Fricas	% 34.68 (120)	% 65.32 (226)
Sympy	% 23.7 (82)	% 76.3 (264)
Giac	% 32.08 (111)	% 67.92 (235)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

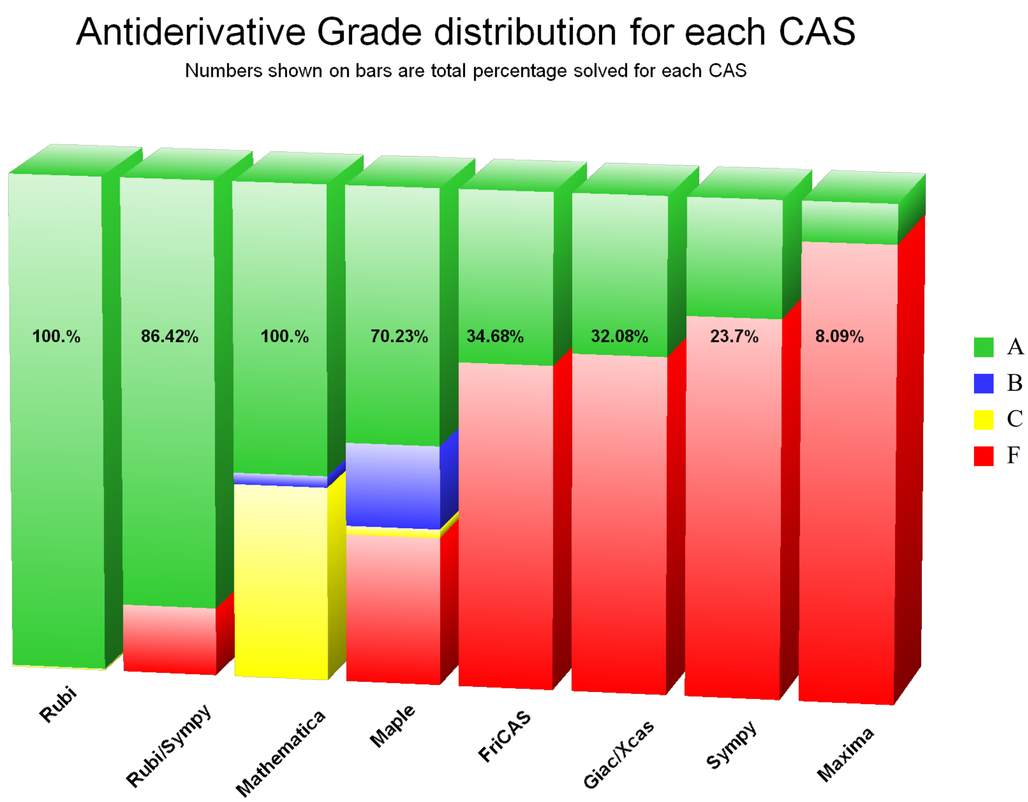
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

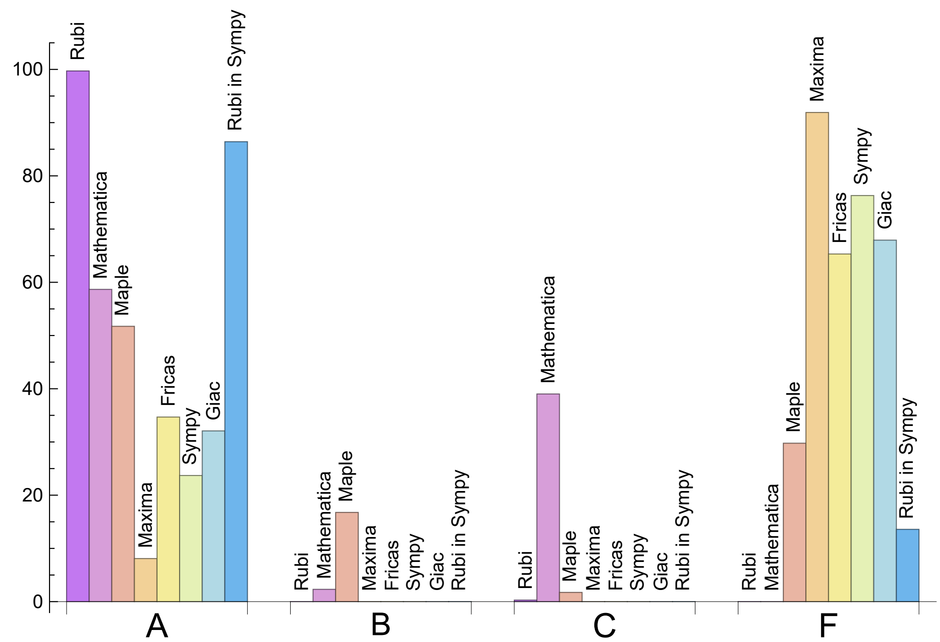
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.71	0.	0.29	0.
Rubi in Sympy	86.42	0.	0.	13.58
Mathematica	58.67	2.31	39.02	0.
Maple	51.73	16.76	1.73	29.77
Maxima	8.09	0.	0.	91.91
Fricas	34.68	0.	0.	65.32
Sympy	23.7	0.	0.	76.3
Giac	32.08	0.	0.	67.92

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.31	179.76	1.	118.5	1.
Rubi in Sympy	41.72	147.89	0.94	88.	0.88
Mathematica	0.25	128.18	1.	94.	0.99
Maple	0.03	869.32	5.26	110.	1.24
Maxima	1.41	104.68	1.6	65.	1.36
Fricas	0.79	22.65	0.63	1.	0.01
Sympy	19.99	230.73	2.33	130.	1.85
Giac	1.54	145.36	1.42	122.	1.26

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 2, 3, 4, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 29, 30, 34, 35, 36, 37, 41, 42, 96, 114, 115, 128, 133, 134, 139, 140, 318, 320, 324, 326, 328, 330, 332, 334, 336, 337, 339}

Not solved by Mathematica {}

Not solved by Maple {109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 294, 295, 296, 297, 298, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345}

Not solved by Maxima {5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346}

Not solved by Fricas {109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 313, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345}

Not solved by Sympy {26, 33, 34, 40, 41, 42, 49, 50, 51, 52, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 222, 223, 224, 225, 226, 227, 229, 230, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346}

Not solved by Giac {109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {112, 114, 115, 119, 120, 121, 127, 128, 132, 133, 134, 139, 140, 298}

Mathematica {112, 114, 115, 119, 120, 121, 126, 127, 128, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 346}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	1	107	132	0
normalized size	1	1.	1.	1.03	1.38	0.01	1.14	1.4	0.
time (sec)	N/A	0.145	0.034	0.002	1.348	0.179	0.148	0.226	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	1	76	99	0
normalized size	1	1.	1.	1.04	1.36	0.01	1.09	1.41	0.
time (sec)	N/A	0.102	0.023	0.001	1.34	0.178	0.13	0.236	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.072	0.017	0.001	1.397	0.177	0.111	0.227	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	1	26	35	0
normalized size	1	1.	1.	0.89	1.14	0.04	0.93	1.25	0.
time (sec)	N/A	0.037	0.009	0.002	1.345	0.176	0.084	0.235	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	45	0	1	82	46	34
normalized size	1	1.	1.	1.12	0.	0.02	2.05	1.15	0.85
time (sec)	N/A	0.052	0.039	0.008	0.	0.207	1.57	0.231	8.762

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	1	112	77	51
normalized size	1	1.	1.	1.08	0.	0.02	1.78	1.22	0.81
time (sec)	N/A	0.062	0.075	0.01	0.	0.213	2.071	0.236	9.607

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	90	0	1	150	105	78
normalized size	1	1.	0.89	0.98	0.	0.01	1.63	1.14	0.85
time (sec)	N/A	0.086	0.102	0.011	0.	0.211	2.746	0.232	12.902

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	1	136	177	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.11	1.45	0.
time (sec)	N/A	0.169	0.039	0.002	1.349	0.18	0.167	0.233	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	1	97	123	0
normalized size	1	1.	1.	1.06	1.35	0.01	1.18	1.5	0.
time (sec)	N/A	0.116	0.03	0.002	1.352	0.18	0.139	0.236	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	1	53	68	0
normalized size	1	1.	1.	0.98	1.3	0.02	1.06	1.36	0.
time (sec)	N/A	0.07	0.014	0.002	1.344	0.179	0.112	0.226	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	1	172	97	0
normalized size	1	1.	0.94	1.51	0.	0.02	2.73	1.54	0.
time (sec)	N/A	0.097	0.082	0.005	0.	0.21	2.105	0.249	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	89	129	0	1	236	128	0
normalized size	1	1.	1.09	1.57	0.	0.01	2.88	1.56	0.
time (sec)	N/A	0.218	0.099	0.012	0.	0.21	3.395	0.235	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	121	147	0	1	223	170	105
normalized size	1	1.	1.04	1.27	0.	0.01	1.92	1.47	0.91
time (sec)	N/A	0.182	0.172	0.012	0.	0.246	4.61	0.232	23.06

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	161	177	225	1	189	252	0
normalized size	1	1.	1.05	1.15	1.46	0.01	1.23	1.64	0.
time (sec)	N/A	0.23	0.051	0.002	1.352	0.18	0.194	0.226	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	1	136	177	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.11	1.45	0.
time (sec)	N/A	0.168	0.039	0.002	1.367	0.179	0.163	0.225	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	1	76	99	0
normalized size	1	1.	1.	1.04	1.36	0.01	1.09	1.41	0.
time (sec)	N/A	0.104	0.023	0.	1.359	0.178	0.135	0.236	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	93	161	0	1	240	176	0
normalized size	1	1.	0.95	1.64	0.	0.01	2.45	1.8	0.
time (sec)	N/A	0.147	0.099	0.004	0.	0.21	2.653	0.236	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	205	0	1	313	205	0
normalized size	1	1.	1.	1.92	0.	0.01	2.93	1.92	0.
time (sec)	N/A	0.215	0.1	0.013	0.	0.222	4.801	0.236	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	141	266	0	1	422	243	0
normalized size	1	1.	1.08	2.05	0.	0.01	3.25	1.87	0.
time (sec)	N/A	0.365	0.128	0.015	0.	0.248	8.273	0.23	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	136	246	0	1	323	267	0
normalized size	1	1.	0.96	1.73	0.	0.01	2.27	1.88	0.
time (sec)	N/A	0.206	0.146	0.007	0.	0.218	3.387	0.235	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	92	161	0	1	240	174	0
normalized size	1	1.	0.94	1.64	0.	0.01	2.45	1.78	0.
time (sec)	N/A	0.136	0.116	0.005	0.	0.212	2.734	0.233	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	95	0	1	172	97	0
normalized size	1	1.	0.94	1.51	0.	0.02	2.73	1.54	0.
time (sec)	N/A	0.095	0.083	0.004	0.	0.211	2.163	0.232	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	1	82	45	34
normalized size	1	1.	1.03	1.15	0.	0.03	2.1	1.15	0.87
time (sec)	N/A	0.048	0.042	0.004	0.	0.21	1.601	0.23	8.525

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	0	1	712	257	60
normalized size	1	1.	0.87	0.79	0.	0.01	10.17	3.67	0.86
time (sec)	N/A	0.071	0.079	0.01	0.	0.23	8.098	0.258	15.186

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	144	0	1	2033	165	94
normalized size	1	1.	0.87	1.32	0.	0.01	18.65	1.51	0.86
time (sec)	N/A	0.213	0.316	0.016	0.	0.318	46.026	0.224	41.21

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	310	0	1	0	293	146
normalized size	1	1.	0.99	1.94	0.	0.01	0.	1.83	0.91
time (sec)	N/A	0.464	0.513	0.017	0.	0.968	0.	0.234	95.697

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	192	402	0	1	498	413	0
normalized size	1	1.	1.	2.09	0.	0.01	2.59	2.15	0.
time (sec)	N/A	0.336	0.158	0.016	0.	0.215	9.143	0.234	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	296	0	1	398	297	0
normalized size	1	1.	1.	2.08	0.	0.01	2.8	2.09	0.
time (sec)	N/A	0.26	0.143	0.016	0.	0.214	6.652	0.244	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	205	0	1	313	205	0
normalized size	1	1.	1.	1.93	0.	0.01	2.95	1.93	0.
time (sec)	N/A	0.205	0.1	0.014	0.	0.211	5.18	0.235	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	88	129	0	1	236	127	0
normalized size	1	1.	1.07	1.57	0.	0.01	2.88	1.55	0.
time (sec)	N/A	0.223	0.101	0.012	0.	0.212	3.687	0.233	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	68	0	1	112	77	51
normalized size	1	1.	1.	1.08	0.	0.02	1.78	1.22	0.81
time (sec)	N/A	0.062	0.075	0.01	0.	0.208	2.166	0.234	9.346

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	144	0	1	2033	163	94
normalized size	1	1.	1.01	1.33	0.	0.01	18.82	1.51	0.87
time (sec)	N/A	0.212	0.242	0.016	0.	0.321	47.682	0.237	41.249

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	136	238	0	1	0	1	141
normalized size	1	1.	0.81	1.43	0.	0.01	0.	0.01	0.84
time (sec)	N/A	0.455	0.566	0.02	0.	0.929	0.	0.388	95.521

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	197	403	0	1	0	448	0
normalized size	1	1.	0.86	1.75	0.	0.	0.	1.95	0.
time (sec)	N/A	0.723	0.983	0.023	0.	4.165	0.	0.24	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	484	0	1	614	459	0
normalized size	1	1.	1.	2.47	0.	0.01	3.13	2.34	0.
time (sec)	N/A	0.476	0.202	0.019	0.	0.219	21.413	0.233	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	160	367	0	1	513	343	0
normalized size	1	1.	1.	2.29	0.	0.01	3.21	2.14	0.
time (sec)	N/A	0.416	0.156	0.016	0.	0.218	13.583	0.233	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	139	266	0	1	422	240	0
normalized size	1	1.	1.07	2.05	0.	0.01	3.25	1.85	0.
time (sec)	N/A	0.362	0.132	0.015	0.	0.219	8.401	0.25	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	124	147	0	1	223	170	105
normalized size	1	1.	1.07	1.27	0.	0.01	1.92	1.47	0.91
time (sec)	N/A	0.181	0.163	0.012	0.	0.215	4.729	0.233	23.471

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	89	0	1	150	105	78
normalized size	1	1.	0.91	0.97	0.	0.01	1.63	1.14	0.85
time (sec)	N/A	0.085	0.105	0.011	0.	0.209	2.797	0.231	12.602

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	158	309	0	1	0	294	146
normalized size	1	1.	0.98	1.92	0.	0.01	0.	1.83	0.91
time (sec)	N/A	0.446	0.548	0.017	0.	0.984	0.	0.23	98.386

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	197	403	0	1	0	450	0
normalized size	1	1.	0.83	1.71	0.	0.	0.	1.91	0.
time (sec)	N/A	0.717	0.775	0.021	0.	4.169	0.	0.247	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	233	568	0	1	0	1	0
normalized size	1	1.	0.74	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.037	1.714	0.024	0.	13.618	0.	0.494	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	23	45	45	31	27	27
normalized size	1	1.	0.71	0.68	1.32	1.32	0.91	0.79	0.79
time (sec)	N/A	0.026	0.012	0.01	1.343	0.194	0.319	0.234	8.89

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	33	65	90	46	73	39
normalized size	1	1.	0.87	0.7	1.38	1.91	0.98	1.55	0.83
time (sec)	N/A	0.044	0.021	0.012	1.513	0.201	0.449	0.233	11.203

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	181	310	0	1	484	271	230
normalized size	1	1.	0.78	1.34	0.	0.	2.1	1.17	1.
time (sec)	N/A	0.413	0.187	0.016	0.	0.412	60.359	0.241	50.338

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	123	190	0	1	291	174	143
normalized size	1	1.	0.83	1.28	0.	0.01	1.95	1.17	0.96
time (sec)	N/A	0.205	0.121	0.012	0.	0.264	33.619	0.237	24.803

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	96	0	1	144	95	75
normalized size	1	1.	0.9	1.1	0.	0.01	1.66	1.09	0.86
time (sec)	N/A	0.079	0.066	0.007	0.	0.232	17.121	0.232	9.625

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	1	41	50	39
normalized size	1	1.	1.07	0.78	0.	0.02	0.89	1.09	0.85
time (sec)	N/A	0.026	0.029	0.001	0.	0.221	6.342	0.228	3.129

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	932	0	1	0	150	70
normalized size	1	1.	1.02	11.37	0.	0.01	0.	1.83	0.85
time (sec)	N/A	0.133	0.069	0.055	0.	0.251	0.	0.26	20.412

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	2521	0	1	0	4	68
normalized size	1	1.	1.	30.74	0.	0.01	0.	0.05	0.83
time (sec)	N/A	0.098	0.152	0.039	0.	0.28	0.	2.649	17.221

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	128	5101	0	1	0	4	129
normalized size	1	1.	0.86	34.23	0.	0.01	0.	0.03	0.87
time (sec)	N/A	0.263	0.197	0.037	0.	0.357	0.	3.949	31.151

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	184	7922	0	1	0	4	190
normalized size	1	1.	0.88	38.09	0.	0.	0.	0.02	0.91
time (sec)	N/A	0.554	0.436	0.047	0.	1.256	0.	32.137	82.834

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	220	393	0	1	665	351	269
normalized size	1	1.	0.81	1.44	0.	0.	2.44	1.29	0.99
time (sec)	N/A	0.495	0.208	0.018	0.	0.69	151.033	0.284	53.83

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	159	249	0	1	440	236	192
normalized size	1	1.	0.81	1.27	0.	0.01	2.24	1.2	0.98
time (sec)	N/A	0.262	0.169	0.013	0.	0.349	87.222	0.255	28.596

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	131	0	1	253	139	102
normalized size	1	1.	0.83	1.11	0.	0.01	2.14	1.18	0.86
time (sec)	N/A	0.106	0.125	0.007	0.	0.248	43.635	0.286	12.268

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	51	0	1	70	66	60
normalized size	1	1.	0.95	0.78	0.	0.02	1.08	1.02	0.92
time (sec)	N/A	0.038	0.052	0.003	0.	0.223	10.058	0.265	4.313

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	110	1845	0	1	0	205	102
normalized size	1	1.	0.97	16.33	0.	0.01	0.	1.81	0.9
time (sec)	N/A	0.29	0.318	0.025	0.	0.412	0.	0.283	39.987

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	142	4621	0	1	0	4	114
normalized size	1	1.	1.08	35.27	0.	0.01	0.	0.03	0.87
time (sec)	N/A	0.249	0.199	0.033	0.	0.366	0.	0.642	40.672

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	9059	0	1	0	4	100
normalized size	1	1.	0.91	80.17	0.	0.01	0.	0.04	0.88
time (sec)	N/A	0.161	0.206	0.041	0.	0.319	0.	1.485	26.018

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	178	13766	0	1	0	4	175
normalized size	1	1.	0.89	69.18	0.	0.01	0.	0.02	0.88
time (sec)	N/A	0.307	0.372	0.059	0.	0.564	0.	28.901	43.454

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	260	18791	0	1	0	4	277
normalized size	1	1.	0.87	62.64	0.	0.	0.	0.01	0.92
time (sec)	N/A	0.915	0.452	0.075	0.	2.456	0.	1.588	158.453

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	270	476	0	1	0	433	352
normalized size	1	1.	0.77	1.36	0.	0.	0.	1.24	1.01
time (sec)	N/A	0.562	0.308	0.018	0.	1.24	0.	0.241	65.835

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	191	308	0	1	537	298	238
normalized size	1	1.	0.79	1.28	0.	0.	2.23	1.24	0.99
time (sec)	N/A	0.324	0.211	0.011	0.	0.556	170.682	0.367	32.758

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	121	166	0	1	316	182	134
normalized size	1	1.	0.81	1.11	0.	0.01	2.12	1.22	0.9
time (sec)	N/A	0.135	0.133	0.007	0.	0.325	88.521	0.337	14.711

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	66	0	1	97	85	78
normalized size	1	1.	0.85	0.79	0.	0.01	1.15	1.01	0.93
time (sec)	N/A	0.052	0.069	0.	0.	0.233	14.318	0.285	5.829

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	140	3053	0	1	0	290	146
normalized size	1	1.	0.89	19.45	0.	0.01	0.	1.85	0.93
time (sec)	N/A	0.482	0.189	0.027	0.	1.324	0.	0.408	69.389

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	144	7345	0	1	0	4	155
normalized size	1	1.	0.82	41.97	0.	0.01	0.	0.02	0.89
time (sec)	N/A	0.527	0.279	0.037	0.	1.001	0.	0.794	75.19

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	184	14133	0	1	0	4	177
normalized size	1	1.	0.95	72.85	0.	0.01	0.	0.02	0.91
time (sec)	N/A	0.483	0.344	0.048	0.	0.615	0.	0.65	73.234

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	139	21220	0	1	0	4	131
normalized size	1	1.	0.97	147.36	0.	0.01	0.	0.03	0.91
time (sec)	N/A	0.198	0.296	0.086	0.	0.465	0.	29.169	35.28

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	235	28625	0	1	0	4	226
normalized size	1	1.	0.94	114.96	0.	0.	0.	0.02	0.91
time (sec)	N/A	0.377	0.487	0.095	0.	1.396	0.	2.01	54.969

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	84	0	128	24
normalized size	1	1.	1.	1.1	0.	2.8	0.	4.27	0.8
time (sec)	N/A	0.046	0.042	0.024	0.	0.213	0.	0.265	10.54

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	64	84	80	128	0	95	24
normalized size	1	1.	2.37	3.11	2.96	4.74	0.	3.52	0.89
time (sec)	N/A	0.039	0.042	0.017	1.521	0.209	0.	0.255	8.652

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	187	143	100	0	159	19
normalized size	1	1.	1.	7.48	5.72	4.	0.	6.36	0.76
time (sec)	N/A	0.043	0.013	0.104	1.498	0.21	0.	0.24	11.258

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	140	228	0	1	400	203	163
normalized size	1	1.	0.83	1.35	0.	0.01	2.37	1.2	0.96
time (sec)	N/A	0.334	0.156	0.017	0.	0.27	39.827	0.242	42.704

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	131	0	1	238	122	102
normalized size	1	1.	0.84	1.21	0.	0.01	2.2	1.13	0.94
time (sec)	N/A	0.144	0.09	0.01	0.	0.228	20.518	0.24	20.442

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	61	62	0	1	126	66	49
normalized size	1	1.	1.05	1.07	0.	0.02	2.17	1.14	0.84
time (sec)	N/A	0.054	0.043	0.006	0.	0.219	8.542	0.234	7.968

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	17	31	22
normalized size	1	1.	1.	0.84	0.	0.04	0.68	1.24	0.88
time (sec)	N/A	0.017	0.011	0.	0.	0.213	3.614	0.236	2.454

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	300	0	1	0	95	42
normalized size	1	1.	1.	6.12	0.	0.02	0.	1.94	0.86
time (sec)	N/A	0.058	0.044	0.023	0.	0.261	0.	0.242	10.42

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	809	0	1	0	327	83
normalized size	1	1.	1.	8.01	0.	0.01	0.	3.24	0.82
time (sec)	N/A	0.14	0.169	0.029	0.	0.363	0.	0.249	20.468

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	143	1815	0	1	0	726	146
normalized size	1	1.	0.88	11.13	0.	0.01	0.	4.45	0.9
time (sec)	N/A	0.339	0.276	0.039	0.	0.617	0.	0.967	55.935

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	172	340	0	1	0	317	262
normalized size	1	1.	0.67	1.32	0.	0.	0.	1.23	1.02
time (sec)	N/A	0.633	0.4	0.024	0.	0.452	0.	0.248	85.096

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	122	219	0	1	0	212	163
normalized size	1	1.	0.72	1.3	0.	0.01	0.	1.25	0.96
time (sec)	N/A	0.434	0.156	0.013	0.	0.278	0.	0.258	44.942

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	89	123	0	1	0	124	95
normalized size	1	1.17	0.99	1.37	0.	0.01	0.	1.38	1.06
time (sec)	N/A	0.154	0.154	0.01	0.	0.23	0.	0.244	20.424

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	58	54	0	1	60	68	46
normalized size	1	1.	1.07	1.	0.	0.02	1.11	1.26	0.85
time (sec)	N/A	0.05	0.063	0.007	0.	0.22	11.148	0.229	8.098

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	31	17	19	12
normalized size	1	1.	1.	0.94	1.19	1.94	1.06	1.19	0.75
time (sec)	N/A	0.009	0.012	0.	1.355	0.207	1.806	0.225	1.274

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	618	0	1	0	143	66
normalized size	1	1.	1.	7.82	0.	0.01	0.	1.81	0.84
time (sec)	N/A	0.119	0.159	0.025	0.	0.306	0.	0.231	18.837

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	133	1439	0	1	0	4	121
normalized size	1	1.	0.93	10.06	0.	0.01	0.	0.03	0.85
time (sec)	N/A	0.323	0.292	0.031	0.	0.579	0.	3.906	54.396

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	181	2919	0	1	0	4	196
normalized size	1	1.	0.8	12.97	0.	0.	0.	0.02	0.87
time (sec)	N/A	0.679	0.794	0.04	0.	1.307	0.	12.073	115.598

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	157	351	0	1	0	320	260
normalized size	1	1.	0.62	1.38	0.	0.	0.	1.25	1.02
time (sec)	N/A	0.615	0.278	0.024	0.	0.561	0.	0.234	91.401

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	125	228	0	1	0	213	162
normalized size	1	1.	0.73	1.33	0.	0.01	0.	1.24	0.94
time (sec)	N/A	0.369	0.157	0.012	0.	0.303	0.	0.233	50.925

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	136	0	1	0	139	94
normalized size	1	1.	0.92	1.3	0.	0.01	0.	1.32	0.9
time (sec)	N/A	0.133	0.22	0.01	0.	0.236	0.	0.232	23.838

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	92	73	144	54	41
normalized size	1	1.	0.79	0.72	1.96	1.55	3.06	1.15	0.87
time (sec)	N/A	0.036	0.043	0.005	1.349	0.214	36.389	0.226	6.519

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	63	95	36	32
normalized size	1	1.	0.74	0.67	1.08	1.62	2.44	0.92	0.82
time (sec)	N/A	0.02	0.016	0.004	1.345	0.212	2.771	0.226	2.09

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	1070	0	1	0	432	107
normalized size	1	1.	0.92	8.77	0.	0.01	0.	3.54	0.88
time (sec)	N/A	0.346	0.36	0.027	0.	0.546	0.	0.229	55.144

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	170	2371	0	1	0	4	178
normalized size	1	1.	0.84	11.74	0.	0.	0.	0.02	0.88
time (sec)	N/A	0.62	1.028	0.036	0.	1.225	0.	4.944	116.464

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	221	4495	0	1	0	4	0
normalized size	1	1.	0.71	14.36	0.	0.	0.	0.01	0.
time (sec)	N/A	1.084	1.222	0.046	0.	3.904	0.	1.165	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	163	190	628	309	0	294	204
normalized size	1	1.	0.73	0.85	2.8	1.38	0.	1.31	0.91
time (sec)	N/A	0.279	0.158	0.011	1.365	0.802	0.	0.234	41.474

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	107	115	336	204	0	186	156
normalized size	1	1.	0.61	0.66	1.93	1.17	0.	1.07	0.9
time (sec)	N/A	0.201	0.106	0.01	1.368	0.332	0.	0.232	30.056

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	57	139	117	566	97	78
normalized size	1	1.	0.65	0.63	1.53	1.29	6.22	1.07	0.86
time (sec)	N/A	0.082	0.06	0.006	1.35	0.228	127.945	0.229	10.601

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	63	95	36	32
normalized size	1	1.	0.74	0.67	1.08	1.62	2.44	0.92	0.82
time (sec)	N/A	0.019	0.022	0.005	1.35	0.211	2.737	0.228	2.026

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	628	0	1	0	146	66
normalized size	1	1.	0.99	7.95	0.	0.01	0.	1.85	0.84
time (sec)	N/A	0.122	0.152	0.054	0.	0.31	0.	0.23	20.604

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	823	0	1	0	304	83
normalized size	1	1.	1.	8.23	0.	0.01	0.	3.04	0.83
time (sec)	N/A	0.144	0.176	0.041	0.	0.356	0.	0.233	21.817

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	130	5177	0	1	0	4	129
normalized size	1	1.	0.87	34.74	0.	0.01	0.	0.03	0.87
time (sec)	N/A	0.229	0.262	0.05	0.	0.404	0.	1.243	32.714

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	179	13964	0	1	0	4	175
normalized size	1	1.	0.9	70.17	0.	0.01	0.	0.02	0.88
time (sec)	N/A	0.298	0.413	0.074	0.	0.62	0.	19.528	44.854

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	36	0	24	15
normalized size	1	1.	1.	0.95	0.	1.8	0.	1.2	0.75
time (sec)	N/A	0.014	0.017	0.004	0.	0.211	0.	0.241	4.939

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	0	65	0	69	22
normalized size	1	1.	1.	1.12	0.	2.6	0.	2.76	0.88
time (sec)	N/A	0.027	0.04	0.006	0.	0.213	0.	0.237	6.097

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	1	0	95	42
normalized size	1	1.	1.	6.24	0.	0.02	0.	1.94	0.86
time (sec)	N/A	0.056	0.043	0.015	0.	0.259	0.	0.239	11.279

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	66	31	34	14
normalized size	1	1.	1.	0.93	1.2	4.4	2.07	2.27	0.93
time (sec)	N/A	0.016	0.02	0.008	1.491	0.203	10.816	0.229	4.065

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	99	0	0	0	136	0	525
normalized size	1	1.	0.15	0.	0.	0.	0.21	0.	0.81
time (sec)	N/A	1.205	0.085	0.049	0.	0.	11.488	0.	90.845

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	88	0	0	0	99	0	496
normalized size	1	1.	0.14	0.	0.	0.	0.16	0.	0.8
time (sec)	N/A	0.961	0.063	0.039	0.	0.	7.902	0.	59.535

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	76	0	0	0	63	0	469
normalized size	1	1.	0.13	0.	0.	0.	0.11	0.	0.8
time (sec)	N/A	0.806	0.05	0.038	0.	0.	5.371	0.	39.191

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	740	740	162	0	0	0	0	0	784
normalized size	1	1.	0.22	0.	0.	0.	0.	0.	1.06
time (sec)	N/A	0.779	0.236	0.066	0.	0.	0.	0.	112.322

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	86	0	0	0	0	0	452
normalized size	1	1.	0.15	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.803	0.091	0.074	0.	0.	0.	0.	46.811

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	350	0	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.304	0.735	0.07	0.	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	364	0	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.471	0.361	0.073	0.	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	110	0	0	0	139	0	544
normalized size	1	1.	0.16	0.	0.	0.	0.21	0.	0.81
time (sec)	N/A	1.293	0.085	0.045	0.	0.	21.027	0.	102.624

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	637	637	99	0	0	0	131	0	515
normalized size	1	1.	0.16	0.	0.	0.	0.21	0.	0.81
time (sec)	N/A	1.064	0.072	0.039	0.	0.	15.453	0.	69.578

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	88	0	0	0	100	0	488
normalized size	1	1.	0.14	0.	0.	0.	0.16	0.	0.8
time (sec)	N/A	0.928	0.061	0.036	0.	0.	10.476	0.	48.147

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	765	765	333	0	0	0	0	0	811
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	1.06
time (sec)	N/A	1.072	0.272	0.085	0.	0.	0.	0.	163.58

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	775	775	320	0	0	0	0	0	814
normalized size	1	1.	0.41	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	1.055	0.228	0.067	0.	0.	0.	0.	158.993

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	815	815	346	0	0	0	0	0	78
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	0.1
time (sec)	N/A	1.386	0.436	0.069	0.	0.	0.	0.	50.406

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	98	0	0	0	165	0	537
normalized size	1	1.	0.15	0.	0.	0.	0.25	0.	0.81
time (sec)	N/A	1.165	0.103	0.045	0.	0.	12.509	0.	113.285

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	88	0	0	0	129	0	507
normalized size	1	1.	0.14	0.	0.	0.	0.21	0.	0.81
time (sec)	N/A	1.005	0.076	0.04	0.	0.	8.902	0.	81.761

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	597	77	0	0	0	94	0	478
normalized size	1	1.	0.13	0.	0.	0.	0.16	0.	0.8
time (sec)	N/A	0.827	0.065	0.039	0.	0.	6.537	0.	51.672

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	62	0	0	0	60	0	450
normalized size	1	1.	0.11	0.	0.	0.	0.11	0.	0.79
time (sec)	N/A	0.775	0.053	0.036	0.	0.	4.311	0.	33.094

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	355
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	1.74
time (sec)	N/A	0.127	0.064	0.057	0.	0.	0.	0.	73.703

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	322	0	0	0	0	0	813
normalized size	1	1.	0.41	0.	0.	0.	0.	0.	1.03
time (sec)	N/A	1.217	0.282	0.066	0.	0.	0.	0.	151.13

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	352	0	0	0	0	0	0
normalized size	1	1.	0.43	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.42	0.386	0.068	0.	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	623	76	0	0	0	0	0	503
normalized size	1	1.	0.12	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	1.071	0.097	0.082	0.	0.	0.	0.	82.707

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	62	0	0	0	0	0	473
normalized size	1	1.	0.1	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.918	0.065	0.073	0.	0.	0.	0.	53.325

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	561	561	51	0	0	0	60	0	442
normalized size	1	1.	0.09	0.	0.	0.	0.11	0.	0.79
time (sec)	N/A	0.774	0.051	0.037	0.	0.	9.811	0.	32.785

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	325	0	0	0	0	0	809
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	1.04
time (sec)	N/A	1.216	0.332	0.061	0.	0.	0.	0.	150.704

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	807	807	323	0	0	0	0	0	0
normalized size	1	1.	0.4	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.389	0.334	0.066	0.	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	849	849	353	0	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.736	0.392	0.068	0.	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	96	0	0	0	0	0	527
normalized size	1	1.	0.15	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	1.3	0.123	0.093	0.	0.	0.	0.	131.245

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	83	0	0	0	0	0	476
normalized size	1	1.	0.14	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	1.028	0.108	0.075	0.	0.	0.	0.	66.355

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	45	57	0	0	37
normalized size	1	1.	0.55	0.55	1.02	1.3	0.	0.	0.84
time (sec)	N/A	0.051	0.038	0.008	1.483	0.262	0.	0.	21.476

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	590	74	0	0	0	60	0	466
normalized size	1	1.	0.13	0.	0.	0.	0.1	0.	0.79
time (sec)	N/A	0.791	0.087	0.039	0.	0.	36.95	0.	42.61

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	347	0	0	0	0	0	0
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.291	0.328	0.062	0.	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	827	827	346	0	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.527	0.427	0.067	0.	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	163	0	0	0	0	0	355
normalized size	1	1.	0.65	0.	0.	0.	0.	0.	1.41
time (sec)	N/A	0.192	0.25	0.06	0.	0.	0.	0.	90.639

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	166	0	0	0	0	0	348
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	1.72
time (sec)	N/A	0.103	0.234	0.064	0.	0.	0.	0.	35.677

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	153	0	0	0	0	0	376
normalized size	1	1.	0.75	0.	0.	0.	0.	0.	1.84
time (sec)	N/A	0.122	0.232	0.067	0.	0.	0.	0.	38.639

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	355
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	1.74
time (sec)	N/A	0.102	0.067	0.	0.	0.	0.	0.	75.3

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	156	0	0	0	0	0	410
normalized size	1	1.	0.76	0.	0.	0.	0.	0.	2.01
time (sec)	N/A	0.098	0.236	0.062	0.	0.	0.	0.	85.054

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	0	0	0	0	0	144
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	1.27
time (sec)	N/A	0.048	0.169	0.046	0.	0.	0.	0.	13.915

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0	192
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	1.76
time (sec)	N/A	0.046	0.056	0.069	0.	0.	0.	0.	11.585

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	203	0	0	0	0	0	87
normalized size	1	1.	2.11	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.069	0.438	0.047	0.	0.	0.	0.	6.056

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	203	0	0	0	0	0	87
normalized size	1	1.	2.14	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.065	0.237	0.05	0.	0.	0.	0.	5.631

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	169	0	0	0	0	0	54
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.36
time (sec)	N/A	0.088	0.258	0.069	0.	0.	0.	0.	38.064

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	167	0	0	0	0	0	53
normalized size	1	1.	1.09	0.	0.	0.	0.	0.	0.35
time (sec)	N/A	0.087	0.274	0.07	0.	0.	0.	0.	38.08

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	168	0	0	0	0	0	51
normalized size	1	1.	1.11	0.	0.	0.	0.	0.	0.34
time (sec)	N/A	0.089	0.25	0.061	0.	0.	0.	0.	39.375

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	172	0	0	0	0	0	58
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.38
time (sec)	N/A	0.089	0.272	0.061	0.	0.	0.	0.	39.963

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	148	0	0	0	0	0	32
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.21
time (sec)	N/A	0.09	0.233	0.074	0.	0.	0.	0.	16.167

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-2)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	148	0	0	0	0	0	46
normalized size	1	1.	1.01	0.	0.	0.	0.	0.	0.31
time (sec)	N/A	0.08	0.273	0.063	0.	0.	0.	0.	25.286

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	136	0	0	0	0	0	29
normalized size	1	1.	1.11	0.	0.	0.	0.	0.	0.24
time (sec)	N/A	0.063	0.185	0.036	0.	0.	0.	0.	11.7

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	136	0	0	0	0	0	27
normalized size	1	1.	1.11	0.	0.	0.	0.	0.	0.22
time (sec)	N/A	0.058	0.189	0.077	0.	0.	0.	0.	11.879

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	0	0	0	0	0	42
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	0.35
time (sec)	N/A	0.056	0.188	0.074	0.	0.	0.	0.	17.175

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	136	0	0	0	0	0	49
normalized size	1	1.	1.14	0.	0.	0.	0.	0.	0.41
time (sec)	N/A	0.057	0.204	0.034	0.	0.	0.	0.	17.145

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	124	0	0	0	0	0	19
normalized size	1	1.	1.77	0.	0.	0.	0.	0.	0.27
time (sec)	N/A	0.031	0.148	0.035	0.	0.	0.	0.	5.124

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	137	0	0	0	0	0	22
normalized size	1	1.	1.32	0.	0.	0.	0.	0.	0.21
time (sec)	N/A	0.053	0.167	0.056	0.	0.	0.	0.	9.582

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	125	0	0	0	0	0	15
normalized size	1	1.	1.69	0.	0.	0.	0.	0.	0.2
time (sec)	N/A	0.037	0.07	0.063	0.	0.	0.	0.	8.496

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	243	543	0	0	0	0	301
normalized size	1	1.	0.74	1.66	0.	0.	0.	0.	0.92
time (sec)	N/A	0.739	0.799	0.068	0.	0.	0.	0.	91.917

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	198	328	0	0	0	0	214
normalized size	1	1.	0.8	1.32	0.	0.	0.	0.	0.86
time (sec)	N/A	0.421	0.442	0.016	0.	0.	0.	0.	57.139

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0	168
normalized size	1	1.	0.42	0.5	0.	0.	0.	0.	0.82
time (sec)	N/A	0.276	0.082	0.023	0.	0.	0.	0.	40.271

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	133	181	0	0	0	0	71
normalized size	1	1.	1.58	2.15	0.	0.	0.	0.	0.85
time (sec)	N/A	0.057	0.529	0.049	0.	0.	0.	0.	8.984

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	243	617	0	0	0	0	197
normalized size	1	1.	1.03	2.6	0.	0.	0.	0.	0.83
time (sec)	N/A	0.33	0.77	0.055	0.	0.	0.	0.	48.298

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	285	1411	0	0	0	0	274
normalized size	1	1.	0.92	4.57	0.	0.	0.	0.	0.89
time (sec)	N/A	0.588	0.897	0.063	0.	0.	0.	0.	82.839

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	302	780	0	0	0	0	379
normalized size	1	1.	0.74	1.9	0.	0.	0.	0.	0.92
time (sec)	N/A	1.071	1.064	0.028	0.	0.	0.	0.	141.37

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	246	545	0	0	0	0	299
normalized size	1	1.	0.73	1.62	0.	0.	0.	0.	0.89
time (sec)	N/A	0.729	0.716	0.022	0.	0.	0.	0.	98.026

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	199	330	0	0	0	0	233
normalized size	1	1.	0.73	1.21	0.	0.	0.	0.	0.85
time (sec)	N/A	0.472	0.476	0.022	0.	0.	0.	0.	63.056

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	191	332	0	0	0	0	230
normalized size	1	1.	0.72	1.24	0.	0.	0.	0.	0.86
time (sec)	N/A	0.461	0.482	0.033	0.	0.	0.	0.	63.975

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	232	607	0	0	0	0	192
normalized size	1	1.	1.01	2.65	0.	0.	0.	0.	0.84
time (sec)	N/A	0.372	0.8	0.036	0.	0.	0.	0.	46.265

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	285	1414	0	0	0	0	270
normalized size	1	1.	0.9	4.49	0.	0.	0.	0.	0.86
time (sec)	N/A	0.695	0.996	0.042	0.	0.	0.	0.	85.636

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	127	303	0	0	0	0	221
normalized size	1	1.	0.54	1.29	0.	0.	0.	0.	0.94
time (sec)	N/A	0.377	0.172	0.038	0.	0.	0.	0.	48.444

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	32	75	0	0	0	0	32
normalized size	1	1.	0.84	1.97	0.	0.	0.	0.	0.84
time (sec)	N/A	0.035	0.08	0.059	0.	0.	0.	0.	5.062

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	15	38	23	51	17	23	17
normalized size	1	1.	0.75	1.9	1.15	2.55	0.85	1.15	0.85
time (sec)	N/A	0.012	0.004	0.007	1.525	0.219	23.073	0.239	3.462

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0	175
normalized size	1	1.	0.2	0.2	0.	0.	0.	0.	0.96
time (sec)	N/A	0.246	0.039	0.023	0.	0.	0.	0.	35.293

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	60	78	0	0	0	0	75
normalized size	1	1.	0.66	0.86	0.	0.	0.	0.	0.82
time (sec)	N/A	0.186	0.057	0.033	0.	0.	0.	0.	34.149

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	60	53	0	0	0	0	129
normalized size	1	1.	0.4	0.35	0.	0.	0.	0.	0.86
time (sec)	N/A	0.187	0.046	0.035	0.	0.	0.	0.	27.399

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	0	0	0	0	19
normalized size	1	1.	1.	1.45	0.	0.	0.	0.	0.95
time (sec)	N/A	0.027	0.03	0.049	0.	0.	0.	0.	5.199

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	24	0	0	0	0	20
normalized size	1	1.	1.	1.14	0.	0.	0.	0.	0.95
time (sec)	N/A	0.027	0.026	0.118	0.	0.	0.	0.	5.183

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	35	0	0	0	0	19
normalized size	1	1.	1.	1.75	0.	0.	0.	0.	0.95
time (sec)	N/A	0.026	0.028	0.111	0.	0.	0.	0.	5.269

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	0	0	0	5
normalized size	1	1.	1.	1.25	0.	0.	0.	0.	1.25
time (sec)	N/A	0.019	0.025	0.02	0.	0.	0.	0.	5.449

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	0	0	0	0	20
normalized size	1	1.	1.	1.25	0.	0.	0.	0.	1.
time (sec)	N/A	0.024	0.025	0.028	0.	0.	0.	0.	5.144

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	25	0	0	0	0	22
normalized size	1	1.	1.	1.19	0.	0.	0.	0.	1.05
time (sec)	N/A	0.024	0.026	0.025	0.	0.	0.	0.	4.865

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	0	0	0	0	20
normalized size	1	1.	1.	1.25	0.	0.	0.	0.	1.
time (sec)	N/A	0.026	0.025	0.036	0.	0.	0.	0.	5.411

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	0	0	0	14
normalized size	1	1.	0.92	1.08	0.	0.	0.	0.	1.08
time (sec)	N/A	0.051	0.022	0.011	0.	0.	0.	0.	13.137

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	33	0	0	0	0	31
normalized size	1	1.	0.87	1.06	0.	0.	0.	0.	1.
time (sec)	N/A	0.066	0.027	0.024	0.	0.	0.	0.	12.415

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	37	0	0	0	0	31
normalized size	1	1.	0.77	1.06	0.	0.	0.	0.	0.89
time (sec)	N/A	0.067	0.027	0.035	0.	0.	0.	0.	12.424

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	27	37	0	0	0	0	36
normalized size	1	1.	0.77	1.06	0.	0.	0.	0.	1.03
time (sec)	N/A	0.067	0.026	0.029	0.	0.	0.	0.	12.434

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	27	36	0	0	0	0	119
normalized size	1	1.	0.21	0.27	0.	0.	0.	0.	0.91
time (sec)	N/A	0.131	0.024	0.077	0.	0.	0.	0.	19.386

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	27	34	0	0	0	0	114
normalized size	1	1.	0.2	0.25	0.	0.	0.	0.	0.84
time (sec)	N/A	0.144	0.025	0.049	0.	0.	0.	0.	19.085

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	27	26	0	0	0	0	121
normalized size	1	1.	0.18	0.18	0.	0.	0.	0.	0.82
time (sec)	N/A	0.16	0.025	0.113	0.	0.	0.	0.	20.257

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	32	0	0	0	0	37
normalized size	1	1.	0.88	0.8	0.	0.	0.	0.	0.92
time (sec)	N/A	0.051	0.035	0.048	0.	0.	0.	0.	10.403

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	321	852	0	0	0	0	408
normalized size	1	1.	0.76	2.01	0.	0.	0.	0.	0.96
time (sec)	N/A	1.065	2.729	0.033	0.	0.	0.	0.	130.858

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	260	615	0	0	0	0	328
normalized size	1	1.	0.76	1.79	0.	0.	0.	0.	0.95
time (sec)	N/A	0.729	0.845	0.025	0.	0.	0.	0.	91.941

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	216	399	0	0	0	0	236
normalized size	1	1.	0.83	1.53	0.	0.	0.	0.	0.91
time (sec)	N/A	0.461	0.597	0.022	0.	0.	0.	0.	61.749

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0	172
normalized size	1	1.	0.44	0.81	0.	0.	0.	0.	0.89
time (sec)	N/A	0.274	0.081	0.018	0.	0.	0.	0.	39.148

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	0	0	0	73
normalized size	1	1.	0.99	1.15	0.	0.	0.	0.	0.84
time (sec)	N/A	0.061	0.084	0.023	0.	0.	0.	0.	10.583

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	112	248	0	0	0	0	160
normalized size	1	1.	0.41	0.91	0.	0.	0.	0.	0.59
time (sec)	N/A	0.403	0.402	0.037	0.	0.	0.	0.	59.092

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	261	752	0	0	0	0	221
normalized size	1	1.	1.02	2.95	0.	0.	0.	0.	0.87
time (sec)	N/A	0.404	1.071	0.041	0.	0.	0.	0.	58.23

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	301	1607	0	0	0	0	301
normalized size	1	1.	0.9	4.81	0.	0.	0.	0.	0.9
time (sec)	N/A	0.704	1.054	0.049	0.	0.	0.	0.	120.703

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	318	755	0	0	0	0	418
normalized size	1	1.	0.71	1.7	0.	0.	0.	0.	0.94
time (sec)	N/A	1.015	2.098	0.061	0.	0.	0.	0.	143.453

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	256	539	0	0	0	0	318
normalized size	1	1.	0.74	1.56	0.	0.	0.	0.	0.92
time (sec)	N/A	0.717	0.823	0.036	0.	0.	0.	0.	97.81

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	196	345	0	0	0	0	223
normalized size	1	1.	0.76	1.34	0.	0.	0.	0.	0.86
time (sec)	N/A	0.438	0.49	0.033	0.	0.	0.	0.	61.085

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	136	188	0	0	0	0	71
normalized size	1	1.	1.62	2.24	0.	0.	0.	0.	0.85
time (sec)	N/A	0.053	0.52	0.026	0.	0.	0.	0.	8.281

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	112	144	0	0	0	0	228
normalized size	1	1.	0.58	0.74	0.	0.	0.	0.	1.18
time (sec)	N/A	0.403	0.412	0.034	0.	0.	0.	0.	58.687

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	224	354	0	0	0	0	207
normalized size	1	1.	0.93	1.46	0.	0.	0.	0.	0.86
time (sec)	N/A	0.34	1.243	0.038	0.	0.	0.	0.	51.877

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	337	964	0	0	0	0	284
normalized size	1	1.	1.04	2.98	0.	0.	0.	0.	0.88
time (sec)	N/A	0.675	1.844	0.044	0.	0.	0.	0.	96.898

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	0	0	0	73
normalized size	1	1.	0.99	1.15	0.	0.	0.	0.	0.84
time (sec)	N/A	0.062	0.085	0.	0.	0.	0.	0.	10.454

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0	75
normalized size	1	1.	1.	1.22	0.	0.	0.	0.	0.86
time (sec)	N/A	0.162	0.096	0.048	0.	0.	0.	0.	37.32

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	106	0	0	0	0	75
normalized size	1	1.	1.02	1.22	0.	0.	0.	0.	0.86
time (sec)	N/A	0.162	0.102	0.044	0.	0.	0.	0.	37.482

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	108	0	0	0	0	73
normalized size	1	1.	1.	1.23	0.	0.	0.	0.	0.83
time (sec)	N/A	0.171	0.093	0.041	0.	0.	0.	0.	54.8

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	0	0	0	0	14
normalized size	1	1.	1.	1.42	0.	0.	0.	0.	1.17
time (sec)	N/A	0.025	0.033	0.043	0.	0.	0.	0.	5.225

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	58	14	0	0	0	0	12
normalized size	1	1.	5.8	1.4	0.	0.	0.	0.	1.2
time (sec)	N/A	0.027	0.051	0.038	0.	0.	0.	0.	5.249

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	0	0	0	0	14
normalized size	1	1.	1.	1.42	0.	0.	0.	0.	1.17
time (sec)	N/A	0.025	0.031	0.025	0.	0.	0.	0.	5.239

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	0	73	0	12
normalized size	1	1.	1.	1.	0.	0.	7.3	0.	1.2
time (sec)	N/A	0.023	0.032	0.03	0.	0.	33.988	0.	4.955

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	0	0	0	0	14
normalized size	1	1.	1.5	1.17	0.	0.	0.	0.	1.17
time (sec)	N/A	0.023	0.033	0.037	0.	0.	0.	0.	4.947

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	0	12
normalized size	1	1.	1.	1.08	0.	0.	0.	0.	1.
time (sec)	N/A	0.027	0.03	0.035	0.	0.	0.	0.	8.287

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	26	8	22	95	22	28	8
normalized size	1	1.	3.25	1.	2.75	11.88	2.75	3.5	1.
time (sec)	N/A	0.008	0.009	0.052	1.542	0.25	8.932	0.266	2.775

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	16	0	0	0	0	12
normalized size	1	1.	1.67	1.33	0.	0.	0.	0.	1.
time (sec)	N/A	0.026	0.035	0.028	0.	0.	0.	0.	5.282

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	16	11	0	0	0	0	10
normalized size	1	1.	1.6	1.1	0.	0.	0.	0.	1.
time (sec)	N/A	0.028	0.045	0.031	0.	0.	0.	0.	5.387

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	16	0	0	0	0	12
normalized size	1	1.	1.67	1.33	0.	0.	0.	0.	1.
time (sec)	N/A	0.026	0.037	0.062	0.	0.	0.	0.	5.404

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	19	20	0	0	0	0	48
normalized size	1	1.	0.37	0.39	0.	0.	0.	0.	0.94
time (sec)	N/A	0.031	0.029	0.102	0.	0.	0.	0.	5.536

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	17	15	0	0	0	0	46
normalized size	1	1.	0.35	0.31	0.	0.	0.	0.	0.94
time (sec)	N/A	0.033	0.042	0.057	0.	0.	0.	0.	5.19

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	19	20	0	0	0	0	48
normalized size	1	1.	0.37	0.39	0.	0.	0.	0.	0.94
time (sec)	N/A	0.032	0.029	0.022	0.	0.	0.	0.	5.582

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	46	8	9	8
normalized size	1	1.	1.	1.	1.12	5.75	1.	1.12	1.
time (sec)	N/A	0.008	0.007	0.031	1.529	0.245	10.548	0.225	2.524

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	19	15	0	0	0	0	42
normalized size	1	1.	0.4	0.32	0.	0.	0.	0.	0.89
time (sec)	N/A	0.029	0.029	0.034	0.	0.	0.	0.	4.878

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	14	0	0	0	0	12
normalized size	1	1.	1.9	1.4	0.	0.	0.	0.	1.2
time (sec)	N/A	0.023	0.04	0.036	0.	0.	0.	0.	4.99

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	0	76	0	12
normalized size	1	1.	1.	1.	0.	0.	7.6	0.	1.2
time (sec)	N/A	0.023	0.034	0.009	0.	0.	33.279	0.	4.427

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	0	0	0	0	20
normalized size	1	1.	1.	1.25	0.	0.	0.	0.	1.
time (sec)	N/A	0.026	0.033	0.023	0.	0.	0.	0.	5.037

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	0	0	0	14
normalized size	1	1.	1.	0.94	0.	0.	0.	0.	0.88
time (sec)	N/A	0.026	0.039	0.03	0.	0.	0.	0.	4.935

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	25	0	0	0	0	20
normalized size	1	1.	1.	1.25	0.	0.	0.	0.	1.
time (sec)	N/A	0.03	0.034	0.052	0.	0.	0.	0.	4.956

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	43	0	0	0	0	31
normalized size	1	1.	1.	1.34	0.	0.	0.	0.	0.97
time (sec)	N/A	0.046	0.033	0.046	0.	0.	0.	0.	8.694

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	0	0	0	0	29
normalized size	1	1.	1.	1.13	0.	0.	0.	0.	0.97
time (sec)	N/A	0.051	0.043	0.043	0.	0.	0.	0.	8.665

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	43	0	0	0	0	31
normalized size	1	1.	1.	1.34	0.	0.	0.	0.	0.97
time (sec)	N/A	0.047	0.031	0.037	0.	0.	0.	0.	8.611

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	30	0	0	75	0	41
normalized size	1	1.	1.2	1.2	0.	0.	3.	0.	1.64
time (sec)	N/A	0.032	0.038	0.029	0.	0.	33.077	0.	5.252

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	34	0	0	0	0	31
normalized size	1	1.	1.	1.06	0.	0.	0.	0.	0.97
time (sec)	N/A	0.044	0.03	0.035	0.	0.	0.	0.	8.316

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	47	28	0	0	0	0	12
normalized size	1	1.	3.92	2.33	0.	0.	0.	0.	1.
time (sec)	N/A	0.024	0.038	0.032	0.	0.	0.	0.	6.416

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	26	22	46	0	0	22
normalized size	1	1.	1.38	0.9	0.76	1.59	0.	0.	0.76
time (sec)	N/A	0.011	0.019	0.013	1.545	0.239	0.	0.	2.701

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	32	0	0	0	0	29
normalized size	1	1.	1.25	1.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.047	0.037	0.031	0.	0.	0.	0.	8.653

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	0	0	0	0	27
normalized size	1	1.	1.2	0.9	0.	0.	0.	0.	0.9
time (sec)	N/A	0.05	0.044	0.032	0.	0.	0.	0.	8.666

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	32	0	0	0	0	29
normalized size	1	1.	1.25	1.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.047	0.034	0.064	0.	0.	0.	0.	8.727

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	42	0	0	0	0	53
normalized size	1	1.	0.74	0.79	0.	0.	0.	0.	1.
time (sec)	N/A	0.034	0.038	0.09	0.	0.	0.	0.	5.697

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	37	33	0	0	0	0	51
normalized size	1	1.	0.73	0.65	0.	0.	0.	0.	1.
time (sec)	N/A	0.037	0.043	0.06	0.	0.	0.	0.	5.729

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	42	0	0	0	0	53
normalized size	1	1.	0.74	0.79	0.	0.	0.	0.	1.
time (sec)	N/A	0.035	0.034	0.092	0.	0.	0.	0.	5.751

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	24	9	136	0	0	24
normalized size	1	1.	0.93	0.86	0.32	4.86	0.	0.	0.86
time (sec)	N/A	0.011	0.017	0.01	1.53	0.234	0.	0.	3.128

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	53	33	0	0	0	0	48
normalized size	1	1.	1.08	0.67	0.	0.	0.	0.	0.98
time (sec)	N/A	0.031	0.054	0.028	0.	0.	0.	0.	5.453

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	34	0	0	0	0	31
normalized size	1	1.	1.26	1.1	0.	0.	0.	0.	1.
time (sec)	N/A	0.051	0.034	0.03	0.	0.	0.	0.	12.03

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	65	30	30	0	0	73	0	42
normalized size	1	1.55	0.71	0.71	0.	0.	1.74	0.	1.
time (sec)	N/A	0.04	0.048	0.012	0.	0.	35.229	0.	5.645

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	0	0	0	39
normalized size	1	1.	1.	0.92	0.	0.	0.	0.	0.98
time (sec)	N/A	0.051	0.035	0.034	0.	0.	0.	0.	8.883

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	0	0	0	0	32
normalized size	1	1.	1.	0.94	0.	0.	0.	0.	0.89
time (sec)	N/A	0.053	0.045	0.026	0.	0.	0.	0.	8.847

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	0	0	0	39
normalized size	1	1.	1.	0.92	0.	0.	0.	0.	0.98
time (sec)	N/A	0.052	0.038	0.035	0.	0.	0.	0.	8.91

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0	75
normalized size	1	1.	1.	1.22	0.	0.	0.	0.	0.86
time (sec)	N/A	0.153	0.094	0.017	0.	0.	0.	0.	29.82

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	171	0	0	0	0	76
normalized size	1	1.	1.	1.9	0.	0.	0.	0.	0.84
time (sec)	N/A	0.163	0.082	0.024	0.	0.	0.	0.	37.923

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	168	0	0	0	0	75
normalized size	1	1.	1.	1.91	0.	0.	0.	0.	0.85
time (sec)	N/A	0.155	0.077	0.02	0.	0.	0.	0.	30.195

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	110	0	0	0	0	76
normalized size	1	1.	1.	1.21	0.	0.	0.	0.	0.84
time (sec)	N/A	0.162	0.072	0.019	0.	0.	0.	0.	37.188

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	109	0	0	0	0	73
normalized size	1	1.	1.	1.24	0.	0.	0.	0.	0.83
time (sec)	N/A	0.16	0.086	0.018	0.	0.	0.	0.	42.163

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	165	0	0	0	0	73
normalized size	1	1.	1.	1.85	0.	0.	0.	0.	0.82
time (sec)	N/A	0.16	0.072	0.025	0.	0.	0.	0.	38.938

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	166	0	0	0	0	73
normalized size	1	1.	1.	1.87	0.	0.	0.	0.	0.82
time (sec)	N/A	0.158	0.072	0.02	0.	0.	0.	0.	40.64

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	111	0	0	0	0	73
normalized size	1	1.	1.	1.23	0.	0.	0.	0.	0.81
time (sec)	N/A	0.161	0.065	0.014	0.	0.	0.	0.	40.103

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	0	0	0	172
normalized size	1	1.	0.44	0.81	0.	0.	0.	0.	0.89
time (sec)	N/A	0.271	0.078	0.	0.	0.	0.	0.	38.784

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	89	104	0	0	0	0	177
normalized size	1	1.	0.44	0.51	0.	0.	0.	0.	0.87
time (sec)	N/A	0.291	0.068	0.023	0.	0.	0.	0.	42.327

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	89	108	0	0	0	0	180
normalized size	1	1.	0.44	0.53	0.	0.	0.	0.	0.89
time (sec)	N/A	0.296	0.08	0.019	0.	0.	0.	0.	42.714

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	92	165	0	0	0	0	189
normalized size	1	1.	0.43	0.78	0.	0.	0.	0.	0.89
time (sec)	N/A	0.324	0.065	0.019	0.	0.	0.	0.	55.137

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	89	164	0	0	0	0	162
normalized size	1	1.	0.47	0.87	0.	0.	0.	0.	0.86
time (sec)	N/A	0.391	0.074	0.018	0.	0.	0.	0.	76.148

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	90	109	0	0	0	0	162
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.	0.85
time (sec)	N/A	0.382	0.076	0.022	0.	0.	0.	0.	75.689

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	92	111	0	0	0	0	165
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.	0.85
time (sec)	N/A	0.394	0.079	0.017	0.	0.	0.	0.	99.842

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	93	167	0	0	0	0	167
normalized size	1	1.	0.47	0.84	0.	0.	0.	0.	0.84
time (sec)	N/A	0.396	0.065	0.017	0.	0.	0.	0.	93.126

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	106	0	0	0	0	75
normalized size	1	1.	1.	1.22	0.	0.	0.	0.	0.86
time (sec)	N/A	0.151	0.085	0.017	0.	0.	0.	0.	31.343

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	171	0	0	0	0	76
normalized size	1	1.	1.	1.9	0.	0.	0.	0.	0.84
time (sec)	N/A	0.159	0.069	0.025	0.	0.	0.	0.	39.135

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	168	0	0	0	0	75
normalized size	1	1.	1.	1.91	0.	0.	0.	0.	0.85
time (sec)	N/A	0.15	0.07	0.018	0.	0.	0.	0.	31.364

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	110	0	0	0	0	76
normalized size	1	1.	1.	1.21	0.	0.	0.	0.	0.84
time (sec)	N/A	0.158	0.063	0.016	0.	0.	0.	0.	37.582

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	164	0	0	0	0	73
normalized size	1	1.	1.	1.86	0.	0.	0.	0.	0.83
time (sec)	N/A	0.159	0.083	0.019	0.	0.	0.	0.	42.265

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	110	0	0	0	0	73
normalized size	1	1.	1.	1.24	0.	0.	0.	0.	0.82
time (sec)	N/A	0.156	0.069	0.024	0.	0.	0.	0.	39.08

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	110	0	0	0	0	73
normalized size	1	1.	1.	1.24	0.	0.	0.	0.	0.82
time (sec)	N/A	0.157	0.07	0.021	0.	0.	0.	0.	40.807

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	167	0	0	0	0	73
normalized size	1	1.	1.	1.86	0.	0.	0.	0.	0.81
time (sec)	N/A	0.156	0.062	0.016	0.	0.	0.	0.	39.315

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	0	0	0	168
normalized size	1	1.	0.42	0.5	0.	0.	0.	0.	0.82
time (sec)	N/A	0.276	0.082	0.	0.	0.	0.	0.	36.831

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	89	161	0	0	0	0	177
normalized size	1	1.	0.42	0.75	0.	0.	0.	0.	0.83
time (sec)	N/A	0.299	0.068	0.024	0.	0.	0.	0.	40.403

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	89	162	0	0	0	0	180
normalized size	1	1.	0.42	0.76	0.	0.	0.	0.	0.84
time (sec)	N/A	0.304	0.077	0.019	0.	0.	0.	0.	41.247

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	92	111	0	0	0	0	185
normalized size	1	1.	0.41	0.5	0.	0.	0.	0.	0.83
time (sec)	N/A	0.326	0.063	0.02	0.	0.	0.	0.	52.471

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	89	164	0	0	0	0	162
normalized size	1	1.	0.47	0.87	0.	0.	0.	0.	0.86
time (sec)	N/A	0.387	0.08	0.017	0.	0.	0.	0.	74.586

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	90	109	0	0	0	0	162
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.	0.85
time (sec)	N/A	0.375	0.067	0.022	0.	0.	0.	0.	75.267

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	92	111	0	0	0	0	165
normalized size	1	1.	0.47	0.57	0.	0.	0.	0.	0.85
time (sec)	N/A	0.395	0.071	0.018	0.	0.	0.	0.	97.956

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	93	167	0	0	0	0	167
normalized size	1	1.	0.47	0.84	0.	0.	0.	0.	0.84
time (sec)	N/A	0.397	0.064	0.018	0.	0.	0.	0.	94.77

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	37	38	0	0	0	0	75
normalized size	1	1.	0.47	0.49	0.	0.	0.	0.	0.96
time (sec)	N/A	0.054	0.044	0.032	0.	0.	0.	0.	8.781

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	38	0	0	0	0	32
normalized size	1	1.	1.03	0.97	0.	0.	0.	0.	0.82
time (sec)	N/A	0.073	0.056	0.032	0.	0.	0.	0.	15.894

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	0	0	0	0	53
normalized size	1	1.	0.77	0.87	0.	0.	0.	0.	0.87
time (sec)	N/A	0.043	0.05	0.028	0.	0.	0.	0.	8.127

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	27	25	0	0	0	0	5
normalized size	1	1.	4.5	4.17	0.	0.	0.	0.	0.83
time (sec)	N/A	0.025	0.036	0.025	0.	0.	0.	0.	6.948

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	0	0	0	20
normalized size	1	1.	1.04	1.22	0.	0.	0.	0.	0.87
time (sec)	N/A	0.097	0.039	0.028	0.	0.	0.	0.	27.947

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	0	0	0	175
normalized size	1	1.	0.2	0.2	0.	0.	0.	0.	0.96
time (sec)	N/A	0.253	0.035	0.	0.	0.	0.	0.	32.489

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	0	0	0	0	19
normalized size	1	1.	1.84	1.95	0.	0.	0.	0.	1.
time (sec)	N/A	0.026	0.035	0.027	0.	0.	0.	0.	5.265

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	92	0	0	0	0	0	87
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.297	0.414	0.319	0.	0.	0.	0.	23.694

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	92	0	0	0	0	0	85
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.258	0.358	0.212	0.	0.	0.	0.	38.045

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	478	478	94	0	0	0	0	0	425
normalized size	1	1.	0.2	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.102	0.358	0.203	0.	0.	0.	0.	159.989

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	94	0	0	0	0	0	185
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.662	0.314	0.137	0.	0.	0.	0.	82.535

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	C	C	F	F	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	23	122	0	0	0	0	0	15
normalized size	1	0.37	1.97	0.	0.	0.	0.	0.	0.24
time (sec)	N/A	0.035	0.194	0.053	0.	0.	0.	0.	6.14

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	117	0	0	0	0	41
normalized size	1	1.	1.04	2.54	0.	0.	0.	0.	0.89
time (sec)	N/A	0.099	0.107	0.245	0.	0.	0.	0.	15.727

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	81	207	0	0	0	0	66
normalized size	1	1.	1.72	4.4	0.	0.	0.	0.	1.4
time (sec)	N/A	0.134	0.21	0.601	0.	0.	0.	0.	15.151

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	135	0	0	0	0	0	92
normalized size	1	1.	1.05	0.	0.	0.	0.	0.	0.71
time (sec)	N/A	0.06	0.173	0.036	0.	0.	0.	0.	73.971

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	135	0	0	0	0	0	88
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.055	0.196	0.076	0.	0.	0.	0.	73.619

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	144	0	0	0	0	0	88
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.68
time (sec)	N/A	0.069	0.232	0.055	0.	0.	0.	0.	73.72

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	145	0	0	0	0	0	85
normalized size	1	1.	1.17	0.	0.	0.	0.	0.	0.69
time (sec)	N/A	0.072	0.218	0.057	0.	0.	0.	0.	76.199

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	155	0	0	0	0	0	122
normalized size	1	1.	1.29	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.065	0.23	0.067	0.	0.	0.	0.	46.645

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	155	0	0	0	0	0	119
normalized size	1	1.	1.29	0.	0.	0.	0.	0.	0.99
time (sec)	N/A	0.061	0.249	0.063	0.	0.	0.	0.	45.944

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	165	0	0	0	0	0	116
normalized size	1	1.	1.38	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.072	0.237	0.074	0.	0.	0.	0.	52.765

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	162	0	0	0	0	0	112
normalized size	1	1.	1.31	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.073	0.247	0.064	0.	0.	0.	0.	54.209

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	139	0	0	168
normalized size	1	1.	2.08	0.	0.	2.28	0.	0.	2.75
time (sec)	N/A	0.034	0.211	0.095	0.	2.701	0.	0.	50.299

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	127	0	0	332	0	0	185
normalized size	1	1.	2.08	0.	0.	5.44	0.	0.	3.03
time (sec)	N/A	0.033	0.222	0.036	0.	2.77	0.	0.	51.136

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	132	0	0	1	0	0	170
normalized size	1	1.	1.71	0.	0.	0.01	0.	0.	2.21
time (sec)	N/A	0.046	0.25	0.06	0.	9.332	0.	0.	57.179

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	137	0	0	1	0	0	187
normalized size	1	1.	1.73	0.	0.	0.01	0.	0.	2.37
time (sec)	N/A	0.05	0.239	0.054	0.	9.122	0.	0.	65.557

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	157	0	0	0	0	0	231
normalized size	1	1.	1.85	0.	0.	0.	0.	0.	2.72
time (sec)	N/A	0.054	0.27	0.063	0.	0.	0.	0.	90.765

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	157	0	0	0	0	0	248
normalized size	1	1.	1.85	0.	0.	0.	0.	0.	2.92
time (sec)	N/A	0.049	0.235	0.059	0.	0.	0.	0.	96.136

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	163	0	0	0	0	0	233
normalized size	1	1.	1.61	0.	0.	0.	0.	0.	2.31
time (sec)	N/A	0.068	0.251	0.063	0.	0.	0.	0.	93.729

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	168	0	0	0	0	0	250
normalized size	1	1.	1.63	0.	0.	0.	0.	0.	2.43
time (sec)	N/A	0.067	0.252	0.059	0.	0.	0.	0.	107.999

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	A	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	115	0	0	124	0	0	150
normalized size	1	1.	2.17	0.	0.	2.34	0.	0.	2.83
time (sec)	N/A	0.028	0.228	0.083	0.	2.4	0.	0.	45.86

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	431	0	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.684	1.105	0.097	0.	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	435	0	0	0	0	0	270
normalized size	1	1.	1.44	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.581	1.112	0.085	0.	0.	0.	0.	98.333

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	161	0	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.455	0.25	0.06	0.	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	160	0	0	0	0	0	172
normalized size	1	1.	0.8	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.445	0.25	0.075	0.	0.	0.	0.	82.759

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	160	0	0	0	0	0	146
normalized size	1	1.	0.96	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.347	0.088	0.054	0.	0.	0.	0.	58.21

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	161	0	0	0	0	0	131
normalized size	1	1.	1.06	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.365	0.09	0.059	0.	0.	0.	0.	66.326

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	339	0	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.486	0.548	0.06	0.	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	342	0	0	0	0	0	223
normalized size	1	1.	1.35	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.476	0.511	0.059	0.	0.	0.	0.	88.924

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	404	0	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.965	1.625	0.059	0.	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	408	0	0	0	0	0	277
normalized size	1	1.	1.34	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.874	1.662	0.059	0.	0.	0.	0.	169.644

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	436	0	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.772	0.64	0.069	0.	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	439	0	0	0	0	0	245
normalized size	1	1.	1.57	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.707	0.632	0.066	0.	0.	0.	0.	107.621

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	320	0	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.618	0.294	0.065	0.	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	322	0	0	0	0	0	233
normalized size	1	1.	1.16	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.605	0.282	0.064	0.	0.	0.	0.	106.984

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	358	0	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.718	0.984	0.064	0.	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	340	0	0	0	0	0	250
normalized size	1	1.	1.16	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.646	0.812	0.065	0.	0.	0.	0.	113.161

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	480	0	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.04	0.972	0.065	0.	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	485	0	0	0	0	0	303
normalized size	1	1.	1.41	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.973	1.031	0.064	0.	0.	0.	0.	173.819

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	634	0	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.432	2.301	0.065	0.	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	637	0	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.245	2.388	0.066	0.	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	61
normalized size	1	1.	2.18	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.107	0.349	0.109	0.	0.	0.	0.	29.434

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	136	0	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.639	0.098	0.072	0.	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	88	0	160
normalized size	1	1.	0.6	0.	0.	0.	0.5	0.	0.91
time (sec)	N/A	0.263	0.064	0.061	0.	0.	88.489	0.	37.036

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	75	0	0	0	53	0	73
normalized size	1	0.91	0.81	0.	0.	0.	0.57	0.	0.78
time (sec)	N/A	0.096	0.03	0.044	0.	0.	39.935	0.	12.757

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	34
normalized size	1	1.	1.	0.	0.	0.	0.5	0.	0.77
time (sec)	N/A	0.025	0.009	0.001	0.	0.	9.094	0.	4.911

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	42
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.071	0.307	0.06	0.	0.	0.	0.	29.904

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.07	0.319	0.075	0.	0.	0.	0.	27.656

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	44
normalized size	1	1.	2.84	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.069	0.384	0.111	0.	0.	0.	0.	27.715

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	594	71	0	123	0	0	44
normalized size	1	1.	11.21	1.34	0.	2.32	0.	0.	0.83
time (sec)	N/A	0.058	2.4	0.005	0.	0.277	0.	0.	13.151

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [326] had the largest ratio of [0.4286]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	17	0.059
2	A	2	1	1.	17	0.059
3	A	2	1	1.	17	0.059
4	A	2	1	1.	15	0.067
5	A	2	2	1.	17	0.118
6	A	2	2	1.	17	0.118
7	A	3	3	1.	17	0.176
8	A	2	1	1.	19	0.053
9	A	2	1	1.	19	0.053
10	A	2	1	1.	17	0.059
11	A	3	2	1.	19	0.105
12	A	4	3	1.	19	0.158
13	A	3	3	1.	19	0.158
14	A	2	1	1.	19	0.053
15	A	2	1	1.	19	0.053
16	A	2	1	1.	17	0.059
17	A	3	2	1.	19	0.105
18	A	4	3	1.	19	0.158
19	A	5	4	1.	19	0.21
20	A	3	2	1.	19	0.105
21	A	3	2	1.	19	0.105
22	A	3	2	1.	19	0.105
23	A	2	2	1.	17	0.118
24	A	3	2	1.	19	0.105
25	A	4	3	1.	19	0.158
26	A	5	4	1.	19	0.21
27	A	4	3	1.	19	0.158
28	A	4	3	1.	19	0.158

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
29	A	4	3	1.	19	0.158
30	A	4	3	1.	19	0.158
31	A	2	2	1.	17	0.118
32	A	4	3	1.	19	0.158
33	A	5	4	1.	19	0.21
34	A	6	4	1.	19	0.21
35	A	5	4	1.	19	0.21
36	A	5	4	1.	19	0.21
37	A	5	4	1.	19	0.21
38	A	3	3	1.	19	0.158
39	A	3	3	1.	17	0.176
40	A	5	4	1.	19	0.21
41	A	6	4	1.	19	0.21
42	A	7	4	1.	19	0.21
43	A	3	3	1.	15	0.2
44	A	5	3	1.	15	0.2
45	A	6	6	1.	21	0.286
46	A	5	5	1.	21	0.238
47	A	4	4	1.	19	0.21
48	A	3	3	1.	11	0.273
49	A	5	5	1.	21	0.238
50	A	3	3	1.	21	0.143
51	A	4	4	1.	21	0.19
52	A	6	5	1.	21	0.238
53	A	7	6	1.	21	0.286
54	A	6	5	1.	21	0.238
55	A	5	4	1.	19	0.21
56	A	4	3	1.	11	0.273
57	A	6	6	1.	21	0.286
58	A	6	6	1.	21	0.286
59	A	4	3	1.	21	0.143
60	A	5	4	1.	21	0.19
61	A	7	5	1.	21	0.238
62	A	8	6	1.	21	0.286
63	A	7	5	1.	21	0.238
64	A	6	4	1.	19	0.21

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
65	A	5	3	1.	11	0.273
66	A	7	7	1.	21	0.333
67	A	7	7	1.	21	0.333
68	A	7	7	1.	21	0.333
69	A	5	3	1.	21	0.143
70	A	6	4	1.	21	0.19
71	A	4	4	1.	19	0.21
72	A	4	4	1.	17	0.235
73	A	4	4	1.	21	0.19
74	A	5	5	1.	21	0.238
75	A	4	4	1.	21	0.19
76	A	3	3	1.	19	0.158
77	A	2	2	1.	11	0.182
78	A	2	2	1.	21	0.095
79	A	3	3	1.	21	0.143
80	A	5	5	1.	21	0.238
81	A	6	5	1.	21	0.238
82	A	5	5	1.	21	0.238
83	A	4	4	1.17	21	0.19
84	A	3	3	1.	19	0.158
85	A	1	1	1.	11	0.091
86	A	3	3	1.	21	0.143
87	A	5	5	1.	21	0.238
88	A	6	5	1.	21	0.238
89	A	6	6	1.	21	0.286
90	A	5	5	1.	21	0.238
91	A	4	4	1.	21	0.19
92	A	2	2	1.	19	0.105
93	A	2	2	1.	11	0.182
94	A	5	5	1.	21	0.238
95	A	6	5	1.	21	0.238
96	A	7	5	1.	21	0.238
97	A	5	3	1.	21	0.143
98	A	4	3	1.	21	0.143
99	A	3	3	1.	19	0.158
100	A	2	2	1.	11	0.182

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
101	A	3	3	1.	21	0.143
102	A	3	3	1.	21	0.143
103	A	4	4	1.	21	0.19
104	A	5	4	1.	21	0.19
105	A	2	2	1.	26	0.077
106	A	2	2	1.	19	0.105
107	A	2	2	1.	21	0.095
108	A	2	2	1.	15	0.133
109	A	8	8	1.	24	0.333
110	A	7	7	1.	24	0.292
111	A	6	6	1.	22	0.273
112	A	6	6	1.	24	0.25
113	A	6	6	1.	24	0.25
114	A	8	8	1.	24	0.333
115	A	9	8	1.	24	0.333
116	A	9	8	1.	24	0.333
117	A	8	7	1.	24	0.292
118	A	7	6	1.	22	0.273
119	A	7	7	1.	24	0.292
120	A	7	7	1.	24	0.292
121	A	9	9	1.	24	0.375
122	A	8	7	1.	24	0.292
123	A	7	7	1.	24	0.292
124	A	6	6	1.	24	0.25
125	A	5	5	1.	22	0.227
126	A	1	1	1.	24	0.042
127	A	7	7	1.	24	0.292
128	A	8	8	1.	24	0.333
129	A	7	7	1.	24	0.292
130	A	6	6	1.	24	0.25
131	A	5	5	1.	22	0.227
132	A	7	7	1.	24	0.292
133	A	8	8	1.	24	0.333
134	A	9	8	1.	24	0.333
135	A	8	8	1.	24	0.333
136	A	7	7	1.	24	0.292

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
137	A	2	2	1.	24	0.083
138	A	6	6	1.	22	0.273
139	A	8	8	1.	24	0.333
140	A	9	8	1.	24	0.333
141	A	1	1	1.	26	0.038
142	A	1	1	1.	24	0.042
143	A	1	1	1.	23	0.043
144	A	1	1	1.	24	0.042
145	A	1	1	1.	22	0.045
146	A	1	1	1.	19	0.053
147	A	1	1	1.	19	0.053
148	A	1	1	1.	24	0.042
149	A	1	1	1.	22	0.045
150	A	1	1	1.	27	0.037
151	A	1	1	1.	28	0.036
152	A	1	1	1.	29	0.034
153	A	1	1	1.	30	0.033
154	A	1	1	1.	26	0.038
155	A	1	1	1.	26	0.038
156	A	1	1	1.	23	0.043
157	A	1	1	1.	23	0.043
158	A	1	1	1.	23	0.043
159	A	1	1	1.	23	0.043
160	A	1	1	1.	17	0.059
161	A	1	1	1.	21	0.048
162	A	1	1	1.	21	0.048
163	A	6	6	1.	23	0.261
164	A	5	5	1.	23	0.217
165	A	4	4	1.	23	0.174
166	A	1	1	1.	23	0.043
167	A	4	4	1.	23	0.174
168	A	5	5	1.	23	0.217
169	A	7	6	1.	23	0.261
170	A	6	6	1.	23	0.261
171	A	5	5	1.	23	0.217
172	A	5	5	1.	23	0.217

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
173	A	4	4	1.	23	0.174
174	A	5	5	1.	23	0.217
175	A	5	5	1.	23	0.217
176	A	3	3	1.	23	0.13
177	A	2	1	1.	23	0.043
178	A	4	4	1.	23	0.174
179	A	5	5	1.	23	0.217
180	A	4	4	1.	21	0.19
181	A	1	1	1.	23	0.043
182	A	1	1	1.	23	0.043
183	A	1	1	1.	23	0.043
184	A	1	1	1.	21	0.048
185	A	1	1	1.	21	0.048
186	A	1	1	1.	21	0.048
187	A	1	1	1.	23	0.043
188	A	4	4	1.	21	0.19
189	A	3	3	1.	23	0.13
190	A	3	3	1.	23	0.13
191	A	3	3	1.	23	0.13
192	A	4	4	1.	21	0.19
193	A	4	4	1.	21	0.19
194	A	4	4	1.	23	0.174
195	A	2	2	1.	23	0.087
196	A	7	6	1.	23	0.261
197	A	6	6	1.	23	0.261
198	A	5	5	1.	23	0.217
199	A	4	4	1.	23	0.174
200	A	1	1	1.	23	0.043
201	A	6	6	1.	23	0.261
202	A	4	4	1.	23	0.174
203	A	5	5	1.	23	0.217
204	A	7	6	1.	23	0.261
205	A	6	6	1.	23	0.261
206	A	5	5	1.	23	0.217
207	A	1	1	1.	23	0.043
208	A	6	6	1.	23	0.261

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
209	A	4	4	1.	23	0.174
210	A	5	5	1.	23	0.217
211	A	1	1	1.	23	0.043
212	A	3	2	1.	24	0.083
213	A	3	2	1.	24	0.083
214	A	3	2	1.	25	0.08
215	A	1	1	1.	23	0.043
216	A	1	1	1.	23	0.043
217	A	1	1	1.	23	0.043
218	A	2	2	1.	23	0.087
219	A	1	1	1.	21	0.048
220	A	1	1	1.	23	0.043
221	A	2	2	1.	23	0.087
222	A	1	1	1.	23	0.043
223	A	1	1	1.	23	0.043
224	A	1	1	1.	23	0.043
225	A	1	1	1.	21	0.048
226	A	1	1	1.	21	0.048
227	A	1	1	1.	21	0.048
228	A	2	2	1.	21	0.095
229	A	1	1	1.	19	0.053
230	A	1	1	1.	21	0.048
231	A	2	2	1.	21	0.095
232	A	1	1	1.	21	0.048
233	A	1	1	1.	21	0.048
234	A	1	1	1.	21	0.048
235	A	2	2	1.	21	0.095
236	A	2	2	1.	21	0.095
237	A	2	2	1.	21	0.095
238	A	2	2	1.	21	0.095
239	A	2	2	1.	19	0.105
240	A	1	1	1.	21	0.048
241	A	2	2	1.	21	0.095
242	A	2	2	1.	21	0.095
243	A	2	2	1.	21	0.095
244	A	2	2	1.	21	0.095

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	1	1	1.	23	0.043
246	A	1	1	1.	23	0.043
247	A	1	1	1.	23	0.043
248	A	2	2	1.	23	0.087
249	A	1	1	1.	21	0.048
250	A	2	2	1.	23	0.087
251	A	2	2	1.55	23	0.087
252	A	2	2	1.	23	0.087
253	A	2	2	1.	23	0.087
254	A	2	2	1.	23	0.087
255	A	3	3	1.	24	0.125
256	A	3	3	1.	27	0.111
257	A	3	3	1.	25	0.12
258	A	3	3	1.	28	0.107
259	A	3	3	1.	25	0.12
260	A	3	3	1.	26	0.115
261	A	3	3	1.	26	0.115
262	A	3	3	1.	27	0.111
263	A	4	4	1.	23	0.174
264	A	4	4	1.	26	0.154
265	A	4	4	1.	26	0.154
266	A	4	4	1.	29	0.138
267	A	7	6	1.	24	0.25
268	A	7	6	1.	25	0.24
269	A	7	6	1.	27	0.222
270	A	7	6	1.	28	0.214
271	A	3	3	1.	24	0.125
272	A	3	3	1.	27	0.111
273	A	3	3	1.	25	0.12
274	A	3	3	1.	28	0.107
275	A	3	3	1.	25	0.12
276	A	3	3	1.	26	0.115
277	A	3	3	1.	26	0.115
278	A	3	3	1.	27	0.111
279	A	4	4	1.	23	0.174
280	A	4	4	1.	26	0.154

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
281	A	4	4	1.	26	0.154
282	A	4	4	1.	29	0.138
283	A	7	6	1.	24	0.25
284	A	7	6	1.	25	0.24
285	A	7	6	1.	27	0.222
286	A	7	6	1.	28	0.214
287	A	1	1	1.	23	0.043
288	A	2	2	1.	23	0.087
289	A	1	1	1.	21	0.048
290	A	1	1	1.	23	0.043
291	A	4	4	1.	28	0.143
292	A	4	4	1.	23	0.174
293	A	1	1	1.	23	0.043
294	A	1	1	1.	59	0.017
295	A	1	1	1.	59	0.017
296	A	4	4	1.	59	0.068
297	A	3	3	1.	59	0.051
298	C	1	1	0.37	21	0.048
299	A	2	2	1.	26	0.077
300	A	1	1	1.	41	0.024
301	A	1	1	1.	21	0.048
302	A	1	1	1.	21	0.048
303	A	1	1	1.	21	0.048
304	A	1	1	1.	23	0.043
305	A	1	1	1.	23	0.043
306	A	1	1	1.	23	0.043
307	A	1	1	1.	23	0.043
308	A	1	1	1.	25	0.04
309	A	1	1	1.	21	0.048
310	A	1	1	1.	21	0.048
311	A	1	1	1.	21	0.048
312	A	1	1	1.	23	0.043
313	A	1	1	1.	25	0.04
314	A	1	1	1.	25	0.04
315	A	1	1	1.	25	0.04
316	A	1	1	1.	27	0.037

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
317	A	1	1	1.	19	0.053
318	A	13	8	1.	21	0.381
319	A	12	8	1.	21	0.381
320	A	8	7	1.	21	0.333
321	A	8	7	1.	21	0.333
322	A	4	3	1.	21	0.143
323	A	5	4	1.	21	0.19
324	A	7	6	1.	21	0.286
325	A	9	8	1.	21	0.381
326	A	10	9	1.	21	0.429
327	A	10	9	1.	21	0.429
328	A	9	8	1.	21	0.381
329	A	9	8	1.	21	0.381
330	A	9	8	1.	21	0.381
331	A	9	8	1.	21	0.381
332	A	9	8	1.	21	0.381
333	A	9	8	1.	21	0.381
334	A	10	9	1.	21	0.429
335	A	10	9	1.	21	0.429
336	A	11	9	1.	21	0.429
337	A	11	9	1.	21	0.429
338	A	3	2	1.	19	0.105
339	A	5	5	1.	19	0.263
340	A	4	4	1.	19	0.21
341	A	3	3	0.91	17	0.176
342	A	2	2	1.	9	0.222
343	A	2	2	1.	19	0.105
344	A	2	2	1.	19	0.105
345	A	2	2	1.	19	0.105
346	A	1	1	1.	50	0.02

3 Listing of integrals

3.1 $\int \left(a+bx^2\right) \left(c+dx^2\right)^4 \, dx$

Optimal. Leaf size=94

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^
5)/5 + (2*c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/
9 + (b*d^4*x^11)/11
```

Rubi [A] time = 0.145135, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^
5)/5 + (2*c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/
9 + (b*d^4*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^4x^{11}}{11}+c^4\int a\,dx+\frac{c^3x^3\left(4ad+bc\right)}{3}+\frac{2c^2dx^5\left(3ad+2bc\right)}{5}+\frac{2cd^2x^7\left(2ad+3bc\right)}{7}+\frac{d^3x^9\left(ad+4bc\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] b*d**4*x**11/11 + c**4*Integral(a, x) + c**3*x**3*(4*a*d + b*c)/3
 + 2*c**2*d*x**5*(3*a*d + 2*b*c)/5 + 2*c*d**2*x**7*(2*a*d + 3*b*c
)/7 + d**3*x**9*(a*d + 4*b*c)/9
```

Mathematica [A] time = 0.034363, size = 94, normalized size = 1.

$$\frac{1}{3}c^3x^3(4ad+bc)+\frac{2}{5}c^2dx^5(3ad+2bc)+\frac{1}{9}d^3x^9(ad+4bc)+\frac{2}{7}cd^2x^7(2ad+3bc)+ac^4x+\frac{1}{11}bd^4x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^
5)/5 + (2*c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/
9 + (b*d^4*x^11)/11
```

Maple [A] time = 0.002, size = 97, normalized size = 1.

$$\frac{bd^4x^{11}}{11}+\frac{\left(ad^4+4\,bcd^3\right)x^9}{9}+\frac{\left(4\,acd^3+6\,bc^2d^2\right)x^7}{7}+\frac{\left(6\,ac^2d^2+4\,bc^3d\right)x^5}{5}+\frac{\left(4\,ac^3d+bc^4\right)x^3}{3}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^4,x)
```

```
[Out] 1/11*b*d^4*x^11+1/9*(a*d^4+4*b*c*d^3)*x^9+1/7*(4*a*c*d^3+6*b*c^2*
d^2)*x^7+1/5*(6*a*c^2*d^2+4*b*c^3*d)*x^5+1/3*(4*a*c^3*d+b*c^4)*x^
3+a*c^4*x
```

Maxima [A] time = 1.34763, size = 130, normalized size = 1.38

$$\frac{1}{11}\,bd^4x^{11}+\frac{1}{9}\left(4\,bcd^3+ad^4\right)x^9+\frac{2}{7}\left(3\,bc^2d^2+2\,acd^3\right)x^7+ac^4x+\frac{2}{5}\left(2\,bc^3d+3\,ac^2d^2\right)x^5+\frac{1}{3}\left(bc^4+4\,ac^3d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^4,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2
+ 2*a*c*d^3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 +
1/3*(b*c^4 + 4*a*c^3*d)*x^3
```

Fricas [A] time = 0.178747, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}d^4b+\frac{4}{9}x^9d^3cb+\frac{1}{9}x^9d^4a+\frac{6}{7}x^7d^2c^2b+\frac{4}{7}x^7d^3ca+\frac{4}{5}x^5dc^3b+\frac{6}{5}x^5d^2c^2a+\frac{1}{3}x^3c^4b+\frac{4}{3}x^3dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^4,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^4*b + 4/9*x^9*d^3*c*b + 1/9*x^9*d^4*a + 6/7*x^7*d^2*c
^2*b + 4/7*x^7*d^3*c*a + 4/5*x^5*d*c^3*b + 6/5*x^5*d^2*c^2*a + 1/
3*x^3*c^4*b + 4/3*x^3*d*c^3*a + x*c^4*a
```

Sympy [A] time = 0.147961, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{11}}{11} + x^9\left(\frac{ad^4}{9} + \frac{4bcd^3}{9}\right) + x^7\left(\frac{4acd^3}{7} + \frac{6bc^2d^2}{7}\right) + x^5\left(\frac{6ac^2d^2}{5} + \frac{4bc^3d}{5}\right) + x^3\left(\frac{4ac^3d}{3} + \frac{bc^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**
7*(4*a*c*d**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*
c**3*d/5) + x**3*(4*a*c**3*d/3 + b*c**4/3)
```

GIAC/XCAS [A] time = 0.226178, size = 132, normalized size = 1.4

$$\frac{1}{11}bd^4x^{11} + \frac{4}{9}bcd^3x^9 + \frac{1}{9}ad^4x^9 + \frac{6}{7}bc^2d^2x^7 + \frac{4}{7}acd^3x^7 + \frac{4}{5}bc^3dx^5 + \frac{6}{5}ac^2d^2x^5 + \frac{1}{3}bc^4x^3 + \frac{4}{3}ac^3dx^3 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^4,x, algorithm="giac")
```

```
[Out] 1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2
*x^7 + 4/7*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/
3*b*c^4*x^3 + 4/3*a*c^3*d*x^3 + a*c^4*x
```

3.2 $\int \left(a+b x^2\right) \left(c+d x^2\right)^3 \, dx$

Optimal. Leaf size=70

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 +
 (d^2*(3*b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rubi [A] time = 0.101858, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 +
 (d^2*(3*b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^3x^9}{9}+c^3\int a\,dx+\frac{c^2x^3\left(3ad+bc\right)}{3}+\frac{3cdx^5\left(ad+bc\right)}{5}+\frac{d^2x^7\left(ad+3bc\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] b*d**3*x**9/9 + c**3*Integral(a, x) + c**2*x**3*(3*a*d + b*c)/3 +
 3*c*d*x**5*(a*d + b*c)/5 + d**2*x**7*(a*d + 3*b*c)/7
```

Mathematica [A] time = 0.023338, size = 70, normalized size = 1.

$$\frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 +
 (d^2*(3*b*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^9}{9}+\frac{\left(ad^3+3\,bcd^2\right)x^7}{7}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^5}{5}+\frac{\left(3\,ac^2d+bc^3\right)x^3}{3}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3,x)
```

```
[Out] 1/9*b*d^3*x^9+1/7*(a*d^3+3*b*c*d^2)*x^7+1/5*(3*a*c*d^2+3*b*c^2*d)
*x^5+1/3*(3*a*c^2*d+b*c^3)*x^3+a*c^3*x
```

Maxima [A] time = 1.33996, size = 95, normalized size = 1.36

$$\frac{1}{9}\,bd^3x^9+\frac{1}{7}\left(3\,bcd^2+ad^3\right)x^7+\frac{3}{5}\left(bc^2d+acd^2\right)x^5+ac^3x+\frac{1}{3}\left(bc^3+3\,ac^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*
d^2)*x^5 + a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Fricas [A] time = 0.178077, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9d^3b+\frac{3}{7}x^7d^2cb+\frac{1}{7}x^7d^3a+\frac{3}{5}x^5dc^2b+\frac{3}{5}x^5d^2ca+\frac{1}{3}x^3c^3b+x^3dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b + 3/7*x^7*d^2*c*b + 1/7*x^7*d^3*a + 3/5*x^5*d*c^2*b
 + 3/5*x^5*d^2*c*a + 1/3*x^3*c^3*b + x^3*d*c^2*a + x*c^3*a
```

Sympy [A] time = 0.129599, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^9}{9} + x^7\left(\frac{ad^3}{7} + \frac{3bcd^2}{7}\right) + x^5\left(\frac{3acd^2}{5} + \frac{3bc^2d}{5}\right) + x^3\left(ac^2d + \frac{bc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*
(3*a*c*d**2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

GIAC/XCAS [A] time = 0.235981, size = 99, normalized size = 1.41

$$\frac{1}{9}\,bd^3x^9 + \frac{3}{7}\,bcd^2x^7 + \frac{1}{7}\,ad^3x^7 + \frac{3}{5}\,bc^2dx^5 + \frac{3}{5}\,acd^2x^5 + \frac{1}{3}\,bc^3x^3 + ac^2dx^3 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5
 + 3/5*a*c*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

3.3 $\int (a+bx^2)(c+dx^2)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*
d^2*x^7)/7
```

Rubi [A] time = 0.0715088, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*
d^2*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^2x^7}{7}+c^2\int a\,dx+\frac{cx^3\,(2ad+bc)}{3}+\frac{dx^5\,(ad+2bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] b*d**2*x**7/7 + c**2*Integral(a, x) + c*x**3*(2*a*d + b*c)/3 + d*
x**5*(a*d + 2*b*c)/5
```

Mathematica [A] time = 0.017418, size = 50, normalized size = 1.

$$\frac{1}{5}dx^5(ad+2bc)+\frac{1}{3}cx^3(2ad+bc)+ac^2x+\frac{1}{7}bd^2x^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*
d^2*x^7)/7
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^7}{7} + \frac{\left(ad^2 + 2\,bcd\right)\,x^5}{5} + \frac{\left(2\,acd + c^2b\right)\,x^3}{3} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2,x)
```

```
[Out] 1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c
^2*x
```

Maxima [A] time = 1.39718, size = 65, normalized size = 1.3

$$\frac{1}{7}\,bd^2x^7 + \frac{1}{5}\left(2\,bcd + ad^2\right)x^5 + ac^2x + \frac{1}{3}\left(bc^2 + 2\,acd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2
+ 2*a*c*d)*x^3
```

Fricas [A] time = 0.177117, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7d^2b + \frac{2}{5}x^5dcb + \frac{1}{5}x^5d^2a + \frac{1}{3}x^3c^2b + \frac{2}{3}x^3dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b + 2/5*x^5*d*c*b + 1/5*x^5*d^2*a + 1/3*x^3*c^2*b + 2
/3*x^3*d*c*a + x*c^2*a
```

Sympy [A] time = 0.110744, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^7}{7} + x^5\left(\frac{ad^2}{5} + \frac{2bcd}{5}\right) + x^3\left(\frac{2acd}{3} + \frac{bc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*
a*c*d/3 + b*c**2/3)
```

GIAC/XCAS [A] time = 0.227377, size = 68, normalized size = 1.36

$$\frac{1}{7}\,bd^2x^7 + \frac{2}{5}\,bcdx^5 + \frac{1}{5}\,ad^2x^5 + \frac{1}{3}\,bc^2x^3 + \frac{2}{3}\,acdx^3 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2
/3*a*c*d*x^3 + a*c^2*x
```

3.4 $\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rubi [A] time = 0.0369075, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdx^5}{5}+c\int a\,dx+x^3\left(\frac{ad}{3}+\frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c),x)
```

```
[Out] b*d*x**5/5 + c*Integral(a, x) + x**3*(a*d/3 + b*c/3)
```

Mathematica [A] time = 0.00877041, size = 28, normalized size = 1.

$$\frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Maple [A] time = 0.002, size = 25, normalized size = 0.9

$$acx + \frac{(ad + bc)\,x^3}{3} + \frac{bdx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c),x)
```

```
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Maxima [A] time = 1.34531, size = 32, normalized size = 1.14

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,(bc + ad)x^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Fricas [A] time = 0.176008, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5db + \frac{1}{3}x^3cb + \frac{1}{3}x^3da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b + 1/3*x^3*c*b + 1/3*x^3*d*a + x*c*a
```

Sympy [A] time = 0.084437, size = 26, normalized size = 0.93

$$acx + \frac{bdx^5}{5} + x^3\left(\frac{ad}{3} + \frac{bc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c),x)
```

```
[Out] a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

GIAC/XCAS [A] time = 0.23503, size = 35, normalized size = 1.25

$$\frac{1}{5}\,bdx^5 + \frac{1}{3}\,bcx^3 + \frac{1}{3}\,adx^3 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

3.5 $\int \frac{a+bx^2}{c+dx^2} dx$

Optimal. Leaf size=40

$$\frac{bx}{d} - \frac{(bc - ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3
/2))
```

Rubi [A] time = 0.0524135, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{bx}{d} - \frac{(bc - ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3
/2))
```

Rubi in Sympy [A] time = 8.76221, size = 34, normalized size = 0.85

$$\frac{bx}{d} + \frac{(ad - bc)\,\text{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*d**(3/2))
```

Mathematica [A] time = 0.038996, size = 40, normalized size = 1.

$$\frac{bx}{d} - \frac{(bc - ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3
/2))
```

Maple [A] time = 0.008, size = 45, normalized size = 1.1

$$\frac{bx}{d} + a\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}} - \frac{bc}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c),x)
```

```
[Out] b*x/d+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-1/d/(c*d)^(1/2)*arc
tan(x*d/(c*d)^(1/2))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.206565, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{-cd}bx - (bc - ad)\log\left(\frac{2\,cdx + (dx^2 - c)\sqrt{-cd}}{dx^2 + c}\right)}{2\sqrt{-cd}d}, \frac{\sqrt{cd}bx - (bc - ad)\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{\sqrt{cd}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-c*d)*b*x - (b*c - a*d)*log((2*c*d*x + (d*x^2 - c)*s
qrt(-c*d))/(d*x^2 + c)))/(sqrt(-c*d)*d), (sqrt(c*d)*b*x - (b*c -
a*d)*arctan(sqrt(c*d)*x/c))/(sqrt(c*d)*d)]
```

Sympy [A] time = 1.56986, size = 82, normalized size = 2.05

$$\frac{bx}{d} - \frac{\sqrt{-\frac{1}{cd^3}}\,(ad - bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^3}}\,(ad - bc)\log\left(cd\sqrt{-\frac{1}{cd^3}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3))
+ x)/2 + sqrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3))
+ x)/2
```

GIAC/XCAS [A] time = 0.230823, size = 46, normalized size = 1.15

$$\frac{bx}{d} - \frac{(bc - ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

3.6 $\int \frac{a+bx^2}{(c+dx^2)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0619877, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(3/2))
```

Rubi in Sympy [A] time = 9.60656, size = 51, normalized size = 0.81

$$\frac{x\,(ad-bc)}{2cd\,(c+dx^2)}+\frac{(ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c*d*(c + d*x**2)) + (a*d + b*c)*atan(sqrt(d)*x/s
qrt(c))/(2*c**(3/2)*d**(3/2))
```

Mathematica [A] time = 0.0748312, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(3/2))
```

Maple [A] time = 0.01, size = 68, normalized size = 1.1

$$\frac{(ad-bc)\,x}{2\,cd\,(dx^2+c)}+\frac{a}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^
(1/2))*a+1/2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212666, size = 1, normalized size = 0.02

$$\left[-\frac{2\,(bc-ad)\sqrt{-cd}x-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)}{4\,(cd^2x^2+c^2d)\sqrt{-cd}},\right.$$
$$\left.-\frac{(bc-ad)\sqrt{cd}x-\left(bc^2+acd+\left(bcd+ad^2\right)x^2\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{2\,(cd^2x^2+c^2d)\sqrt{cd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(b*c - a*d)*sqrt(-c*d)*x - (b*c^2 + a*c*d + (b*c*d + a*d
^2)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)))/((c
*d^2*x^2 + c^2*d)*sqrt(-c*d)), -1/2*((b*c - a*d)*sqrt(c*d)*x - (b
*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*arctan(sqrt(c*d)*x/c))/((c*d^
2*x^2 + c^2*d)*sqrt(c*d))]
```

Sympy [A] time = 2.0712, size = 112, normalized size = 1.78

$$\frac{x\left(ad-bc\right)}{2c^{2}d+2cd^{2}x^{2}}-\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\left(ad+bc\right)\log\left(-c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^{3}d^{3}}}\left(ad+bc\right)\log\left(c^{2}d\sqrt{-\frac{1}{c^{3}d^{3}}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(
a*d + b*c)*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**
3*d**3))*(a*d + b*c)*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

GIAC/XCAS [A] time = 0.23582, size = 77, normalized size = 1.22

$$\frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}cd}-\frac{bcx-adx}{2\,(dx^{2}+c)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*
x - a*d*x)/((d*x^2 + c)*c*d)
```

3.7 $\int \frac{a+bx^2}{(c+dx^2)^3} dx$

Optimal. Leaf size=92

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d(c+dx^2)}-\frac{x(bc-ad)}{4cd(c+dx^2)^2}$$

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2
*d*(c + d*x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*
c^(5/2)*d^(3/2))
```

Rubi [A] time = 0.086278, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x(3ad+bc)}{8c^2d(c+dx^2)}-\frac{x(bc-ad)}{4cd(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2
*d*(c + d*x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*
c^(5/2)*d^(3/2))
```

Rubi in Sympy [A] time = 12.9023, size = 78, normalized size = 0.85

$$\frac{x(ad-bc)}{4cd(c+dx^2)^2}+\frac{x(3ad+bc)}{8c^2d(c+dx^2)}+\frac{(3ad+bc)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] x*(a*d - b*c)/(4*c*d*(c + d*x**2)**2) + x*(3*a*d + b*c)/(8*c**2*d
*(c + d*x**2)) + (3*a*d + b*c)*atan(sqrt(d)*x/sqrt(c))/(8*c**(5/2
)*d**(3/2))
```

Mathematica [A] time = 0.101606, size = 82, normalized size = 0.89

$$\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}+\frac{x\left(ad\left(5c+3dx^2\right)+bc\left(dx^2-c\right)\right)}{8c^2d\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)
^2) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3
/2))
```

Maple [A] time = 0.011, size = 90, normalized size = 1.

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{(3\,ad+bc)\,x^3}{8\,c^2}+\frac{(5\,ad-bc)\,x}{8\,cd}\right)+\frac{3\,a}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b}{8\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+3/8/c
^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a+1/8/c/d/(c*d)^(1/2)*arct
an(x*d/(c*d)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211059, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(bcd^2+3\,ad^3\right)x^4+bc^3+3\,ac^2d+2\left(bc^2d+3\,acd^2\right)x^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)+2\left(\left(bcd+3\,ad^2\right)x^3-\left(bc^2-5\,acd\right)x\right)\sqrt{}}{16\left(c^2d^3x^4+2\,c^3d^2x^2+c^4d\right)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d
+ 3*a*c*d^2)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 +
 c)) + 2*((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*a*c*d)*x)*sqrt(-c*d)
)/((c^2*d^3*x^4 + 2*c^3*d^2*x^2 + c^4*d)*sqrt(-c*d)), 1/8*(((b*c*
d^2 + 3*a*d^3)*x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*
x^2)*arctan(sqrt(c*d)*x/c) + ((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*
a*c*d)*x)*sqrt(c*d))/((c^2*d^3*x^4 + 2*c^3*d^2*x^2 + c^4*d)*sqrt(
c*d))]
```

Sympy [A] time = 2.7462, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad+bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^3}}\,(3ad+bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16}+\frac{x^3\left(3ad^2+bcd\right)+x\left(5acd-bc^2\right)}{8c^4d+16c^3d^2x^2+8c^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**
3)) + x)/16 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(
-1/(c**5*d**3)) + x)/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d -
 b*c**2))/(8*c**4*d + 16*c**3*d**2*x**2 + 8*c**2*d**3*x**4)
```

GIAC/XCAS [A] time = 0.232243, size = 105, normalized size = 1.14

$$\frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d}+\frac{bcdx^3+3\,ad^2x^3-bc^2x+5\,acdx}{8\,(dx^2+c)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(
b*c*d*x^3 + 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2
*d)
```

3.8 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi [A] time = 0.169096, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{ac^2x^3\left(3ad+2bc\right)}{3}+\frac{b^2d^3x^{11}}{11}+\frac{bd^2x^9\left(2ad+3bc\right)}{9}+c^3\int a^2\,dx+\frac{cx^5\left(3a^2d^2+6abcd+b^2c^2\right)}{5}+\frac{dx^7\left(a^2d^2+6abcd+3b^2c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a*c**2*x**3*(3*a*d + 2*b*c)/3 + b**2*d**3*x**11/11 + b*d**2*x**9*
(2*a*d + 3*b*c)/9 + c**3*Integral(a**2, x) + c*x**5*(3*a**2*d**2
+ 6*a*b*c*d + b**2*c**2)/5 + d*x**7*(a**2*d**2 + 6*a*b*c*d + 3*b*
*2*c**2)/7
```

Mathematica [A] time = 0.0390952, size = 122, normalized size = 1.

$$\frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c
*d + 3*a^2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7
)/7 + (b*d^2*(3*b*c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A] time = 0.002, size = 125, normalized size = 1.

$$\frac{b^2d^3x^{11}}{11}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^9}{9}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^7}{7}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^5}{5}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x^3}{3}+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^3,x)
```

```
[Out] 1/11*b^2*d^3*x^11+1/9*(2*a*b*d^3+3*b^2*c*d^2)*x^9+1/7*(a^2*d^3+6*
a*b*c*d^2+3*b^2*c^2*d)*x^7+1/5*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*
x^5+1/3*(3*a^2*c^2*d+2*a*b*c^3)*x^3+a^2*c^3*x
```

Maxima [A] time = 1.34891, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 +
```

```
 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x
^3
```

Fricas [A] time = 0.180426, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}d^3b^2 + \frac{1}{3}x^9d^2cb^2 + \frac{2}{9}x^9d^3ba + \frac{3}{7}x^7dc^2b^2 + \frac{6}{7}x^7d^2cba + \frac{1}{7}x^7d^3a^2 + \frac{1}{5}x^5c^3b^2 + \frac{6}{5}x^5dc^2ba + \frac{3}{5}x^5d^2ca^2 + \frac{2}{3}x^3c^3ba + x^3dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^3*b^2 + 1/3*x^9*d^2*c*b^2 + 2/9*x^9*d^3*b*a + 3/7*x^7
*d*c^2*b^2 + 6/7*x^7*d^2*c*b*a + 1/7*x^7*d^3*a^2 + 1/5*x^5*c^3*b^
2 + 6/5*x^5*d*c^2*b*a + 3/5*x^5*d^2*c*a^2 + 2/3*x^3*c^3*b*a + x^3
*d*c^2*a^2 + x*c^3*a^2
```

Sympy [A] time = 0.166929, size = 136, normalized size = 1.11

$$a^2c^3x + \frac{b^2d^3x^{11}}{11} + x^9\left(\frac{2abd^3}{9} + \frac{b^2cd^2}{3}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + x^5\left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^3\left(a^2c^2d + \frac{2abc^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d*
*2/3) + x**7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x
**5*(3*a**2*c*d**2/5 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2
*c**2*d + 2*a*b*c**3/3)
```

GIAC/XCAS [A] time = 0.233217, size = 177, normalized size = 1.45

$$\frac{1}{11}\,b^2d^3x^{11} + \frac{1}{3}\,b^2cd^2x^9 + \frac{2}{9}\,abd^3x^9 + \frac{3}{7}\,b^2c^2dx^7 + \frac{6}{7}\,abcd^2x^7 + \frac{1}{7}\,a^2d^3x^7 + \frac{1}{5}\,b^2c^3x^5 + \frac{6}{5}\,abc^2dx^5 + \frac{3}{5}\,a^2cd^2x^5 + \frac{2}{3}\,abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2
*c^2*d*x^7 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^
5 + 6/5*a*b*c^2*d*x^5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2
*c^2*d*x^3 + a^2*c^3*x
```

3.9 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi [A] time = 0.116459, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2acx^3\,(ad+bc)}{3}+\frac{b^2d^2x^9}{9}+\frac{2bdx^7\,(ad+bc)}{7}+c^2\int a^2\,dx+x^5\left(\frac{a^2d^2}{5}+\frac{4abcd}{5}+\frac{b^2c^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] 2*a*c*x**3*(a*d + b*c)/3 + b**2*d**2*x**9/9 + 2*b*d*x**7*(a*d + b
*c)/7 + c**2*Integral(a**2, x) + x**5*(a**2*d**2/5 + 4*a*b*c*d/5
+ b**2*c**2/5)
```

Mathematica [A] time = 0.0297056, size = 82, normalized size = 1.

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a
^2*d^2)*x^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A] time = 0.002, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^9}{9} + \frac{\left(2\,abd^2 + 2\,b^2cd\right)\,x^7}{7} + \frac{\left(a^2d^2 + 4\,abcd + b^2c^2\right)\,x^5}{5} + \frac{\left(2\,a^2cd + 2\,abc^2\right)\,x^3}{3} + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c)^2,x)
```

```
[Out] 1/9*b^2*d^2*x^9+1/7*(2*a*b*d^2+2*b^2*c*d)*x^7+1/5*(a^2*d^2+4*a*b*
c*d+b^2*c^2)*x^5+1/3*(2*a^2*c*d+2*a*b*c^2)*x^3+a^2*c^2*x
```

Maxima [A] time = 1.35161, size = 111, normalized size = 1.35

$$\frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Fricas [A] time = 0.180148, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9d^2b^2 + \frac{2}{7}x^7dcb^2 + \frac{2}{7}x^7d^2ba + \frac{1}{5}x^5c^2b^2 + \frac{4}{5}x^5dcba + \frac{1}{5}x^5d^2a^2 + \frac{2}{3}x^3c^2ba + \frac{2}{3}x^3dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^2*b^2 + 2/7*x^7*d*c*b^2 + 2/7*x^7*d^2*b*a + 1/5*x^5*c^2
*b^2 + 4/5*x^5*d*c*b*a + 1/5*x^5*d^2*a^2 + 2/3*x^3*c^2*b*a + 2/3*
x^3*d*c*a^2 + x*c^2*a^2
```

Sympy [A] time = 0.13947, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^9}{9} + x^7\left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3\left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/
7) + x**5*(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**
2*c*d/3 + 2*a*b*c**2/3)
```

GIAC/XCAS [A] time = 0.236269, size = 123, normalized size = 1.5

$$\frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\,b^2cdx^7 + \frac{2}{7}\,abd^2x^7 + \frac{1}{5}\,b^2c^2x^5 + \frac{4}{5}\,abcdx^5 + \frac{1}{5}\,a^2d^2x^5 + \frac{2}{3}\,abc^2x^3 + \frac{2}{3}\,a^2cdx^3 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2
*x^5 + 4/5*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*
a^2*c*d*x^3 + a^2*c^2*x
```

3.10 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Rubi [A] time = 0.0704014, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\int c\,dx+\frac{ax^3\,(ad+2bc)}{3}+\frac{b^2dx^7}{7}+\frac{bx^5\,(2ad+bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*Integral(c, x) + a*x**3*(a*d + 2*b*c)/3 + b**2*d*x**7/7 + b*
x**5*(2*a*d + b*c)/5
```

Mathematica [A] time = 0.0138175, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^
2*d*x^7)/7
```

Maple [A] time = 0.002, size = 49, normalized size = 1.

$$\frac{b^2dx^7}{7} + \frac{\left(2\,abd + b^2c\right)x^5}{5} + \frac{\left(a^2d + 2\,abc\right)x^3}{3} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2*(d*x^2+c),x)
```

```
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2
*c*x
```

Maxima [A] time = 1.34434, size = 65, normalized size = 1.3

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\left(b^2c + 2\,abd\right)x^5 + a^2cx + \frac{1}{3}\left(2\,abc + a^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*
c + a^2*d)*x^3
```

Fricas [A] time = 0.179046, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7db^2 + \frac{1}{5}x^5cb^2 + \frac{2}{5}x^5dba + \frac{2}{3}x^3cba + \frac{1}{3}x^3da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b^2 + 1/5*x^5*c*b^2 + 2/5*x^5*d*b*a + 2/3*x^3*c*b*a + 1
/3*x^3*d*a^2 + x*c*a^2
```

Sympy [A] time = 0.111504, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^7}{7} + x^5\left(\frac{2abd}{5} + \frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3} + \frac{2abc}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2*(d*x**2+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a*
*2*d/3 + 2*a*b*c/3)
```

GIAC/XCAS [A] time = 0.225958, size = 68, normalized size = 1.36

$$\frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abdx^5 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2dx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1
/3*a^2*d*x^3 + a^2*c*x
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2} dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^3}{3d}$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi [A] time = 0.0972643, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} - \frac{bx(bc-2ad)}{d^2} + \frac{b^2x^3}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x^3}{3d} + \frac{(2ad-bc)\int b\,dx}{d^2} + \frac{(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + (2*a*d - b*c)*Integral(b, x)/d**2 + (a*d - b*c)
**2*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*d**(5/2))
```

Mathematica [A] time = 0.0818075, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}} + \frac{bx\left(6ad-3bc+bdx^2\right)}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Maple [A] time = 0.005, size = 95, normalized size = 1.5

$$\frac{b^2x^3}{3\,d}+2\,\frac{abx}{d}-\frac{b^2xc}{d^2}+a^2\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-2\,\frac{abc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+\frac{b^2c^2}{d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] 1/3*b^2*x^3/d+2*b/d*a*x-b^2/d^2*x*c+1/(c*d)^(1/2)*arctan(x*d/(c*d
)^(1/2))*a^2-2/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b*c+1/d^2/
(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210304, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)+2\left(b^2dx^3-3\left(b^2c-2\,abd\right)x\right)\sqrt{-cd}}{6\sqrt{-cd}d^2},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\cdots\right.}{3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((2*c*d*x + (d*x^2 - c
)*sqrt(-c*d))/(d*x^2 + c)) + 2*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x
)*sqrt(-c*d))/(sqrt(-c*d)*d^2), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*arctan(sqrt(c*d)*x/c) + (b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)
*sqrt(c*d))/(sqrt(c*d)*d^2)]
```

Sympy [A] time = 2.10464, size = 172, normalized size = 2.73

$$\frac{b^2x^3}{3d} - \frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^5}}\,(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{x\left(2abd-b^2c\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) - sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(-c*d**2*sq
rt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2
) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c
*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
 + x*(2*a*b*d - b**2*c)/d**2
```

GIAC/XCAS [A] time = 0.249393, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2d^2x^3 - 3\,b^2cdx + 6\,abd^2x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*
d^2) + 1/3*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2} dx$

Optimal. Leaf size=82

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a
*d)*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2)
)
```

Rubi [A] time = 0.218373, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{(bc-ad)(ad+3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2(c+dx^2)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a
*d)*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2)
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int b^2\,dx}{d^2}+\frac{x\,(ad-bc)^2}{2cd^2\,(c+dx^2)}+\frac{(ad-bc)\,(ad+3bc)\,\text{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Integral(b**2, x)/d**2 + x*(a*d - b*c)**2/(2*c*d**2*(c + d*x**2))
 + (a*d - b*c)*(a*d + 3*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(3/2)*
d**(5/2))
```

Mathematica [A] time = 0.0986943, size = 89, normalized size = 1.09

$$-\frac{\left(-a^2d^2-2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c
^2 - 2*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)
*d^(5/2))
```

Maple [A] time = 0.012, size = 129, normalized size = 1.6

$$\frac{b^2x}{d^2}+\frac{a^2x}{2\,c\,(dx^2+c)}-\frac{abx}{d\,(dx^2+c)}+\frac{cxb^2}{2\,d^2\,(dx^2+c)}+\frac{a^2}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,b^2c}{2\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] b^2*x/d^2+1/2/c*x/(d*x^2+c)*a^2-1/d*x/(d*x^2+c)*a*b+1/2/d^2*c*x/(
d*x^2+c)*b^2+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2+1/d/(c
*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b-3/2/d^2*c/(c*d)^(1/2)*arcta
n(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.209986, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^{2}c^{3}-2\,abc^{2}d-a^{2}cd^{2}+\left(3\,b^{2}c^{2}d-2\,abcd^{2}-a^{2}d^{3}\right)x^{2}\right)\log\left(\frac{2\,cdx+\left(dx^{2}-c\right)\sqrt{-cd}}{dx^{2}+c}\right)-2\left(2\,b^{2}cdx^{3}+\left(3\,b^{2}c^{2}-2\,abcd+a^{2}\right.\right.}{4\left(cd^{3}x^{2}+c^{2}d^{2}\right)\sqrt{-cd}}\right.$$

$$\left.-\frac{\left(3\,b^{2}c^{3}-2\,abc^{2}d-a^{2}cd^{2}+\left(3\,b^{2}c^{2}d-2\,abcd^{2}-a^{2}d^{3}\right)x^{2}\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)-\left(2\,b^{2}cdx^{3}+\left(3\,b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x\right)\sqrt{cd}}{2\left(cd^{3}x^{2}+c^{2}d^{2}\right)\sqrt{cd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*
b*c*d^2 - a^2*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d
*x^2 + c)) - 2*(2*b^2*c*d*x^3 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*x)*sqrt(-c*d))/((c*d^3*x^2 + c^2*d^2)*sqrt(-c*d)), -1/2*((3*b^2*
c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*
d^3)*x^2)*arctan(sqrt(c*d)*x/c) - (2*b^2*c*d*x^3 + (3*b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x)*sqrt(c*d))/((c*d^3*x^2 + c^2*d^2)*sqrt(c*d
))]
```

Sympy [A] time = 3.39544, size = 236, normalized size = 2.88

$$\frac{b^{2}x}{d^{2}}+\frac{x\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2c^{2}d^{2}+2cd^{3}x^{2}}-\frac{\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)\log\left(-\frac{c^{2}d^{2}\sqrt{-\frac{1}{c^{3}d^{5}}}(ad-bc)(ad+3bc)}{a^{2}d^{2}+2abcd-3b^{2}c^{2}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{c^{3}d^{5}}}\left(ad-bc\right)\left(ad+3bc\right)\log\left(\frac{c^{2}d^{2}\sqrt{-\frac{1}{c^{3}d^{5}}}(ad-bc)(ad+3bc)}{a^{2}d^{2}+2abcd-3b^{2}c^{2}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2
+ 2*c*d**3*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)
*log(-c**2*d**2*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a
**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)/4 + sqrt(-1/(c**3*d**5))
*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2*sqrt(-1/(c**3*d**5))*(a*
d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x)
/4
```

GIAC/XCAS [A] time = 0.234873, size = 128, normalized size = 1.56

$$\frac{b^2x}{d^2} - \frac{\left(3\,b^2c^2 - 2\,abcd - a^2d^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2 + c)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt
(c*d))/(sqrt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2
*x)/((d*x^2 + c)*c*d^2)
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2
- b^2/d^2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi [A] time = 0.181628, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8\left(c+dx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}-\frac{x\left(a+bx^2\right)(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2))/(4*c*d*(c + d*x^2)^2) + (3*(a^2/c^2
- b^2/d^2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rubi in Sympy [A] time = 23.0598, size = 105, normalized size = 0.91

$$\frac{x\left(\frac{3a^2}{8c^2}-\frac{3b^2}{8d^2}\right)}{c+dx^2}+\frac{x\left(a+bx^2\right)(ad-bc)}{4cd\left(c+dx^2\right)^2}+\frac{(ad(3ad+bc)+bc(ad+3bc))\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] x*(3*a**2/(8*c**2) - 3*b**2/(8*d**2))/(c + d*x**2) + x*(a + b*x**
2)*(a*d - b*c)/(4*c*d*(c + d*x**2)**2) + (a*d*(3*a*d + b*c) + b*c
*(a*d + 3*b*c))*atan(sqrt(d)*x/sqrt(c))/(8*c**(5/2)*d**(5/2))
```

Mathematica [A] time = 0.171888, size = 121, normalized size = 1.04

$$\frac{x\left(a^2d^2\left(5c+3dx^2\right)-2abcd\left(c-dx^2\right)-b^2c^2\left(3c+5dx^2\right)\right)}{8c^2d^2\left(c+dx^2\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] (x*(-2*a*b*c*d*(c - d*x^2) + a^2*d^2*(5*c + 3*d*x^2) - b^2*c^2*(3
*c + 5*d*x^2)))/(8*c^2*d^2*(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c
*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A] time = 0.012, size = 147, normalized size = 1.3

$$\frac{1}{\left(dx^2+c\right)^2}\left(\frac{\left(3\,a^2d^2+2\,abcd-5\,b^2c^2\right)x^3}{8\,c^2d}+\frac{\left(5\,a^2d^2-2\,abcd-3\,b^2c^2\right)x}{8\,d^2c}\right)+\frac{3\,a^2}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{ab}{4\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,b^2}{8\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a
*b*c*d-3*b^2*c^2)/d^2/c*x)/(d*x^2+c)^2+3/8/c^2/(c*d)^(1/2)*arctan
(x*d/(c*d)^(1/2))*a^2+1/4/c/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))
*a*b+3/8/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.245758, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,b^2c^4+2\,abc^3d+3\,a^2c^2d^2+\left(3\,b^2c^2d^2+2\,abcd^3+3\,a^2d^4\right)x^4+2\left(3\,b^2c^3d+2\,abc^2d^2+3\,a^2cd^3\right)x^2\right)\log\left(\frac{2\,cdx+(dx^2-c)\sqrt{-cd}}{dx^2+c}\right.}{16\left(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2\right)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2
+ 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 +
 3*a^2*c*d^3)*x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2
+ c)) - 2*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c
^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x)*sqrt(-c*d))/((c^2*d^4*x^4 + 2*
c^3*d^3*x^2 + c^4*d^2)*sqrt(-c*d)), 1/8*((3*b^2*c^4 + 2*a*b*c^3*d
 + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
+ 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*arctan(sqrt(
c*d)*x/c) - ((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2
*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x)*sqrt(c*d))/((c^2*d^4*x^4 + 2
*c^3*d^3*x^2 + c^4*d^2)*sqrt(c*d))]
```

Sympy [A] time = 4.6103, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{c^5d^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16}+\frac{x^3\left(3a^2d^3+2abcd^2-5b^2c^2d\right)+x\left(5a^2cd^2-2abc^2d-3b^2c^3\right)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log
(-c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(
3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**
5*d**5)) + x)/16 + (x**3*(3*a**2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**
2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d - 3*b**2*c**3))/(8*c**4*d*
*2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

GIAC/XCAS [A] time = 0.232259, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^{2}d^{2}}-\frac{5\,b^{2}c^{2}dx^{3}-2\,abcd^{2}x^{3}-3\,a^{2}d^{3}x^{3}+3\,b^{2}c^{3}x+2\,abc^{2}dx-5\,a^{2}cd^{2}x}{8\,\left(dx^{2}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c^2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2
*d^3*x^3 + 3*b^2*c^3*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 +
 c)^2*c^2*d^2)
```

3.14 $\int (a+bx^2)^3 (c+dx^2)^3 \, dx$

Optimal. Leaf size=154

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d
 + a^2*d^2)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)
*x^7)/7 + (b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^
2*(b*c + a*d)*x^11)/11 + (b^3*d^3*x^13)/13
```

Rubi [A] time = 0.229851, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d
 + a^2*d^2)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)
*x^7)/7 + (b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^
2*(b*c + a*d)*x^11)/11 + (b^3*d^3*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2x^3\left(ad+bc\right)+\frac{3acx^5\left(a^2d^2+3abcd+b^2c^2\right)}{5}+\frac{b^3d^3x^{13}}{13}+\frac{3b^2d^2x^{11}\left(ad+bc\right)}{11}+\frac{bdx^9\left(a^2d^2+3abcd+b^2c^2\right)}{3}+c^3\int a^3\,dx+\frac{x^7\left(ad+bc\right)\left(a^2d^2+8abcd+b^2c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**2*x**3*(a*d + b*c) + 3*a*c*x**5*(a**2*d**2 + 3*a*b*c*d +
b**2*c**2)/5 + b**3*d**3*x**13/13 + 3*b**2*d**2*x**11*(a*d + b*c)
```

```
/11 + b*d*x**9*(a**2*d**2 + 3*a*b*c*d + b**2*c**2)/3 + c**3*Integ
ral(a**3, x) + x**7*(a*d + b*c)*(a**2*d**2 + 8*a*b*c*d + b**2*c**
2)/7
```

Mathematica [A] time = 0.0510546, size = 161, normalized size = 1.05

$$a^3c^3x+\frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right)+a^2c^2x^3(ad+bc)+\frac{1}{7}x^7\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{3}{11}b^2d^2x^{11}(ad+bc)+\frac{1}{13}b^3d^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d
 + a^2*d^2)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 +
a^3*d^3)*x^7)/7 + (b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (
3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*d^3*x^13)/13
```

Maple [A] time = 0.002, size = 177, normalized size = 1.2

$$\frac{b^3d^3x^{13}}{13}+\frac{\left(3\,ab^2d^3+3\,b^3cd^2\right)x^{11}}{11}+\frac{\left(3\,a^2bd^3+9\,ab^2cd^2+3\,b^3c^2d\right)x^9}{9}+\frac{\left(a^3d^3+9\,a^2bcd^2+9\,ab^2c^2d+b^3c^3\right)x^7}{7}+\frac{\left(3\,a^3cd^2+9\,a^2bc^2d+3\,ab^2c^3\right)x^5}{5}+\frac{\left(3\,a^3c^2d+3\,a^2bc^3\right)x^3}{3}+a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(d*x^2+c)^3,x)
```

```
[Out] 1/13*b^3*d^3*x^13+1/11*(3*a*b^2*d^3+3*b^3*c*d^2)*x^11+1/9*(3*a^2*
b*d^3+9*a*b^2*c*d^2+3*b^3*c^2*d)*x^9+1/7*(a^3*d^3+9*a^2*b*c*d^2+9
*a*b^2*c^2*d+b^3*c^3)*x^7+1/5*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*
c^3)*x^5+1/3*(3*a^3*c^2*d+3*a^2*b*c^3)*x^3+a^3*c^3*x
```

Maxima [A] time = 1.35202, size = 225, normalized size = 1.46

$$\frac{1}{13}\,b^3d^3x^{13}+\frac{3}{11}\,\left(b^3cd^2+ab^2d^3\right)x^{11}+\frac{1}{3}\,\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9+\frac{1}{7}\,\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x+\frac{3}{5}\,\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^5+\left(a^2bc^3+a^3c^2d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c
^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3
+ 3*a^2*b*c^2*d + a^3*c*d^2)*x^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Fricas [A] time = 0.179988, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}d^3b^3+\frac{3}{11}x^{11}d^2cb^3+\frac{3}{11}x^{11}d^3b^2a+\frac{1}{3}x^9dc^2b^3+x^9d^2cb^2a+\frac{1}{3}x^9d^3ba^2+\frac{1}{7}x^7c^3b^3+\frac{9}{7}x^7dc^2b^2a+\frac{9}{7}x^7d^2cba^2+\frac{1}{7}x^7d^3a^3+\frac{3}{5}x^5c^3b^2a+\frac{9}{5}x^5dc^2ba^2+\frac{3}{5}x^5d^2ca^3+x^3c^3ba^2+x^3dc^2a^3+xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b^3 + 3/11*x^11*d^2*c*b^3 + 3/11*x^11*d^3*b^2*a + 1
/3*x^9*d*c^2*b^3 + x^9*d^2*c*b^2*a + 1/3*x^9*d^3*b*a^2 + 1/7*x^7*
c^3*b^3 + 9/7*x^7*d*c^2*b^2*a + 9/7*x^7*d^2*c*b*a^2 + 1/7*x^7*d^3
*a^3 + 3/5*x^5*c^3*b^2*a + 9/5*x^5*d*c^2*b*a^2 + 3/5*x^5*d^2*c*a^
3 + x^3*c^3*b*a^2 + x^3*d*c^2*a^3 + x*c^3*a^3
```

Sympy [A] time = 0.193799, size = 189, normalized size = 1.23

$$a^3c^3x+\frac{b^3d^3x^{13}}{13}+x^{11}\left(\frac{3ab^2d^3}{11}+\frac{3b^3cd^2}{11}\right)+x^9\left(\frac{a^2bd^3}{3}+ab^2cd^2+\frac{b^3c^2d}{3}\right)+x^7\left(\frac{a^3d^3}{7}+\frac{9a^2bcd^2}{7}+\frac{9ab^2c^2d}{7}+\frac{b^3c^3}{7}\right)+x^5\left(\frac{3a^3cd^2}{5}+\frac{9a^2bc^2d}{5}+\frac{3ab^2c^3}{5}\right)+x^3\left(a^3c^2d+a^2bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b*
*3*c*d**2/11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d
/3) + x**7*(a**3*d**3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 +
 b**3*c**3/7) + x**5*(3*a**3*c*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b
**2*c**3/5) + x**3*(a**3*c**2*d + a**2*b*c**3)
```

GIAC/XCAS [A] time = 0.226071, size = 252, normalized size = 1.64

$$\frac{1}{13}\,b^{3}d^{3}x^{13}+\frac{3}{11}\,b^{3}cd^{2}x^{11}+\frac{3}{11}\,ab^{2}d^{3}x^{11}+\frac{1}{3}\,b^{3}c^{2}dx^{9}+ab^{2}cd^{2}x^{9}+\frac{1}{3}\,a^{2}bd^{3}x^{9}+\frac{1}{7}\,b^{3}c^{3}x^{7}+\frac{9}{7}\,ab^{2}c^{2}dx^{7}+\frac{9}{7}\,a^{2}bcd^{2}x^{7}+\frac{1}{7}\,a^{3}d^{3}x^{7}+\frac{3}{5}\,ab^{2}c^{3}x^{5}+\frac{9}{5}\,a^{2}bc^{2}dx^{5}+\frac{3}{5}\,a^{3}cd^{2}x^{5}+a^{2}bc^{3}x^{3}+a^{3}c^{2}dx^{3}+a^{3}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1
/3*b^3*c^2*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*
c^3*x^7 + 9/7*a*b^2*c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3
*x^7 + 3/5*a*b^2*c^3*x^5 + 9/5*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^
5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 + a^3*c^3*x
```

3.15 $\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=122

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7
)/7 + (b^2*d*(2*b*c + 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rubi [A] time = 0.167675, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7
)/7 + (b^2*d*(2*b*c + 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2cx^3\left(2ad+3bc\right)}{3}+\frac{ax^5\left(a^2d^2+6abcd+3b^2c^2\right)}{5}+\frac{b^3d^2x^{11}}{11}+\frac{b^2dx^9\left(3ad+2bc\right)}{9}+\frac{bx^7\left(3a^2d^2+6abcd+b^2c^2\right)}{7}+c^2\int a^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**2*c*x**3*(2*a*d + 3*b*c)/3 + a*x**5*(a**2*d**2 + 6*a*b*c*d + 3
*b**2*c**2)/5 + b**3*d**2*x**11/11 + b**2*d*x**9*(3*a*d + 2*b*c)/
9 + b*x**7*(3*a**2*d**2 + 6*a*b*c*d + b**2*c**2)/7 + c**2*Integra
l(a**3, x)
```

Mathematica [A] time = 0.0391154, size = 122, normalized size = 1.

$$a^3c^2x+\frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}a^2cx^3(2ad+3bc)+\frac{1}{9}b^2dx^9(3ad+2bc)+\frac{1}{11}b^3d^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b
*c*d + a^2*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7
)/7 + (b^2*d*(2*b*c + 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Maple [A] time = 0.002, size = 125, normalized size = 1.

$$\frac{b^3d^2x^{11}}{11}+\frac{\left(3\,ab^2d^2+2\,b^3cd\right)x^9}{9}+\frac{\left(3\,a^2bd^2+6\,ab^2cd+b^3c^2\right)x^7}{7}+\frac{\left(a^3d^2+6\,a^2bcd+3\,ab^2c^2\right)x^5}{5}+\frac{\left(2\,a^3cd+3\,a^2bc^2\right)x^3}{3}+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(d*x^2+c)^2,x)
```

```
[Out] 1/11*b^3*d^2*x^11+1/9*(3*a*b^2*d^2+2*b^3*c*d)*x^9+1/7*(3*a^2*b*d^
2+6*a*b^2*c*d+b^3*c^2)*x^7+1/5*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*
x^5+1/3*(2*a^3*c*d+3*a^2*b*c^2)*x^3+a^3*c^2*x
```

Maxima [A] time = 1.36673, size = 167, normalized size = 1.37

$$\frac{1}{11}\,b^3d^2x^{11}+\frac{1}{9}\left(2\,b^3cd+3\,ab^2d^2\right)x^9+\frac{1}{7}\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^7+a^3c^2x+\frac{1}{5}\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^5+\frac{1}{3}\left(3\,a^2bc^2+2\,a^3cd\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c
```

```
^2 + 6*a^2*b*c*d + a^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x
^3
```

Fricas [A] time = 0.179498, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}d^2b^3 + \frac{2}{9}x^9dcb^3 + \frac{1}{3}x^9d^2b^2a + \frac{1}{7}x^7c^2b^3 + \frac{6}{7}x^7dcb^2a + \frac{3}{7}x^7d^2ba^2 + \frac{3}{5}x^5c^2b^2a + \frac{6}{5}x^5dcba^2 + \frac{1}{5}x^5d^2a^3 + x^3c^2ba^2 + \frac{2}{3}x^3dca^3 + xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^2*b^3 + 2/9*x^9*d*c*b^3 + 1/3*x^9*d^2*b^2*a + 1/7*x^7
*c^2*b^3 + 6/7*x^7*d*c*b^2*a + 3/7*x^7*d^2*b*a^2 + 3/5*x^5*c^2*b^
2*a + 6/5*x^5*d*c*b*a^2 + 1/5*x^5*d^2*a^3 + x^3*c^2*b*a^2 + 2/3*x
^3*d*c*a^3 + x*c^2*a^3
```

Sympy [A] time = 0.163172, size = 136, normalized size = 1.11

$$a^3c^2x + \frac{b^3d^2x^{11}}{11} + x^9\left(\frac{ab^2d^2}{3} + \frac{2b^3cd}{9}\right) + x^7\left(\frac{3a^2bd^2}{7} + \frac{6ab^2cd}{7} + \frac{b^3c^2}{7}\right) + x^5\left(\frac{a^3d^2}{5} + \frac{6a^2bcd}{5} + \frac{3ab^2c^2}{5}\right) + x^3\left(\frac{2a^3cd}{3} + a^2bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c
*d/9) + x**7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x
**5*(a**3*d**2/5 + 6*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a*
*3*c*d/3 + a**2*b*c**2)
```

GIAC/XCAS [A] time = 0.225019, size = 177, normalized size = 1.45

$$\frac{1}{11}b^3d^2x^{11} + \frac{2}{9}b^3cdx^9 + \frac{1}{3}ab^2d^2x^9 + \frac{1}{7}b^3c^2x^7 + \frac{6}{7}ab^2cdx^7 + \frac{3}{7}a^2bd^2x^7 + \frac{3}{5}ab^2c^2x^5 + \frac{6}{5}a^2bcdx^5 + \frac{1}{5}a^3d^2x^5 + a^2bc^2x^3 + \frac{2}{3}a^3cdx^3 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3
*c^2*x^7 + 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*
x^5 + 6/5*a^2*b*c*d*x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a
^3*c*d*x^3 + a^3*c^2*x
```

3.16 $\int (a+bx^2)^3 (c+dx^2)\, dx$

Optimal. Leaf size=70

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 +
 (b^2*(b*c + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rubi [A] time = 0.104099, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 +
 (b^2*(b*c + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\int c\,dx+\frac{a^2x^3\,(ad+3bc)}{3}+\frac{3abx^5\,(ad+bc)}{5}+\frac{b^3dx^9}{9}+\frac{b^2x^7\,(3ad+bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3*(d*x**2+c),x)
```

```
[Out] a**3*Integral(c, x) + a**2*x**3*(a*d + 3*b*c)/3 + 3*a*b*x**5*(a*d
 + b*c)/5 + b**3*d*x**9/9 + b**2*x**7*(3*a*d + b*c)/7
```

Mathematica [A] time = 0.0230202, size = 70, normalized size = 1.

$$a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 +
 (b^2*(b*c + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Maple [A] time = 0., size = 73, normalized size = 1.

$$\frac{b^3dx^9}{9}+\frac{\left(3\,ab^2d+b^3c\right)x^7}{7}+\frac{\left(3\,a^2bd+3\,ab^2c\right)x^5}{5}+\frac{\left(a^3d+3\,a^2bc\right)x^3}{3}+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3*(d*x^2+c),x)
```

```
[Out] 1/9*b^3*d*x^9+1/7*(3*a*b^2*d+b^3*c)*x^7+1/5*(3*a^2*b*d+3*a*b^2*c)
*x^5+1/3*(a^3*d+3*a^2*b*c)*x^3+a^3*c*x
```

Maxima [A] time = 1.35944, size = 95, normalized size = 1.36

$$\frac{1}{9}\,b^3dx^9+\frac{1}{7}\left(b^3c+3\,ab^2d\right)x^7+\frac{3}{5}\left(ab^2c+a^2bd\right)x^5+a^3cx+\frac{1}{3}\left(3\,a^2bc+a^3d\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*
b*d)*x^5 + a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Fricas [A] time = 0.177811, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9db^3+\frac{1}{7}x^7cb^3+\frac{3}{7}x^7db^2a+\frac{3}{5}x^5cb^2a+\frac{3}{5}x^5dba^2+x^3cba^2+\frac{1}{3}x^3da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b^3 + 1/7*x^7*c*b^3 + 3/7*x^7*d*b^2*a + 3/5*x^5*c*b^2*a
 + 3/5*x^5*d*b*a^2 + x^3*c*b*a^2 + 1/3*x^3*d*a^3 + x*c*a^3
```

<u>**Sympy**</u> **[A]** time = 0.134674, size = 76, normalized size = 1.09

$$a^3cx + \frac{b^3dx^9}{9} + x^7\left(\frac{3ab^2d}{7} + \frac{b^3c}{7}\right) + x^5\left(\frac{3a^2bd}{5} + \frac{3ab^2c}{5}\right) + x^3\left(\frac{a^3d}{3} + a^2bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3*(d*x**2+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*
(3*a**2*b*d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

<u>**GIAC/XCAS**</u> **[A]** time = 0.2356, size = 99, normalized size = 1.41

$$\frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bdx^5 + a^2bcx^3 + \frac{1}{3}\,a^3dx^3 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5
 + 3/5*a^2*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2}\,dx$

Optimal. Leaf size=98

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}+\frac{b^3x^5}{5d}$$

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*
x^3)/(3*d^2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(7/2))
```

Rubi [A] time = 0.146696, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{bx\left(3a^2d^2-3abcd+b^2c^2\right)}{d^3}-\frac{b^2x^3(bc-3ad)}{3d^2}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}+\frac{b^3x^5}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*
x^3)/(3*d^2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^3x^5}{5d}+\frac{b^2x^3\,(3ad-bc)}{3d^2}+\frac{\left(3a^2d^2-3abcd+b^2c^2\right)\int b\,dx}{d^3}+\frac{(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) + b**2*x**3*(3*a*d - b*c)/(3*d**2) + (3*a**2*d**2
 - 3*a*b*c*d + b**2*c**2)*Integral(b, x)/d**3 + (a*d - b*c)**3*at
an(sqrt(d)*x/sqrt(c))/(sqrt(c)*d**(7/2))
```

Mathematica [A] time = 0.0987528, size = 93, normalized size = 0.95

$$\frac{bx\left(45a^2d^2+15abd\left(dx^2-3c\right)+b^2\left(15c^2-5cdx^2+3d^2x^4\right)\right)}{15d^3}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*
x^2 + 3*d^2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(Sqrt[c]*d^(7/2))
```

Maple [A] time = 0.004, size = 161, normalized size = 1.6

$$\frac{b^3x^5}{5\,d}+\frac{ab^2x^3}{d}-\frac{b^3x^3c}{3\,d^2}+3\,\frac{a^2bx}{d}-3\,\frac{ab^2cx}{d^2}+\frac{b^3c^2x}{d^3}+a^3\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$
$$-3\,\frac{a^2bc}{d\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)+3\,\frac{ab^2c^2}{d^2\sqrt{cd}}\arctan\left(\frac{dx}{\sqrt{cd}}\right)-\frac{b^3c^3}{d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] 1/5*b^3*x^5/d+b^2/d*x^3*a-1/3*b^3/d^2*x^3*c+3*b/d*a^2*x-3*b^2/d^2
*a*c*x+b^3/d^3*c^2*x+1/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^3-3/
d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*b*c+3/d^2/(c*d)^(1/2)*a
rctan(x*d/(c*d)^(1/2))*a*b^2*c^2-1/d^3/(c*d)^(1/2)*arctan(x*d/(c*
d)^(1/2))*b^3*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.209798, size = 1, normalized size = 0.01

$$\left[-\frac{15\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)-2\,\left(3\,b^3d^2x^5-5\,\left(b^3cd-3\,ab^2d^2\right)x^3+15\,\left(b^3c^2-3\,ab^2c\right.\right.}{30\,\sqrt{-cd}d^3}\right.$$

$$-\frac{15\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{cd}x}{c}\right)-\left(3\,b^3d^2x^5-5\,\left(b^3cd-3\,ab^2d^2\right)x^3+15\,\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)\right.}{15\,\sqrt{cd}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*(3*b^3*d^2*
x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 15*(b^3*c^2 - 3*a*b^2*c*d +
 3*a^2*b*d^2)*x)*sqrt(-c*d))/(sqrt(-c*d)*d^3), -1/15*(15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(c*d)*x/c)
 - (3*b^3*d^2*x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 15*(b^3*c^2 -
 3*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(c*d))/(sqrt(c*d)*d^3)]
```

Sympy [A] time = 2.65285, size = 240, normalized size = 2.45

$$\frac{b^3x^5}{5d}-\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{x^3\left(3ab^2d-b^3c\right)}{3d^2}+\frac{x\left(3a^2bd^2-3ab^2cd+b^3c^2\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d**3*sq
rt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a
*b**2*c**2*d - b**3*c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)*
*3*log(c*d**3*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**
2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + x**3*(3*a*b**2
*d - b**3*c)/(3*d**2) + x*(3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c*
*2)/d**3
```

GIAC/XCAS [A] time = 0.235751, size = 176, normalized size = 1.8

$$-\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^{3}}+\frac{3\,b^{3}d^{4}x^{5}-5\,b^{3}cd^{3}x^{3}+15\,ab^{2}d^{4}x^{3}+15\,b^{3}c^{2}d^{2}x-45\,ab^{2}cd^{3}x+45\,a^{2}bd^{4}x}{15\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/s
qrt(c*d))/(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3
 + 15*a*b^2*d^4*x^3 + 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^
2*b*d^4*x)/d^5
```

3.18 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)
^3*x)/(2*c*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(7/2))
```

Rubi [A] time = 0.215367, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)
^3*x)/(2*c*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^3x^3}{3d^2}+\frac{(3ad-2bc)\int b^2\,dx}{d^3}+\frac{x\,(ad-bc)^3}{2cd^3\left(c+dx^2\right)}+\frac{(ad-bc)^2\,(ad+5bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + (3*a*d - 2*b*c)*Integral(b**2, x)/d**3 + x*(
a*d - b*c)**3/(2*c*d**3*(c + d*x**2)) + (a*d - b*c)**2*(a*d + 5*b
*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(3/2)*d**(7/2))
```

Mathematica [A] time = 0.0995282, size = 107, normalized size = 1.

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(ad+5bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}-\frac{x(bc-ad)^3}{2cd^3\left(c+dx^2\right)}+\frac{b^3x^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)
^3*x)/(2*c*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(7/2))
```

Maple [B] time = 0.013, size = 205, normalized size = 1.9

$$\frac{b^3x^3}{3\,d^2}+3\,\frac{ab^2x}{d^2}-2\,\frac{b^3xc}{d^3}+\frac{xa^3}{2\,c\,(dx^2+c)}-\frac{3\,a^2bx}{2\,d\,(dx^2+c)}+\frac{3\,acxb^2}{2\,d^2\,(dx^2+c)}-\frac{c^2xb^3}{2\,d^3\,(dx^2+c)}+\frac{a^3}{2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,a^2b}{2\,d}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{9\,ab^2c}{2\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{5\,b^3c^2}{2\,d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/3*b^3*x^3/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c+1/2/c*x/(d*x^2+c)*a^3
-3/2/d*x/(d*x^2+c)*a^2*b+3/2/d^2*c*x/(d*x^2+c)*a*b^2-1/2/d^3*c^2*
x/(d*x^2+c)*b^3+1/2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^3+3/2
/d/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2*b-9/2/d^2*c/(c*d)^(1/2
)*arctan(x*d/(c*d)^(1/2))*a*b^2+5/2/d^3*c^2/(c*d)^(1/2)*arctan(x*
d/(c*d)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221784, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,b^3c^4-9\,ab^2c^3d+3\,a^2bc^2d^2+a^3cd^3+\left(5\,b^3c^3d-9\,ab^2c^2d^2+3\,a^2bcd^3+a^3d^4\right)x^2\right)\log\left(\frac{2\,cdx+\left(dx^2-c\right)\sqrt{-cd}}{dx^2+c}\right)+2\left(2\,b^3cd^2\right.}{12\left(cd^4x^2+c^2d^3\right)\sqrt{-cd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3
 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)
*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) + 2*(2*b^3*c
*d^2*x^5 - 2*(5*b^3*c^2*d - 9*a*b^2*c*d^2)*x^3 - 3*(5*b^3*c^3 - 9
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)*sqrt(-c*d))/((c*d^4*x^
2 + c^2*d^3)*sqrt(-c*d)), 1/6*(3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a
^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2
*b*c*d^3 + a^3*d^4)*x^2)*arctan(sqrt(c*d)*x/c) + (2*b^3*c*d^2*x^5
 - 2*(5*b^3*c^2*d - 9*a*b^2*c*d^2)*x^3 - 3*(5*b^3*c^3 - 9*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)*sqrt(c*d))/((c*d^4*x^2 + c^2*d
^3)*sqrt(c*d))]
```

Sympy [A] time = 4.80084, size = 313, normalized size = 2.93

$$\frac{b^3x^3}{3d^2}+\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2c^2d^3+2cd^4x^2}-\frac{\sqrt{-\frac{1}{c^3d^7}}\left(ad-bc\right)^2\left(ad+5bc\right)\log\left(-\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{c^3d^7}}\left(ad-bc\right)^2\left(ad+5bc\right)\log\left(\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}+\frac{x\left(3ab^2d-2b^3c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c*
*2*d - b**3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2) - sqrt(-1/(c**3*d
**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-1/(c**3*d
**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 -
 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(c**3*d**7))*(a*
d - b*c)**2*(a*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d
 - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*
```

```
c**2*d + 5*b**3*c**3) + x)/4 + x*(3*a*b**2*d - 2*b**3*c)/d**3
```

GIAC/XCAS [A] time = 0.235833, size = 205, normalized size = 1.92

$$\frac{\left(5\,b^{3}c^{3}-9\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^{3}}-\frac{b^{3}c^{3}x-3\,ab^{2}c^{2}dx+3\,a^{2}bcd^{2}x-a^{3}d^{3}x}{2\,(dx^{2}+c)cd^{3}}+\frac{b^{3}d^{4}x^{3}-6\,b^{3}cd^{3}x+9\,ab^{2}d^{4}x}{3\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(
d*x/sqrt(c*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d
*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*
d^4*x^3 - 6*b^3*c*d^3*x + 9*a*b^2*d^4*x)/d^6
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} \, dx$

Optimal. Leaf size=130

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c
 - a*d)^2*(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*
d)*(4*b^2*c^2 + (b*c + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^
(5/2)*d^(7/2))
```

Rubi [A] time = 0.365222, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{3(bc-ad)\left((ad+bc)^2+4b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c
 - a*d)^2*(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*
d)*(4*b^2*c^2 + (b*c + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^
(5/2)*d^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int b^3\,dx}{d^3}+\frac{x\,(ad-bc)^3}{4cd^3\,(c+dx^2)^2}+\frac{3x\,(ad-bc)^2\,(ad+3bc)}{8c^2d^3\,(c+dx^2)}+\frac{3\,(ad-bc)\left(4b^2c^2+(ad+bc)^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Integral(b**3, x)/d**3 + x*(a*d - b*c)**3/(4*c*d**3*(c + d*x**2)*
*2) + 3*x*(a*d - b*c)**2*(a*d + 3*b*c)/(8*c**2*d**3*(c + d*x**2))
 + 3*(a*d - b*c)*(4*b**2*c**2 + (a*d + b*c)**2)*atan(sqrt(d)*x/sq
rt(c))/(8*c**(5/2)*d**(7/2))
```

Mathematica [A] time = 0.127599, size = 141, normalized size = 1.08

$$-\frac{3\left(-a^3d^3-a^2bcd^2-3ab^2c^2d+5b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}+\frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3\left(c+dx^2\right)}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^2\right)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c
 - a*d)^2*(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^
3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(8*c^(5/2)*d^(7/2))
```

Maple [B] time = 0.015, size = 266, normalized size = 2.1

$$\frac{b^3x}{d^3}+\frac{3\,dx^3a^3}{8\,(dx^2+c)^2\,c^2}+\frac{3\,x^3a^2b}{8\,(dx^2+c)^2\,c}-\frac{15\,ab^2x^3}{8\,d\,(dx^2+c)^2}+\frac{9\,cx^3b^3}{8\,d^2\,(dx^2+c)^2}+\frac{5\,xa^3}{8\,(dx^2+c)^2\,c}-\frac{3\,a^2bx}{8\,d\,(dx^2+c)^2}-\frac{9\,acxb^2}{8\,d^2\,(dx^2+c)^2}+\frac{7\,c^2xb^3}{8\,d^3\,(dx^2+c)^2}+\frac{3\,a^3}{8\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{3\,a^2b}{8\,cd}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{9\,ab^2}{8\,d^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15\,b^3c}{8\,d^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] b^3*x/d^3+3/8*d/(d*x^2+c)^2/c^2*x^3*a^3+3/8/(d*x^2+c)^2/c*x^3*a^2
*b-15/8/d/(d*x^2+c)^2*x^3*a*b^2+9/8/d^2/(d*x^2+c)^2*c*x^3*b^3+5/8
/(d*x^2+c)^2/c*x*a^3-3/8/d/(d*x^2+c)^2*x*a^2*b-9/8/d^2/(d*x^2+c)^
2*c*x*a*b^2+7/8/d^3/(d*x^2+c)^2*c^2*x*b^3+3/8/c^2/(c*d)^(1/2)*arc
tan(x*d/(c*d)^(1/2))*a^3+3/8/d/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/
2))*a^2*b+9/8/d^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b^2-15/8/
d^3*c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247622, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^3c^5-3\,ab^2c^4d-a^2bc^3d^2-a^3c^2d^3+\left(5\,b^3c^3d^2-3\,ab^2c^2d^3-a^2bcd^4-a^3d^5\right)x^4+2\left(5\,b^3c^4d-3\,ab^2c^3d^2-a^2bc^2d^3\right.}{}\right.$$

$$-\frac{3\left(5\,b^3c^5-3\,ab^2c^4d-a^2bc^3d^2-a^3c^2d^3+\left(5\,b^3c^3d^2-3\,ab^2c^2d^3-a^2bcd^4-a^3d^5\right)x^4+2\left(5\,b^3c^4d-3\,ab^2c^3d^2-a^2bc^2d^3-\right.}{8\left(c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 - a^3*c^2*d^
3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)*x^4
 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*
x^2)*log((2*c*d*x + (d*x^2 - c)*sqrt(-c*d))/(d*x^2 + c)) - 2*(8*b
^3*c^2*d^2*x^5 + (25*b^3*c^3*d - 15*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
 + 3*a^3*d^4)*x^3 + (15*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2
 + 5*a^3*c*d^3)*x)*sqrt(-c*d))/((c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^
4*d^3)*sqrt(-c*d)), -1/8*(3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^
3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*
d^4 - a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2
*d^3 - a^3*c*d^4)*x^2)*arctan(sqrt(c*d)*x/c) - (8*b^3*c^2*d^2*x^5
 + (25*b^3*c^3*d - 15*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + 3*a^3*d^4)*
x^3 + (15*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 5*a^3*c*d^3
)*x)*sqrt(c*d))/((c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)*sqrt(c*d
))]
```

Sympy [A] time = 8.27276, size = 422, normalized size = 3.25

$$\frac{b^3x}{d^3}-\frac{3\sqrt{-\frac{1}{c^5d^7}}\left(ad-bc\right)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}\left(ad-bc\right)\left(a^2d^2+2abcd+5b^2c^2\right)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{c^5d^7}}\left(ad-bc\right)\left(a^2d^2+2abcd+5b^2c^2\right)\log\left(\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}\left(ad-bc\right)\left(a^2d^2+2abcd+5b^2c^2\right)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16}+\frac{x^3\left(3a^3d^4+3a^2bcd^3-15ab^2c^2d^2+9b^3c^3d\right)+x\left(5a^3cd^3-3a^2bc^2d^2-9ab^2c^3d+7b^3c^4\right)}{8c^4d^3+16c^3d^4x^2+8c^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a
*b*c*d + 5*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d
- b*c)*(a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**
2*b*c*d**2 + 9*a*b**2*c**2*d - 15*b**3*c**3) + x)/16 + 3*sqrt(-1/
(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5*b**2*c**2)*lo
g(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b
*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**
2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d**
3 - 15*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a
**2*b*c**2*d**2 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 +
16*c**3*d**4*x**2 + 8*c**2*d**5*x**4)
```

GIAC/XCAS [A] time = 0.229855, size = 243, normalized size = 1.87

$$\frac{b^3x}{d^3} - \frac{3\left(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3} + \frac{9\,b^3c^3dx^3 - 15\,ab^2c^2d^2x^3 + 3\,a^2bcd^3x^3 + 3\,a^3d^4x^3 + 7\,b^3c^4x - 9\,ab^2c^3dx - 3\,a^2bc^2d^2x + 5\,a^3cd^3x}{8\,(dx^2+c)^2c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^
3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x
^3 - 15*a*b^2*c^2*d^2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7
*b^3*c^4*x - 9*a*b^2*c^3*d*x - 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)
/((d*x^2 + c)^2*c^2*d^3)
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$

Optimal. Leaf size=142

$$\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{d^4x^7}{7b}$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*
d)*x^5)/(5*b^2) + (d^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2))
```

Rubi [A] time = 0.205817, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}+\frac{d^3x^5(4bc-ad)}{5b^2}+\frac{d^4x^7}{7b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*
(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*
d)*x^5)/(5*b^2) + (d^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^4x^7}{7b}-\frac{d^3x^5\,(ad-4bc)}{5b^2}+\frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3}-\frac{(ad-2bc)\left(a^2d^2-2abcd+2b^2c^2\right)\int d\,dx}{b^4}+\frac{(ad-bc)^4\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] d**4*x**7/(7*b) - d**3*x**5*(a*d - 4*b*c)/(5*b**2) + d**2*x**3*(a
**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/(3*b**3) - (a*d - 2*b*c)*(a**
2*d**2 - 2*a*b*c*d + 2*b**2*c**2)*Integral(d, x)/b**4 + (a*d - b*
c)**4*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(9/2))
```

Mathematica [A] time = 0.146117, size = 136, normalized size = 0.96

$$\frac{dx\left(-105a^3d^3+35a^2bd^2\left(12c+dx^2\right)-7ab^2d\left(90c^2+20cdx^2+3d^2x^4\right)+3b^3\left(140c^3+70c^2dx^2+28cd^2x^4+5d^3x^6\right)\right)}{105b^4}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*
c^2 + 20*c*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 2
8*c*d^2*x^4 + 5*d^3*x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2))
```

Maple [A] time = 0.007, size = 246, normalized size = 1.7

$$\frac{d^4x^7}{7\,b}-\frac{d^4x^5a}{5\,b^2}+\frac{4\,d^3x^5c}{5\,b}+\frac{d^4x^3a^2}{3\,b^3}-\frac{4\,d^3x^3ac}{3\,b^2}+2\,\frac{d^2x^3c^2}{b}-\frac{d^4a^3x}{b^4}+4\,\frac{a^2d^3cx}{b^3}-6\,\frac{ac^2d^2x}{b^2}+4\,\frac{dc^3x}{b}+\frac{a^4d^4}{b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-4\,\frac{a^3cd^3}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+6\,\frac{a^2c^2d^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-4\,\frac{ac^3d}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^4\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^4/(b*x^2+a),x)
```

```
[Out] 1/7*d^4*x^7/b-1/5*d^4/b^2*x^5*a+4/5*d^3/b*x^5*c+1/3*d^4/b^3*x^3*a
^2-4/3*d^3/b^2*x^3*a*c+2*d^2/b*x^3*c^2-d^4/b^4*a^3*x+4*d^3/b^3*a^
2*c*x-6*d^2/b^2*a*c^2*x+4*d/b*c^3*x+1/b^4/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*a^4*d^4-4/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^
3*c*d^3+6/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*c^2*d^2-4/b
/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*c^3*d+1/(a*b)^(1/2)*arctan
(x*b/(a*b)^(1/2))*c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218328, size = 1, normalized size = 0.01

$$\left[\frac{105\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right) + 2\left(15\,b^3d^4x^7 + 21\left(4\,b^3cd^3 - ab^2d^4\right)x^5 + 3\right.}{210\,\sqrt{-ab}b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2
+ a)) + 2*(15*b^3*d^4*x^7 + 21*(4*b^3*c*d^3 - a*b^2*d^4)*x^5 + 35
*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3 + 105*(4*b^3*c^3
*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-a*b))/(s
qrt(-a*b)*b^4), 1/105*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c
^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(sqrt(a*b)*x/a) + (15*b^3
*d^4*x^7 + 21*(4*b^3*c*d^3 - a*b^2*d^4)*x^5 + 35*(6*b^3*c^2*d^2 -
 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3 + 105*(4*b^3*c^3*d - 6*a*b^2*c^2*
d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(a*b))/(sqrt(a*b)*b^4)]
```

Sympy [A] time = 3.38733, size = 323, normalized size = 2.27

$$-\frac{\sqrt{-\frac{1}{ab^9}}\left(ad-bc\right)^4\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}\left(ad-bc\right)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^9}}\left(ad-bc\right)^4\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}\left(ad-bc\right)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{d^4x^7}{7b} - \frac{x^5\left(ad^4-4bcd^3\right)}{5b^2} + \frac{x^3\left(a^2d^4-4abcd^3+6b^2c^2d^2\right)}{3b^3} - \frac{x\left(a^3d^4-4a^2bcd^3+6ab^2c^2d^2-4b^3c^3d\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(-a*b**4*sqrt(-1/(a*b**9))*(
a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + sqrt(-1/(a*b**9))*(a*d
- b*c)**4*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4
- 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**
4*c**4) + x)/2 + d**4*x**7/(7*b) - x**5*(a*d**4 - 4*b*c*d**3)/(5*
b**2) + x**3*(a**2*d**4 - 4*a*b*c*d**3 + 6*b**2*c**2*d**2)/(3*b**
3) - x*(a**3*d**4 - 4*a**2*b*c*d**3 + 6*a*b**2*c**2*d**2 - 4*b**3
*c**3*d)/b**4
```

GIAC/XCAS [A] time = 0.234794, size = 267, normalized size = 1.88

$$\frac{\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^4x^7 + 84\,b^6cd^3x^5 - 21\,ab^5d^4x^5 + 210\,b^6c^2d^2x^3 - 140\,ab^5cd^3x^3 + 35\,a^2b^4d^4x^3 + 420\,b^6c^3dx - 630\,ab^5c^2d^2x + 420\,a^2b}{105\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*
x^7 + 84*b^6*c*d^3*x^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 -
 140*a*b^5*c*d^3*x^3 + 35*a^2*b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630
*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3*x - 105*a^3*b^3*d^4*x)/b^7
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

Optimal. Leaf size=98

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^3)/(3*b^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi [A] time = 0.136129, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*
x^3)/(3*b^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^3x^5}{5b}-\frac{d^2x^3\,(ad-3bc)}{3b^2}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)\int d\,dx}{b^3}-\frac{(ad-bc)^3\,\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] d**3*x**5/(5*b) - d**2*x**3*(a*d - 3*b*c)/(3*b**2) + (a**2*d**2 -
 3*a*b*c*d + 3*b**2*c**2)*Integral(d, x)/b**3 - (a*d - b*c)**3*at
an(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(7/2))
```

Mathematica [A] time = 0.115642, size = 92, normalized size = 0.94

$$\frac{dx\left(15a^2d^2-5abd\left(9c+dx^2\right)+3b^2\left(15c^2+5cdx^2+d^2x^4\right)\right)}{15b^3}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*
x^2 + d^2*x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.005, size = 161, normalized size = 1.6

$$\frac{d^3x^5}{5\,b}-\frac{d^3x^3a}{3\,b^2}+\frac{d^2x^3c}{b}+\frac{a^2d^3x}{b^3}-3\,\frac{acd^2x}{b^2}+3\,\frac{dc^2x}{b}-\frac{a^3d^3}{b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+3\,\frac{a^2cd^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-3\,\frac{ac^2d}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^3\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a),x)
```

```
[Out] 1/5*d^3*x^5/b-1/3*d^3/b^2*x^3*a+d^2/b*x^3*c+d^3/b^3*a^2*x-3*d^2/b
^2*a*c*x+3*d/b*c^2*x-1/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^
3*d^3+3/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a^2*c*d^2-3/b/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*c^2*d+1/(a*b)^(1/2)*arctan(x*b
/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211889, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(3\,b^2d^3x^5+5\left(3\,b^2cd^2-abd^3\right)x^3+15\left(3\,b^2c^2d-3\right.\right.}{30\sqrt{-ab}b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(3*b^2*d^3
*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(3*b^2*c^2*d - 3*a*b*c*
d^2 + a^2*d^3)*x)*sqrt(-a*b))/(sqrt(-a*b)*b^3), 1/15*(15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(a*b)*x/a)
 + (3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(3*b^2*c^2
*d - 3*a*b*c*d^2 + a^2*d^3)*x)*sqrt(a*b))/(sqrt(a*b)*b^3)]
```

Sympy [A] time = 2.73443, size = 240, normalized size = 2.45

$$\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^7}}\,(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}-\frac{x^3\left(ad^3-3bcd^2\right)}{3b^2}+\frac{x\left(a^2d^3-3abcd^2+3b^2c^2d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**
3*c**3) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt
(-1/(a*b**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b
**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**5/(5*b) - x**3*(a*d**3 -
 3*b*c*d**2)/(3*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2
*d)/b**3
```

GIAC/XCAS [A] time = 0.233042, size = 174, normalized size = 1.78

$$\frac{\left(b^{3} c^{3} - 3 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} - a^{3} d^{3}\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b} b^{3}} + \frac{3 \, b^{4} d^{3} x^{5} + 15 \, b^{4} c d^{2} x^{3} - 5 \, a b^{3} d^{3} x^{3} + 45 \, b^{4} c^{2} d x - 45 \, a b^{3} c d^{2} x + 15 \, a^{2} b^{2} d^{3} x}{15 \, b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3
 - 5*a*b^3*d^3*x^3 + 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b
^2*d^3*x)/b^5
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2} \, dx$

Optimal. Leaf size=63

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi [A] time = 0.0946087, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d^2x^3}{3b}-\frac{(ad-2bc)\int d\,dx}{b^2}+\frac{(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] d**2*x**3/(3*b) - (a*d - 2*b*c)*Integral(d, x)/b**2 + (a*d - b*c)
**2*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(5/2))
```

Mathematica [A] time = 0.0833748, size = 59, normalized size = 0.94

$$\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}+\frac{dx\left(-3ad+6bc+bdx^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A] time = 0.004, size = 95, normalized size = 1.5

$$\frac{d^2x^3}{3\,b}-\frac{ad^2x}{b^2}+2\,\frac{dxc}{b}+\frac{a^2d^2}{b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-2\,\frac{acd}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+c^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a),x)
```

```
[Out] 1/3*d^2*x^3/b-d^2/b^2*a*x+2*d/b*x*c+1/b^2/(a*b)^(1/2)*arctan(x*b/
(a*b)^(1/2))*a^2*d^2-2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*c*
d+1/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210575, size = 1, normalized size = 0.02

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(bd^2x^3+3\left(2\,bcd-ad^2\right)x\right)\sqrt{-ab}}{6\,\sqrt{-ab}b^2},\ \frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log((2*a*b*x + (b*x^2 - a
)*sqrt(-a*b))/(b*x^2 + a)) + 2*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x
)*sqrt(-a*b))/(sqrt(-a*b)*b^2), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*arctan(sqrt(a*b)*x/a) + (b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)
*sqrt(a*b))/(sqrt(a*b)*b^2)]
```

Sympy [A] time = 2.16255, size = 172, normalized size = 2.73

$$-\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}+\frac{d^2x^3}{3b}-\frac{x\left(ad^2-2bcd\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*sqrt(-1/(a*b**5))*(
a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-
1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*
c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b
) - x*(a*d**2 - 2*b*c*d)/b**2
```

GIAC/XCAS [A] time = 0.23178, size = 97, normalized size = 1.54

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b^2d^2x^3+6\,b^2cdx-3\,abd^2x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^2) + 1/3*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.23 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi [A] time = 0.0475927, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3
/2))
```

Rubi in Sympy [A] time = 8.52485, size = 34, normalized size = 0.87

$$\frac{dx}{b}-\frac{(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*b**(3/2))
```

Mathematica [A] time = 0.0418032, size = 40, normalized size = 1.03

$$\frac{dx}{b}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b
^(3/2))
```

Maple [A] time = 0.004, size = 45, normalized size = 1.2

$$\frac{dx}{b}-\frac{ad}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+c\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a),x)
```

```
[Out] d*x/b-1/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*a*d+1/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210324, size = 1, normalized size = 0.03

$$\left[\frac{2\sqrt{-ab}dx-(bc-ad)\log\left(-\frac{2\,abx-(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{2\sqrt{-ab}b},\frac{\sqrt{ab}dx+(bc-ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(-a*b)*d*x - (b*c - a*d)*log(-(2*a*b*x - (b*x^2 - a)*
sqrt(-a*b))/(b*x^2 + a)))/(sqrt(-a*b)*b), (sqrt(a*b)*d*x + (b*c -
 a*d)*arctan(sqrt(a*b)*x/a))/(sqrt(a*b)*b)]
```

Sympy [A] time = 1.60144, size = 82, normalized size = 2.1

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
 sqrt(-1/(a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 +
 d*x/b
```

GIAC/XCAS [A] time = 0.23033, size = 45, normalized size = 1.15

$$\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sq
rt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi [A] time = 0.0706849, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sq
rt[d]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rubi in Sympy [A] time = 15.1862, size = 60, normalized size = 0.86

$$\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\,(ad-bc)}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(d)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*(a*d - b*c)) - sqrt(b)*a
tan(sqrt(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c))
```

Mathematica [A] time = 0.0785773, size = 61, normalized size = 0.87

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[
(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A] time = 0.01, size = 55, normalized size = 0.8

$$\frac{d}{ad-bc}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b}{ad-bc}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c),x)
```

```
[Out] d/(a*d-b*c)/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-b/(a*d-b*c)/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230279, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$

$$-\frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)},\frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
 + sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b
*c - a*d), -1/2*(2*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + sqrt(-b/
a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(b*c - a*d),
1/2*(2*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - sqrt(-d/c)*log((d*x^
2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a)*a
rctan(b*x/(a*sqrt(b/a))) - sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))))/(
b*c - a*d)]
```

Sympy [A] time = 8.09849, size = 712, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a
**3*b*c**2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(
-b/a)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) -
a*b**3*c**4*(-b/a)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(
a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-b/a)*log(x + (a**4*c*d
**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-b/a)**(3/2)
/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)**3 +
 a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*
d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d -
b*c)) + sqrt(-d/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c
)**3 + a**3*b*c**2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*
c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d -
 b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3 - b**2*c**2*sqrt
(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)*log(x + (
a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b
*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(
3/2)/(a*d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2
*(a*d - b*c))
```

GIAC/XCAS [A] time = 0.257996, size = 257, normalized size = 3.67

$$-\frac{2\,\sqrt{cd}b|d|\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad+\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{bcd|bc-ad|+ad^2|bc-ad|+(bc-ad)^2d}+\frac{2\,\sqrt{ab}d|b|\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}x}{\sqrt{\frac{bc+ad-\sqrt{-4\,abcd+(bc+ad)^2}}{bd}}}\right)}{b^2c|bc-ad|+abd|bc-ad|-(bc-ad)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -2*sqrt(c*d)*b*abs(d)*arctan(2*sqrt(1/2)*x/sqrt((b*c + a*d + sqrt
(-4*a*b*c*d + (b*c + a*d)^2))/(b*d)))/(b*c*d*abs(b*c - a*d) + a*d
^2*abs(b*c - a*d) + (b*c - a*d)^2*d) + 2*sqrt(a*b)*d*abs(b)*arcta
n(2*sqrt(1/2)*x/sqrt((b*c + a*d - sqrt(-4*a*b*c*d + (b*c + a*d)^2
))/(b*d)))/(b^2*c*abs(b*c - a*d) + a*b*d*abs(b*c - a*d) - (b*c -
a*d)^2*b)
```

3.25 $\int \frac{1}{(a+bx^2)(c+dx^2)^2} \, dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi [A] time = 0.212676, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c\left(c+dx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 41.2097, size = 94, normalized size = 0.86

$$\frac{dx}{2c\left(c+dx^2\right)(ad-bc)}+\frac{\sqrt{d}\,(ad-3bc)\,\mathrm{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}\,(ad-bc)^2}+\frac{b^{\frac{3}{2}}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*(c + d*x**2)*(a*d - b*c)) + sqrt(d)*(a*d - 3*b*c)*atan(s
qrt(d)*x/sqrt(c))/(2*c**(3/2)*(a*d - b*c)**2) + b**(3/2)*atan(sqr
t(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.316056, size = 95, normalized size = 0.87

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}(ad-3bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}+\frac{dx(ad-bc)}{c(c+dx^2)}}{2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/c^(3/2))/(2*(b*c - a*d)^2)
```

Maple [A] time = 0.016, size = 144, normalized size = 1.3

$$\frac{d^2xa}{2\,(ad-bc)^2\,c\,(dx^2+c)}-\frac{dxb}{2\,(ad-bc)^2\,(dx^2+c)}+\frac{ad^2}{2\,(ad-bc)^2\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2}{(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^2/(a*d-b*c)^2/c*x/(d*x^2+c)*a-1/2*d/(a*d-b*c)^2*x/(d*x^2+c)
*b+1/2*d^2/(a*d-b*c)^2/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-3/
2*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+b^2/(a*d-b*
c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.317974, size = 1, normalized size = 0.01

$$\left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\left(bcd-ad^2\right)x}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2\right)}\right.$$

$$-\frac{\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(\frac{dx}{c\sqrt{\frac{d}{c}}}\right)-\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\left(bcd-ad^2\right)x}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2\right)},\ 4\left(bcdx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)
*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(
b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3
*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1/2*((3*b*c^2 - a*c*d + (3
*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - (b*c*d
*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^
2 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2
 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^
2 + b*c^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - (3*b*c^2 - a*c*d
 + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c
) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/
2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - (3
*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(d*x/(c*s
qrt(d/c))) - (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*
d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

Sympy [A] time = 46.0257, size = 2033, normalized size = 18.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*a*c**2*d - 2*b*c**3 + x**2*(2*a*c*d**2 - 2*b*c**2*d)) - sq
rt(-b**3/a)*log(x + (-4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*
c)**6 + 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 64*
a**5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 - a**5*d**5*s
qrt(-b**3/a)/(a*d - b*c)**2 + 56*a**4*b**3*c**6*d**3*(-b**3/a)**(
```

```
3/2)/(a*d - b*c)**6 + 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**
2 - 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a*
*3*b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 - 20*a**2*b**5*c**
8*d*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**2*b**3*c**3*d**2*sqrt
(-b**3/a)/(a*d - b*c)**2 + 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d -
b*c)**6 + 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d*
*3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) + sqrt
(-b**3/a)*log(x + (4*a**7*c**3*d**6*(-b**3/a)**(3/2)/(a*d - b*c)*
*6 - 28*a**6*b*c**4*d**5*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 64*a**
5*b**2*c**5*d**4*(-b**3/a)**(3/2)/(a*d - b*c)**6 + a**5*d**5*sqrt
(-b**3/a)/(a*d - b*c)**2 - 56*a**4*b**3*c**6*d**3*(-b**3/a)**(3/2
)/(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-b**3/a)/(a*d - b*c)**2 +
 4*a**3*b**4*c**7*d**2*(-b**3/a)**(3/2)/(a*d - b*c)**6 + 27*a**3*
b**2*c**2*d**3*sqrt(-b**3/a)/(a*d - b*c)**2 + 20*a**2*b**5*c**8*d
*(-b**3/a)**(3/2)/(a*d - b*c)**6 - 27*a**2*b**3*c**3*d**2*sqrt(-b
**3/a)/(a*d - b*c)**2 - 8*a*b**6*c**9*(-b**3/a)**(3/2)/(a*d - b*c
)**6 - 8*b**5*c**5*sqrt(-b**3/a)/(a*d - b*c)**2)/(a**2*b**2*d**3
- 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/(2*(a*d - b*c)**2) - sqrt(-d
/c**3)*(a*d - 3*b*c)*log(x + (-a**7*c**3*d**6*(-d/c**3)**(3/2)*(a
*d - 3*b*c)**3/(2*(a*d - b*c)**6) + 7*a**6*b*c**4*d**5*(-d/c**3)*
*(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 8*a**5*b**2*c**5*d**
4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - a**5*d**5*sq
rt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) + 7*a**4*b**3*c**6*d
**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 + 9*a**4*b*c
*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a**3*b**4*
c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) -
27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)
**2) - 5*a**2*b**5*c**8*d*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a
*d - b*c)**6) + 27*a**2*b**3*c**3*d**2*sqrt(-d/c**3)*(a*d - 3*b*c
)/(2*(a*d - b*c)**2) + a*b**6*c**9*(-d/c**3)**(3/2)*(a*d - 3*b*c)
**3/(a*d - b*c)**6 + 4*b**5*c**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(a*d
 - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3*c*d**2 + 12*b**4*c**2*d))/
(4*(a*d - b*c)**2) + sqrt(-d/c**3)*(a*d - 3*b*c)*log(x + (a**7*c*
*3*d**6*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 7*
a**6*b*c**4*d**5*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)
**6) + 8*a**5*b**2*c**5*d**4*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/(a
*d - b*c)**6 + a**5*d**5*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*
c)**2) - 7*a**4*b**3*c**6*d**3*(-d/c**3)**(3/2)*(a*d - 3*b*c)**3/
(a*d - b*c)**6 - 9*a**4*b*c*d**4*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(
a*d - b*c)**2) + a**3*b**4*c**7*d**2*(-d/c**3)**(3/2)*(a*d - 3*b*
c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt(-d/c**3)*(
a*d - 3*b*c)/(2*(a*d - b*c)**2) + 5*a**2*b**5*c**8*d*(-d/c**3)**(
3/2)*(a*d - 3*b*c)**3/(2*(a*d - b*c)**6) - 27*a**2*b**3*c**3*d**2
*sqrt(-d/c**3)*(a*d - 3*b*c)/(2*(a*d - b*c)**2) - a*b**6*c**9*(-d
/c**3)**(3/2)*(a*d - 3*b*c)**3/(a*d - b*c)**6 - 4*b**5*c**5*sqrt(
-d/c**3)*(a*d - 3*b*c)/(a*d - b*c)**2)/(a**2*b**2*d**3 - 7*a*b**3
*c*d**2 + 12*b**4*c**2*d))/(4*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.223712, size = 165, normalized size = 1.51

$$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{ab}} - \frac{\left(3\,bcd - ad^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\sqrt{cd}} - \frac{dx}{2\,(bc^2 - acd)(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a
*b)) - 1/2*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*
a*b*c^2*d + a^2*c*d^2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x
^2 + c))
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8
*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d
)^3)
```

Rubi [A] time = 0.463686, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3}-\frac{dx(7bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8
*c^2*(b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(Sqrt[a]*(b*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*
d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d
)^3)
```

Rubi in Sympy [A] time = 95.6967, size = 146, normalized size = 0.91

$$\frac{dx}{4c\left(c+dx^2\right)^2(ad-bc)}+\frac{dx\,(3ad-7bc)}{8c^2\left(c+dx^2\right)(ad-bc)^2}+\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{2}}(ad-bc)^3}-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] d*x/(4*c*(c + d*x**2)**2*(a*d - b*c)) + d*x*(3*a*d - 7*b*c)/(8*c*
*2*(c + d*x**2)*(a*d - b*c)**2) + sqrt(d)*(3*a**2*d**2 - 10*a*b*c
*d + 15*b**2*c**2)*atan(sqrt(d)*x/sqrt(c))/(8*c**(5/2)*(a*d - b*c
)**3) - b**(5/2)*atan(sqrt(b)*x/sqrt(a))/(sqrt(a)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.513175, size = 158, normalized size = 0.99

$$\frac{1}{8}\left(-\frac{\sqrt{d}\left(3a^2d^2-10abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(ad-bc)^3}+\frac{dx(3ad-7bc)}{c^2\left(c+dx^2\right)(bc-ad)^2}-\frac{2dx}{c\left(c+dx^2\right)^2(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((-2*d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (d*(-7*b*c + 3*a*d)*x)/
(c^2*(b*c - a*d)^2*(c + d*x^2)) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(Sqrt[a]*(-(b*c) + a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a
*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c -
a*d)^3))/8
```

Maple [B] time = 0.017, size = 310, normalized size = 1.9

$$\frac{3\,d^4x^3a^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2c^2}-\frac{5\,d^3x^3ab}{4\,(ad-bc)^3\left(dx^2+c\right)^2c}+\frac{7\,d^2x^3b^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{5\,d^3xa^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2c}-\frac{7\,d^2xab}{4\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{9\,cdxb^2}{8\,(ad-bc)^3\left(dx^2+c\right)^2}+\frac{3\,a^2d^3}{8\,(ad-bc)^3c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{5\,abd^2}{4\,(ad-bc)^3c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{15\,db^2}{8\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3}{(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^4/(a*d-b*c)^3/(d*x^2+c)^2/c^2*x^3*a^2-5/4*d^3/(a*d-b*c)^3/(
d*x^2+c)^2/c*x^3*a*b+7/8*d^2/(a*d-b*c)^3/(d*x^2+c)^2*x^3*b^2+5/8*
```

```
d^3/(a*d-b*c)^3/(d*x^2+c)^2/c*x*a^2-7/4*d^2/(a*d-b*c)^3/(d*x^2+c)
^2*x*a*b+9/8*d/(a*d-b*c)^3/(d*x^2+c)^2*c*x*b^2+3/8*d^3/(a*d-b*c)^
3/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-5/4*d^2/(a*d-b*c)^3
/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+15/8*d/(a*d-b*c)^3/(c*
d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-b^3/(a*d-b*c)^3/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.968237, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d
+ 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c^3*
d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3
*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3
*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^
2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 1
0*a*b*c*d^3 + 3*a^2*d^4)*x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2
*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(1
5*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(d/c)*arctan
(d*x/(c*sqrt(d/c))) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*
c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b
^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2
*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*
a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/16*(2*(7*
b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 16*(b^2*c^2*d^2*x^4
 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a)))
 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 -
```

```
10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2
 + 3*a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c
)/(d*x^2 + c)) + 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3
*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 +
 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*
x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b
^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(b*x/
(a*sqrt(b/a))) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15
*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d -
10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d
/c))) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7
- 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6
*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.234435, size = 293, normalized size = 1.83

$$\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{ab}} - \frac{\left(15\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3)\sqrt{cd}} - \frac{7\,bcd^2x^3 - 3\,ad^3x^3 + 9\,bc^2dx - 5\,acd^2x}{8\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)(dx^2 + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a
^2*d^3)*arctan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*sqrt(c*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3
*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c
^2*d^2)*(d*x^2 + c)^2)
```

3.27 $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=192

$$\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{d^5x^7}{7b^2}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c -
a*d)^5*x)/(2*a*b^5*(a + b*x^2)) + ((b*c - a*d)^4*(b*c + 9*a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(11/2))
```

Rubi [A] time = 0.33636, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{d^5x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c -
a*d)^5*x)/(2*a*b^5*(a + b*x^2)) + ((b*c - a*d)^4*(b*c + 9*a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(11/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^2\left(4a^3d^3-15a^2bcd^2+20ab^2c^2d-10b^3c^3\right)\int\frac{1}{b^5}\,dx+\frac{d^5x^7}{7b^2}-\frac{d^4x^5\left(2ad-5bc\right)}{5b^3}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}-\frac{x\left(ad-bc\right)^5}{2ab^5\left(a+bx^2\right)}+\frac{\left(ad-bc\right)^4\left(9ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] -d**2*(4*a**3*d**3 - 15*a**2*b*c*d**2 + 20*a*b**2*c**2*d - 10*b**
3*c**3)*Integral(b**(-5), x) + d**5*x**7/(7*b**2) - d**4*x**5*(2*
a*d - 5*b*c)/(5*b**3) + d**3*x**3*(3*a**2*d**2 - 10*a*b*c*d + 10*
b**2*c**2)/(3*b**4) - x*(a*d - b*c)**5/(2*a*b**5*(a + b*x**2)) +
(a*d - b*c)**4*(9*a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*
b**(11/2))
```

Mathematica [A] time = 0.158329, size = 192, normalized size = 1.

$$\frac{(bc-ad)^4(9ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}+\frac{d^3x^3\left(3a^2d^2-10abcd+10b^2c^2\right)}{3b^4}+\frac{d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)}{b^5}+\frac{x(bc-ad)^5}{2ab^5\left(a+bx^2\right)}+\frac{d^4x^5(5bc-2ad)}{5b^3}+\frac{d^5x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x
)/b^5 + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) +
 (d^4*(5*b*c - 2*a*d)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c -
a*d)^5*x)/(2*a*b^5*(a + b*x^2)) + ((b*c - a*d)^4*(b*c + 9*a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(11/2))
```

Maple [B] time = 0.016, size = 402, normalized size = 2.1

$$\frac{d^5x^7}{7\,b^2}-\frac{2\,d^5x^5a}{5\,b^3}+\frac{d^4x^5c}{b^2}+\frac{d^5x^3a^2}{b^4}-\frac{10\,d^4x^3ac}{3\,b^3}+\frac{10\,d^3x^3c^2}{3\,b^2}-4\,\frac{a^3d^5x}{b^5}+15\,\frac{a^2cd^4x}{b^4}-20\,\frac{ac^2d^3x}{b^3}+10\,\frac{c^3d^2x}{b^2}-\frac{xa^4d^5}{2\,b^5\left(bx^2+a\right)}+\frac{5\,a^3cxd^4}{2\,b^4\left(bx^2+a\right)}-5\,\frac{a^2c^2xd^3}{b^3\left(bx^2+a\right)}+5\,\frac{ac^3xd^2}{b^2\left(bx^2+a\right)}-\frac{5\,xc^4d}{2\,b\left(bx^2+a\right)}+\frac{xc^5}{2\,a\left(bx^2+a\right)}+\frac{9\,a^4d^5}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{35\,a^3cd^4}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+25\,\frac{a^2c^2d^3}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-15\,\frac{ac^3d^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{5\,c^4d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^5}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^5/(b*x^2+a)^2,x)
```

```
[Out] 1/7*d^5*x^7/b^2-2/5*d^5/b^3*x^5*a+d^4/b^2*x^5*c+d^5/b^4*x^3*a^2-1
0/3*d^4/b^3*x^3*a*c+10/3*d^3/b^2*x^3*c^2-4*d^5/b^5*a^3*x+15*d^4/b
^4*a^2*c*x-20*d^3/b^3*a*c^2*x+10*d^2/b^2*c^3*x-1/2/b^5*x*a^4/(b*x
^2+a)*d^5+5/2/b^4*x*a^3/(b*x^2+a)*c*d^4-5/b^3*x*a^2/(b*x^2+a)*c^2
*d^3+5/b^2*x*a/(b*x^2+a)*c^3*d^2-5/2/b*x/(b*x^2+a)*c^4*d+1/2*x/a/
(b*x^2+a)*c^5+9/2/b^5*a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^5
-35/2/b^4*a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^4+25/b^3*a^
2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d^3-15/b^2*a/(a*b)^(1/2
)*arctan(x*b/(a*b)^(1/2))*c^3*d^2+5/2/b/(a*b)^(1/2)*arctan(x*b/(a
*b)^(1/2))*c^4*d+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215377, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 5
0*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b
^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c
*d^4 + 9*a^5*b*d^5)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(
b*x^2 + a)) + 2*(30*a*b^4*d^5*x^9 + 6*(35*a*b^4*c*d^4 - 9*a^2*b^3
*d^5)*x^7 + 14*(50*a*b^4*c^2*d^3 - 35*a^2*b^3*c*d^4 + 9*a^3*b^2*d
^5)*x^5 + 70*(30*a*b^4*c^3*d^2 - 50*a^2*b^3*c^2*d^3 + 35*a^3*b^2*
c*d^4 - 9*a^4*b*d^5)*x^3 + 105*(b^5*c^5 - 5*a*b^4*c^4*d + 30*a^2*
b^3*c^3*d^2 - 50*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 9*a^5*d^5)*x)
*sqrt(-a*b))/((a*b^6*x^2 + a^2*b^5)*sqrt(-a*b)), 1/210*(105*(a*b^
5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3
 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2
*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^
5)*x^2)*arctan(sqrt(a*b)*x/a) + (30*a*b^4*d^5*x^9 + 6*(35*a*b^4*c
*d^4 - 9*a^2*b^3*d^5)*x^7 + 14*(50*a*b^4*c^2*d^3 - 35*a^2*b^3*c*d
^4 + 9*a^3*b^2*d^5)*x^5 + 70*(30*a*b^4*c^3*d^2 - 50*a^2*b^3*c^2*d
^3 + 35*a^3*b^2*c*d^4 - 9*a^4*b*d^5)*x^3 + 105*(b^5*c^5 - 5*a*b^4
*c^4*d + 30*a^2*b^3*c^3*d^2 - 50*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4
```

```
- 9*a^5*d^5)*x)*sqrt(a*b))/((a*b^6*x^2 + a^2*b^5)*sqrt(a*b))]
```

Sympy [A] time = 9.14336, size = 498, normalized size = 2.59

$$-\frac{x\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{2a^2b^5+2ab^6x^2}-\frac{\sqrt{-\frac{1}{a^3b^{11}}}\left(ad-bc\right)^4\left(9ad+bc\right)\log\left(-\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^{11}}}\left(ad-bc\right)^4\left(9ad+bc\right)\log\left(\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4}+\frac{d^5x^7}{7b^2}-\frac{x^5\left(2ad^5-5bcd^4\right)}{5b^3}+\frac{x^3\left(3a^2d^5-10abcd^4+10b^2c^2d^3\right)}{3b^4}-\frac{x\left(4a^3d^5-15a^2bcd^4+20ab^2c^2d^3-10b^3c^3d^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*a**3*b**2*c**2*d**3 - 10*a**
2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)/(2*a**2*b**5 + 2*
a*b**6*x**2) - sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)
*log(-a**2*b**5*sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c
)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*a
**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + sqrt(-
1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*sqrt(-
1/(a**3*b**11))*(a*d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a*
*4*b*c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 + 5
*a*b**4*c**4*d + b**5*c**5) + x)/4 + d**5*x**7/(7*b**2) - x**5*(2
*a*d**5 - 5*b*c*d**4)/(5*b**3) + x**3*(3*a**2*d**5 - 10*a*b*c*d**
4 + 10*b**2*c**2*d**3)/(3*b**4) - x*(4*a**3*d**5 - 15*a**2*b*c*d*
*4 + 20*a*b**2*c**2*d**3 - 10*b**3*c**3*d**2)/b**5
```

GIAC/XCAS [A] time = 0.234203, size = 413, normalized size = 2.15

$$\frac{\left(b^5c^5+5\,ab^4c^4d-30\,a^2b^3c^3d^2+50\,a^3b^2c^2d^3-35\,a^4bcd^4+9\,a^5d^5\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^5}+\frac{b^5c^5x-5\,ab^4c^4dx+10\,a^2b^3c^3d^2x-10\,a^3b^2c^2d^3x+5\,a^4bcd^4x-a^5d^5x}{2\,(bx^2+a)ab^5}+\frac{15\,b^{12}d^5x^7+105\,b^{12}cd^4x^5-42\,ab^{11}d^5x^5+350\,b^{12}c^2d^3x^3-350\,ab^{11}cd^4x^3+105\,a^2b^{10}d^5x^3+1050\,b^{12}c^3d^2x-2100\,ab^{11}c^2d}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^
2*d^3 - 35*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^5) + 1/2*(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^
2*x - 10*a^3*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2
 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^7 + 105*b^12*c*d^4*x^5 - 42*a
*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*b^11*c*d^4*x^3 + 105
*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*c^2*d^3*x +
 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

3.28 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=142

$$\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c -
 a*d)*x^3)/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b
^4*(a + b*x^2)) + ((b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.260008, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c -
 a*d)*x^3)/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b
^4*(a + b*x^2)) + ((b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(3a^2d^2-8abcd+6b^2c^2\right)\int\frac{1}{b^4}\,dx+\frac{d^4x^5}{5b^2}-\frac{2d^3x^3\left(ad-2bc\right)}{3b^3}+\frac{x\left(ad-bc\right)^4}{2ab^4\left(a+bx^2\right)}-\frac{\left(ad-bc\right)^3\left(7ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] d**2*(3*a**2*d**2 - 8*a*b*c*d + 6*b**2*c**2)*Integral(b**(-4), x)
 + d**4*x**5/(5*b**2) - 2*d**3*x**3*(a*d - 2*b*c)/(3*b**3) + x*(a
*d - b*c)**4/(2*a*b**4*(a + b*x**2)) - (a*d - b*c)**3*(7*a*d + b*
c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*b**(9/2))
```

Mathematica [A] time = 0.143361, size = 142, normalized size = 1.

$$\frac{(bc-ad)^3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}+\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}+\frac{x(bc-ad)^4}{2ab^4\left(a+bx^2\right)}+\frac{2d^3x^3(2bc-ad)}{3b^3}+\frac{d^4x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c -
 a*d)*x^3)/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b
^4*(a + b*x^2)) + ((b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Maple [B] time = 0.016, size = 296, normalized size = 2.1

$$\frac{d^4x^5}{5\,b^2}-\frac{2\,d^4x^3a}{3\,b^3}+\frac{4\,d^3x^3c}{3\,b^2}+3\,\frac{a^2d^4x}{b^4}-8\,\frac{acd^3x}{b^3}+6\,\frac{c^2d^2x}{b^2}+\frac{xa^3d^4}{2\,b^4\,(bx^2+a)}-2\,\frac{a^2cxd^3}{b^3\,(bx^2+a)}+3\,\frac{ac^2xd^2}{b^2\,(bx^2+a)}-2\,\frac{xc^3d}{b\,(bx^2+a)}+\frac{xc^4}{2\,a\,(bx^2+a)}-\frac{7\,a^3d^4}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+10\,\frac{a^2cd^3}{b^3\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)-9\,\frac{ac^2d^2}{b^2\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+2\,\frac{c^3d}{b\sqrt{ab}}\arctan\left(\frac{bx}{\sqrt{ab}}\right)+\frac{c^4}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^4/(b*x^2+a)^2,x)
```

```
[Out] 1/5*d^4*x^5/b^2-2/3*d^4/b^3*x^3*a+4/3*d^3/b^2*x^3*c+3*d^4/b^4*a^2
*x-8*d^3/b^3*a*c*x+6*d^2/b^2*c^2*x+1/2/b^4*x*a^3/(b*x^2+a)*d^4-2/
b^3*x*a^2/(b*x^2+a)*c*d^3+3/b^2*x*a/(b*x^2+a)*c^2*d^2-2/b*x/(b*x^
2+a)*c^3*d+1/2*x/a/(b*x^2+a)*c^4-7/2/b^4*a^3/(a*b)^(1/2)*arctan(x
*b/(a*b)^(1/2))*d^4+10/b^3*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*c*d^3-9/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d^2+2/b/(
a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3*d+1/2/a/(a*b)^(1/2)*arctan
(x*b/(a*b)^(1/2))*c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.213542, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(ab^4c^4 + 4\,a^2b^3c^3d - 18\,a^3b^2c^2d^2 + 20\,a^4bcd^3 - 7\,a^5d^4 + \left(b^5c^4 + 4\,ab^4c^3d - 18\,a^2b^3c^2d^2 + 20\,a^3b^2cd^3 - 7\,a^4bd^4\right)x^2\right)\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20
*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*
c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*log(-(2*a*b*x - (b
*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(6*a*b^3*d^4*x^7 + 2*(20*a
*b^3*c*d^3 - 7*a^2*b^2*d^4)*x^5 + 10*(18*a*b^3*c^2*d^2 - 20*a^2*b
^2*c*d^3 + 7*a^3*b*d^4)*x^3 + 15*(b^4*c^4 - 4*a*b^3*c^3*d + 18*a^
2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 7*a^4*d^4)*x)*sqrt(-a*b))/((a*b^
5*x^2 + a^2*b^4)*sqrt(-a*b)), 1/30*(15*(a*b^4*c^4 + 4*a^2*b^3*c^3
*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 +
 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*
d^4)*x^2)*arctan(sqrt(a*b)*x/a) + (6*a*b^3*d^4*x^7 + 2*(20*a*b^3*
c*d^3 - 7*a^2*b^2*d^4)*x^5 + 10*(18*a*b^3*c^2*d^2 - 20*a^2*b^2*c*
d^3 + 7*a^3*b*d^4)*x^3 + 15*(b^4*c^4 - 4*a*b^3*c^3*d + 18*a^2*b^2
*c^2*d^2 - 20*a^3*b*c*d^3 + 7*a^4*d^4)*x)*sqrt(a*b))/((a*b^5*x^2
+ a^2*b^4)*sqrt(a*b))]
```

Sympy [A] time = 6.65158, size = 398, normalized size = 2.8

$$\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{2a^2b^4+2ab^5x^2}+\frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\,(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^3b^9}}\,(ad-bc)^3\,(7ad+bc)\log\left(\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}+\frac{d^4x^5}{5b^2}-\frac{x^3\left(2ad^4-4bcd^3\right)}{3b^3}+\frac{x\left(3a^2d^4-8abcd^3+6b^2c^2d^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3
*c**3*d + b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2) + sqrt(-1/(a**
3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(-1/(a**
3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*
d**3 + 18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)
/4 - sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(a**2*b
**4*sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**
4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d -
 b**4*c**4) + x)/4 + d**4*x**5/(5*b**2) - x**3*(2*a*d**4 - 4*b*c*
d**3)/(3*b**3) + x*(3*a**2*d**4 - 8*a*b*c*d**3 + 6*b**2*c**2*d**2
)/b**4
```

GIAC/XCAS [A] time = 0.244342, size = 297, normalized size = 2.09

$$\frac{\left(b^4c^4+4\,ab^3c^3d-18\,a^2b^2c^2d^2+20\,a^3bcd^3-7\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^4}+\frac{b^4c^4x-4\,ab^3c^3dx+6\,a^2b^2c^2d^2x-4\,a^3bcd^3x+a^4d^4x}{2\,(bx^2+a)ab^4}+\frac{3\,b^8d^4x^5+20\,b^8cd^3x^3-10\,ab^7d^4x^3+90\,b^8c^2d^2x-120\,ab^7cd^3x+45\,a^2b^6d^4x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^
3 - 7*a^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4
*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x
```

```
+ a^4*d^4*x)/((b*x^2 + a)*a*b^4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c
*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8*c^2*d^2*x - 120*a*b^7*c*d^3*
x + 45*a^2*b^6*d^4*x)/b^10
```

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=106

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi [A] time = 0.204606, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^2(2ad-3bc)\int\frac{1}{b^3}\,dx+\frac{d^3x^3}{3b^2}-\frac{x(ad-bc)^3}{2ab^3(a+bx^2)}+\frac{(ad-bc)^2(5ad+bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -d**2*(2*a*d - 3*b*c)*Integral(b**(-3), x) + d**3*x**3/(3*b**2) -
 x*(a*d - b*c)**3/(2*a*b**3*(a + b*x**2)) + (a*d - b*c)**2*(5*a*d
 + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*b**(7/2))
```

Mathematica [A] time = 0.100365, size = 106, normalized size = 1.

$$\frac{(5ad+bc)(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*
x)/(2*a*b^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Maple [B] time = 0.014, size = 205, normalized size = 1.9

$$\frac{d^3x^3}{3\,b^2}-2\,\frac{ad^3x}{b^3}+3\,\frac{d^2xc}{b^2}-\frac{a^2xd^3}{2\,b^3\,(bx^2+a)}+\frac{3\,acxd^2}{2\,b^2\,(bx^2+a)}-\frac{3\,xc^2d}{2\,b\,(bx^2+a)}+\frac{xc^3}{2\,a\,(bx^2+a)}+\frac{5\,a^2d^3}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{9\,acd^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^2d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^3}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^2,x)
```

```
[Out] 1/3*d^3*x^3/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/2/b^3*x*a^2/(b*x^2+
a)*d^3+3/2/b^2*x*a/(b*x^2+a)*c*d^2-3/2/b*x/(b*x^2+a)*c^2*d+1/2*x/
a/(b*x^2+a)*c^3+5/2/b^3*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d
^3-9/2/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^2+3/2/b/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d+1/2/a/(a*b)^(1/2)*arctan(x*
b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210617, size = 1, normalized size = 0.01

$$\left[\frac{3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a^2b^2cd^2+5\,a^3bd^3\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(2\,ab^2d\right.}{12\left(ab^4x^2+a^2b^3\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3
 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)
*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(2*a*b^2
*d^3*x^5 + 2*(9*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(b^3*c^3 - 3*a
*b^2*c^2*d + 9*a^2*b*c*d^2 - 5*a^3*d^3)*x)*sqrt(-a*b))/((a*b^4*x^
2 + a^2*b^3)*sqrt(-a*b)), 1/6*(3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9
*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c
*d^2 + 5*a^3*b*d^3)*x^2)*arctan(sqrt(a*b)*x/a) + (2*a*b^2*d^3*x^5
 + 2*(9*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(b^3*c^3 - 3*a*b^2*c^2
*d + 9*a^2*b*c*d^2 - 5*a^3*d^3)*x)*sqrt(a*b))/((a*b^4*x^2 + a^2*b
^3)*sqrt(a*b))]
```

Sympy [A] time = 5.17971, size = 313, normalized size = 2.95

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{2a^2b^3+2ab^4x^2}-\frac{\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}\left(ad-bc\right)^2\left(5ad+bc\right)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4}+\frac{d^3x^3}{3b^2}-\frac{x\left(2ad^3-3bcd^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2
*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(a*d - b*c)**2
*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2
*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d +
 b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d +
 b*c)*log(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d +
b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3
```

```
) + x)/4 + d**3*x**3/(3*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

GIAC/XCAS [A] time = 0.235221, size = 205, normalized size = 1.93

$$\frac{\left(b^3c^3 + 3\,ab^2c^2d - 9\,a^2bcd^2 + 5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3} + \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\,(bx^2 + a)ab^3} + \frac{b^4d^3x^3 + 9\,b^4cd^2x - 6\,ab^3d^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d
*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*
d^3*x^3 + 9*b^4*c*d^2*x - 6*a*b^3*d^3*x)/b^6
```

3.30 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a
*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2)
)
```

Rubi [A] time = 0.222816, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(bc-ad)(3ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a
*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2)
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\int \frac{1}{b^2}\,dx+\frac{x\,(ad-bc)^2}{2ab^2\,(a+bx^2)}-\frac{(ad-bc)\,(3ad+bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] d**2*Integral(b**(-2), x) + x*(a*d - b*c)**2/(2*a*b**2*(a + b*x**
2)) - (a*d - b*c)*(3*a*d + b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/
2)*b**(5/2))
```

Mathematica [A] time = 0.101111, size = 88, normalized size = 1.07

$$\frac{\left(-3a^2d^2+2abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2\left(a+bx^2\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2
 + 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)
*b^(5/2))
```

Maple [A] time = 0.012, size = 129, normalized size = 1.6

$$\frac{d^2x}{b^2}+\frac{axd^2}{2\,b^2\left(bx^2+a\right)}-\frac{cxd}{b\left(bx^2+a\right)}+\frac{xc^2}{2\,a\left(bx^2+a\right)}-\frac{3\,ad^2}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{cd}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^2}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^2,x)
```

```
[Out] d^2*x/b^2+1/2/b^2*x*a/(b*x^2+a)*d^2-1/b*x/(b*x^2+a)*c*d+1/2*x/a/(
b*x^2+a)*c^2-3/2/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2+1/
b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d+1/2/a/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.211786, size = 1, normalized size = 0.01

$$\left[-\frac{\left(ab^2c^2 + 2\,a^2bcd - 3\,a^3d^2 + \left(b^3c^2 + 2\,ab^2cd - 3\,a^2bd^2\right)x^2\right)\log\left(-\frac{2\,abx-\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right) - 2\left(2\,abd^2x^3 + \left(b^2c^2 - 2\,abcd + 3\right.\right.}{4\left(ab^3x^2 + a^2b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(
b*x^2 + a)) - 2*(2*a*b*d^2*x^3 + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2
)*x)*sqrt(-a*b))/((a*b^3*x^2 + a^2*b^2)*sqrt(-a*b)), 1/2*((a*b^2*
c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*x^2)*arctan(sqrt(a*b)*x/a) + (2*a*b*d^2*x^3 + (b^2*c^2 - 2*a
*b*c*d + 3*a^2*d^2)*x)*sqrt(a*b))/((a*b^3*x^2 + a^2*b^2)*sqrt(a*b
))]
```

Sympy [A] time = 3.68655, size = 236, normalized size = 2.88

$$\frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{2a^2b^2 + 2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\left(ad - bc\right)\left(3ad + bc\right)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}\left(ad - bc\right)\left(3ad + bc\right)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2} + x\right)}{4} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**
2) + sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**
2*sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2
*a*b*c*d - b**2*c**2) + x)/4 - sqrt(-1/(a**3*b**5))*(a*d - b*c)*(
3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*
d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 + d**2*x/b*
*2
```

GIAC/XCAS [A] time = 0.233258, size = 127, normalized size = 1.55

$$\frac{d^2x}{b^2} + \frac{\left(b^2c^2 + 2\,abcd - 3\,a^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\left(bx^2 + a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2
*x)/((b*x^2 + a)*a*b^2)
```

3.31 $\int \frac{c+dx^2}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=63

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi [A] time = 0.0622521, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2)^2,x]
```


```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Rubi in Sympy [A] time = 9.3458, size = 51, normalized size = 0.81

$$-\frac{x\left(ad-bc\right)}{2ab\left(a+bx^2\right)}+\frac{\left(ad+bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a*b*(a + b*x**2)) + (a*d + b*c)*atan(sqrt(b)*x/
sqrt(a))/(2*a**(3/2)*b**(3/2))
```

Mathematica [A] time = 0.0745221, size = 63, normalized size = 1.

$$\frac{(ad+bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{x(ad-bc)}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] -((-(b*c) + a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] time = 0.01, size = 68, normalized size = 1.1

$$-\frac{(ad-bc)\,x}{2\,ab\,(bx^2+a)}+\frac{d}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))*d+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.207925, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{-ab}(bc-ad)x+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{4\,(ab^2x^2+a^2b)\sqrt{-ab}},\ \frac{\sqrt{ab}(bc-ad)x+\left(abc+a^2d+\left(b^2c+abd\right)x^2\right)}{2\,(ab^2x^2+a^2b)\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-a*b)*(b*c - a*d)*x + (a*b*c + a^2*d + (b^2*c + a*b*
d)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)))/((a*
b^2*x^2 + a^2*b)*sqrt(-a*b)), 1/2*(sqrt(a*b)*(b*c - a*d)*x + (a*b
*c + a^2*d + (b^2*c + a*b*d)*x^2)*arctan(sqrt(a*b)*x/a))/((a*b^2*
x^2 + a^2*b)*sqrt(a*b))]
```

Sympy [A] time = 2.16558, size = 112, normalized size = 1.78

$$-\frac{x\left(ad-bc\right)}{2a^2b+2ab^2x^2}-\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad+bc\right)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b^3}}\left(ad+bc\right)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*
(a*d + b*c)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a*
*3*b**3))*(a*d + b*c)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

GIAC/XCAS [A] time = 0.234475, size = 77, normalized size = 1.22

$$\frac{\left(bc+ad\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}+\frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*
x - a*d*x)/((b*x^2 + a)*a*b)
```

3.32 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi [A] time = 0.212001, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}+\frac{bx}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rubi in Sympy [A] time = 41.2487, size = 94, normalized size = 0.87

$$\frac{d^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\left(ad-bc\right)^2}-\frac{bx}{2a\left(a+bx^2\right)\left(ad-bc\right)}-\frac{\sqrt{b}\left(3ad-bc\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] d**(3/2)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*(a*d - b*c)**2) - b*x/(
2*a*(a + b*x**2)*(a*d - b*c)) - sqrt(b)*(3*a*d - b*c)*atan(sqrt(b
)*x/sqrt(a))/(2*a**(3/2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.241835, size = 109, normalized size = 1.01

$$-\frac{\sqrt{b}(3ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(ad-bc)^2}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}-\frac{bx}{2a\left(a+bx^2\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] -(b*x)/(2*a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*
d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d
^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A] time = 0.016, size = 144, normalized size = 1.3

$$\frac{d^2}{(ad-bc)^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{bxd}{2\,(ad-bc)^2\,(bx^2+a)}+\frac{b^2xc}{2\,(ad-bc)^2\,a\,(bx^2+a)}-\frac{3\,bd}{2\,(ad-bc)^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^2c}{2\,(ad-bc)^2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c),x)
```

```
[Out] d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-1/2*b/(a*d-b*
c)^2*x/(b*x^2+a)*d+1/2*b^2/(a*d-b*c)^2*x/a/(b*x^2+a)*c-3/2*b/(a*d
-b*c)^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+1/2*b^2/(a*d-b*c)^2
/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.321059, size = 1, normalized size = 0.01

$$\left[-\frac{\left(abc-3\,a^2d+\left(b^2c-3\,abd\right)x^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,\left(abdx^2+a^2d\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-2\,\left(b^2c-abd\right.}{4\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((
b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d
)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*
(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*
c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/4*(4*(a*b*d*x^2 + a^2*d)
*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - (a*b*c - 3*a^2*d + (b^2*c
- 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c
- 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b
/a))) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d
/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/
2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(b*x
/(a*sqrt(b/a))) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(d*x/(c*s
qrt(d/c))) + (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*
d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

Sympy [A] time = 47.682, size = 2033, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] -b*x/(2*a**3*d - 2*a**2*b*c + x**2*(2*a**2*b*d - 2*a*b**2*c)) + s
qrt(-b/a**3)*(3*a*d - b*c)*log(x + (-a**9*c*d**6*(-b/a**3)**(3/2)
*(3*a*d - b*c)**3/(a*d - b*c)**6 + 5*a**8*b*c**2*d**5*(-b/a**3)**
(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + a**7*b**2*c**3*d**4*(
-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)**6) - 7*a**6*b**3
*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 + 8*a
**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)*
*6 - 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d - b*c)/(a*d - b*c)**2 - 7*a
**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)*
*6) + a**3*b**6*c**7*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d -
b*c)**6) - 27*a**3*b**2*c**2*d**3*sqrt(-b/a**3)*(3*a*d - b*c)/(2*
```

```
(a*d - b*c)**2) + 27*a**2*b**3*c**3*d**2*sqrt(-b/a**3)*(3*a*d - b
*c)/(2*(a*d - b*c)**2) - 9*a*b**4*c**4*d*sqrt(-b/a**3)*(3*a*d - b
*c)/(2*(a*d - b*c)**2) + b**5*c**5*sqrt(-b/a**3)*(3*a*d - b*c)/(2
*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*a*b**2*c*d**3 + b**3*c**2*d
**2))/(4*(a*d - b*c)**2) - sqrt(-b/a**3)*(3*a*d - b*c)*log(x + (a
**9*c*d**6*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(a*d - b*c)**6 - 5*a
**8*b*c**2*d**5*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a*d - b*c)*
*6) - a**7*b**2*c**3*d**4*(-b/a**3)**(3/2)*(3*a*d - b*c)**3/(2*(a
*d - b*c)**6) + 7*a**6*b**3*c**4*d**3*(-b/a**3)**(3/2)*(3*a*d - b
*c)**3/(a*d - b*c)**6 - 8*a**5*b**4*c**5*d**2*(-b/a**3)**(3/2)*(3
*a*d - b*c)**3/(a*d - b*c)**6 + 4*a**5*d**5*sqrt(-b/a**3)*(3*a*d
- b*c)/(a*d - b*c)**2 + 7*a**4*b**5*c**6*d*(-b/a**3)**(3/2)*(3*a*
d - b*c)**3/(2*(a*d - b*c)**6) - a**3*b**6*c**7*(-b/a**3)**(3/2)*
(3*a*d - b*c)**3/(2*(a*d - b*c)**6) + 27*a**3*b**2*c**2*d**3*sqrt
(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - 27*a**2*b**3*c**3*d*
*2*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) + 9*a*b**4*c**4
*d*sqrt(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2) - b**5*c**5*sqr
t(-b/a**3)*(3*a*d - b*c)/(2*(a*d - b*c)**2))/(12*a**2*b*d**4 - 7*
a*b**2*c*d**3 + b**3*c**2*d**2))/(4*(a*d - b*c)**2) + sqrt(-d**3/
c)*log(x + (-8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 20*a
**8*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**7*b**2*c**
3*d**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 56*a**6*b**3*c**4*d**3*(
-d**3/c)**(3/2)/(a*d - b*c)**6 + 64*a**5*b**4*c**5*d**2*(-d**3/c)
**(3/2)/(a*d - b*c)**6 - 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2
 - 28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 4*a**3*b
**6*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 27*a**3*b**2*c**2*d**3
*sqrt(-d**3/c)/(a*d - b*c)**2 + 27*a**2*b**3*c**3*d**2*sqrt(-d**3
/c)/(a*d - b*c)**2 - 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2
 + b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*
b**2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2) - sqrt(-d**3/c)
*log(x + (8*a**9*c*d**6*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 20*a**8
*b*c**2*d**5*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**7*b**2*c**3*d
**4*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 56*a**6*b**3*c**4*d**3*(-d*
*3/c)**(3/2)/(a*d - b*c)**6 - 64*a**5*b**4*c**5*d**2*(-d**3/c)**(
3/2)/(a*d - b*c)**6 + 8*a**5*d**5*sqrt(-d**3/c)/(a*d - b*c)**2 +
28*a**4*b**5*c**6*d*(-d**3/c)**(3/2)/(a*d - b*c)**6 - 4*a**3*b**6
*c**7*(-d**3/c)**(3/2)/(a*d - b*c)**6 + 27*a**3*b**2*c**2*d**3*sq
rt(-d**3/c)/(a*d - b*c)**2 - 27*a**2*b**3*c**3*d**2*sqrt(-d**3/c)
/(a*d - b*c)**2 + 9*a*b**4*c**4*d*sqrt(-d**3/c)/(a*d - b*c)**2 -
b**5*c**5*sqrt(-d**3/c)/(a*d - b*c)**2)/(12*a**2*b*d**4 - 7*a*b**
2*c*d**3 + b**3*c**2*d**2))/(2*(a*d - b*c)**2)
```

GIAC/XCAS [A] time = 0.236578, size = 163, normalized size = 1.51

$$\frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{cd}} + \frac{(b^2c - 3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2 - 2\,a^2bcd + a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc - a^2d)(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c
*d)) + 1/2*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 -
2*a^2*b*c*d + a^3*d^2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x
^2 + a))
```

3.33 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx$

Optimal. Leaf size=167

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{dx(ad+bc)}{2ac\,(c+dx^2)\,(bc-ad)^2}$$

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5
*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3
)
```

Rubi [A] time = 0.454722, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3}+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}+\frac{bx}{2a\,(a+bx^2)\,(c+dx^2)\,(bc-ad)}+\frac{dx(ad+bc)}{2ac\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*
(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5
*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3
)
```

Rubi in Sympy [A] time = 95.5205, size = 141, normalized size = 0.84

$$\frac{dx}{2c\,(a+bx^2)\,(c+dx^2)\,(ad-bc)}+\frac{d^{\frac{3}{2}}\,(ad-5bc)\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{\frac{3}{2}}\,(ad-bc)^3}+\frac{bx\,(ad+bc)}{2ac\,(a+bx^2)\,(ad-bc)^2}+\frac{b^{\frac{3}{2}}\,(5ad-bc)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*(a + b*x**2)*(c + d*x**2)*(a*d - b*c)) + d**(3/2)*(a*d -
 5*b*c)*atan(sqrt(d)*x/sqrt(c))/(2*c**(3/2)*(a*d - b*c)**3) + b*x
*(a*d + b*c)/(2*a*c*(a + b*x**2)*(a*d - b*c)**2) + b**(3/2)*(5*a*
d - b*c)*atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.565728, size = 136, normalized size = 0.81

$$\frac{1}{2}\left(\frac{b^{3/2}(5ad-bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(ad-bc)^3}+\frac{x(bc-ad)\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*
(-(b*c) + a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^
2 + c*d*x^2)) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A] time = 0.02, size = 238, normalized size = 1.4

$$\frac{d^3xa}{2\,(ad-bc)^3\,c\,(dx^2+c)}-\frac{d^2xb}{2\,(ad-bc)^3\,(dx^2+c)}+\frac{ad^3}{2\,(ad-bc)^3\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{5\,d^2b}{2\,(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{b^2xd}{2\,(ad-bc)^3\,(bx^2+a)}-\frac{b^3xc}{2\,(ad-bc)^3\,a\,(bx^2+a)}+\frac{5\,db^2}{2\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b^3c}{2\,(ad-bc)^3\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^3/(a*d-b*c)^3/c*x/(d*x^2+c)*a-1/2*d^2/(a*d-b*c)^3*x/(d*x^2+
c)*b+1/2*d^3/(a*d-b*c)^3/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-
5/2*d^2/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+1/2*b^2
/(a*d-b*c)^3*x/(b*x^2+a)*d-1/2*b^3/(a*d-b*c)^3*x/a/(b*x^2+a)*c+5/
2*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d-1/2*b^3/(
a*d-b*c)^3/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.929467, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d
+ (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*
a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b
*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-
d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3
 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b
^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 -
2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*
c^2*d - a^2*b*d^3)*x^3 + 2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*
c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3
)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (a*b^2*c^3 - 5*a^2*b
*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2
*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3
*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c
^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a
^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3
- a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + 2*(a*b^2*
c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3
- 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqr
t(b/a))) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^
3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 -
a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*
c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*
c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a
^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d
 - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a
*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*
sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + (5*a^2*b*c^2*d - a^3*c*d^2
```

```
+ (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d^
2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (b^3*c^3
- a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^
2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^
3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2
*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 0.388294, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^2),x, algorithm="giac")

[Out] Done
```

3.34 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal. Leaf size=230

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*
c + 3*a*d)*x)/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c
 - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^4)
+ (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.723006, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}+\frac{dx(4bc-ad)(3ad+bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}+\frac{dx(ad+2bc)}{4ac\left(c+dx^2\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(
2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*
c + 3*a*d)*x)/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c
 - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^4)
+ (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.983252, size = 197, normalized size = 0.86

$$\frac{1}{8}\left(\frac{4b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}-\frac{4b^3x}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{d^2x(11bc-3ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{2d^2x}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c
- a*d)^2*(c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*
d)^3*(c + d*x^2)) + (4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*
c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d
)^4))/8
```

Maple [A] time = 0.023, size = 403, normalized size = 1.8

$$\frac{3\,d^5x^3a^2}{8\,(ad-bc)^4\,(dx^2+c)^2\,c^2}-\frac{7\,d^4x^3ab}{4\,(ad-bc)^4\,(dx^2+c)^2\,c}+\frac{11\,d^3x^3b^2}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{5\,d^4xa^2}{8\,(ad-bc)^4\,(dx^2+c)^2\,c}-\frac{9\,d^3xab}{4\,(ad-bc)^4\,(dx^2+c)^2}+\frac{13\,cd^2xb^2}{8\,(ad-bc)^4\,(dx^2+c)^2}+\frac{3\,d^4a^2}{8\,(ad-bc)^4\,c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,abd^3}{4\,(ad-bc)^4\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{35\,d^2b^2}{8\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{b^3xd}{2\,(ad-bc)^4\,(bx^2+a)}+\frac{b^4xc}{2\,(ad-bc)^4\,a\,(bx^2+a)}-\frac{7\,b^3d}{2\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{b^4c}{2\,(ad-bc)^4\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^5/(a*d-b*c)^4/(d*x^2+c)^2/c^2*x^3*a^2-7/4*d^4/(a*d-b*c)^4/(
d*x^2+c)^2/c*x^3*a*b+11/8*d^3/(a*d-b*c)^4/(d*x^2+c)^2*x^3*b^2+5/8
*d^4/(a*d-b*c)^4/(d*x^2+c)^2/c*x*a^2-9/4*d^3/(a*d-b*c)^4/(d*x^2+c
)^2*x*a*b+13/8*d^2/(a*d-b*c)^4/(d*x^2+c)^2*c*x*b^2+3/8*d^4/(a*d-b
```

```
*c)^4/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-7/4*d^3/(a*d-b*
c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+35/8*d^2/(a*d-b*c)
^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-1/2*b^3/(a*d-b*c)^4*x/
(b*x^2+a)*d+1/2*b^4/(a*d-b*c)^4*x/a/(b*x^2+a)*c-7/2*b^3/(a*d-b*c)
^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+1/2*b^4/(a*d-b*c)^4/a/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.16452, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*
a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2
*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*
c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a
*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d
- 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a
^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*
x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a
^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2
*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c)
 - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^
3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b
^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a
*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2
 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*
c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 +
 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*(
(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^
3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
```

```
 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b
*d^5)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4
 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3
*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))
) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d
^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^
4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a
)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 -
4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d
^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5
*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6
*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*
b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4
*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^
4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
 + 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3
- 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*
c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*
(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^
6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b
^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b/a)*arcta
n(b*x/(a*sqrt(b/a))) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a
^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*
x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a
^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2
*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c)
 - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^
3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b
^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a
*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2
 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*
c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 +
 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*(
(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^
3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^
4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^
2 - 7*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^
2*c^3*d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + (35*a^2*b^2
*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 1
4*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^
2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4*d + 56
*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)
*arctan(d*x/(c*sqrt(d/c))) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*
b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4
*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^
4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^
4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c
^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5
+ a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6
*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.240079, size = 448, normalized size = 1.95

$$\frac{b^3x}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)(bx^2+a)}+\frac{\left(b^4c-7\,ab^3d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}}+\frac{\left(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}}+\frac{11\,bcd^3x^3-3\,ad^4x^3+13\,bc^2d^2x-5\,acd^3x}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)(dx^2+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3
)*(b*x^2 + a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2
*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2
*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/8*(11*b*
c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^3*c^5
 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

3.35 $\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=196

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5(a+bx^2)}+\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(bc-ad)^5}{4ab^5(a+bx^2)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}$$

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c -
 3*a*d)*x^3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a
*b^5*(a + b*x^2)^2) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b
^5*(a + b*x^2)) + ((b*c - a*d)^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2
*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(11/2))
```

Rubi [A] time = 0.476364, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5(a+bx^2)}+\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(bc-ad)^5}{4ab^5(a+bx^2)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c -
 3*a*d)*x^3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a
*b^5*(a + b*x^2)^2) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b
^5*(a + b*x^2)) + ((b*c - a*d)^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2
*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(11/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(6a^2d^2-15abcd+10b^2c^2\right)\int\frac{1}{b^5}\,dx+\frac{d^5x^5}{5b^3}-\frac{d^4x^3\left(3ad-5bc\right)}{3b^4}-\frac{x\left(ad-bc\right)^5}{4ab^5\left(a+bx^2\right)^2}+\frac{x\left(ad-bc\right)^4\left(17ad+3bc\right)}{8a^2b^5\left(a+bx^2\right)}-\frac{\left(ad-bc\right)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] d**3*(6*a**2*d**2 - 15*a*b*c*d + 10*b**2*c**2)*Integral(b**(-5),
x) + d**5*x**5/(5*b**3) - d**4*x**3*(3*a*d - 5*b*c)/(3*b**4) - x*
(a*d - b*c)**5/(4*a*b**5*(a + b*x**2)**2) + x*(a*d - b*c)**4*(17*
a*d + 3*b*c)/(8*a**2*b**5*(a + b*x**2)) - (a*d - b*c)**3*(63*a**2
*d**2 + 14*a*b*c*d + 3*b**2*c**2)*atan(sqrt(b)*x/sqrt(a))/(8*a**(
5/2)*b**(11/2))
```

Mathematica [A] time = 0.201818, size = 196, normalized size = 1.

$$\frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^3x\left(6a^2d^2-15abcd+10b^2c^2\right)}{b^5}+\frac{\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^2\right)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c -
 3*a*d)*x^3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a
*b^5*(a + b*x^2)^2) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b
^5*(a + b*x^2)) + ((b*c - a*d)^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2
*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(11/2))
```

Maple [B] time = 0.019, size = 484, normalized size = 2.5

$$\frac{d^5x^5}{5\,b^3}-\frac{d^5x^3a}{b^4}+\frac{5\,d^4x^3c}{3\,b^3}+6\,\frac{a^2d^5x}{b^5}-15\,\frac{acd^4x}{b^4}+10\,\frac{c^2d^3x}{b^3}+\frac{17\,a^3x^3d^5}{8\,b^4\left(bx^2+a\right)^2}-\frac{65\,a^2x^3cd^4}{8\,b^3\left(bx^2+a\right)^2}+\frac{45\,ax^3c^2d^3}{4\,b^2\left(bx^2+a\right)^2}-\frac{25\,x^3c^3d^2}{4\,b\left(bx^2+a\right)^2}+\frac{5\,x^3c^4d}{8\left(bx^2+a\right)^2a}+\frac{3\,bx^3c^5}{8\left(bx^2+a\right)^2a^2}+\frac{15\,a^4xd^5}{8\,b^5\left(bx^2+a\right)^2}-\frac{55\,a^3cxd^4}{8\,b^4\left(bx^2+a\right)^2}+\frac{35\,a^2c^2xd^3}{4\,b^3\left(bx^2+a\right)^2}-\frac{15\,ac^3xd^2}{4\,b^2\left(bx^2+a\right)^2}-\frac{5\,xc^4d}{8\,b\left(bx^2+a\right)^2}+\frac{5\,xc^5}{8\left(bx^2+a\right)^2a}-\frac{63\,a^3d^5}{8\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{175\,a^2cd^4}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{75\,ac^2d^3}{4\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{15\,c^3d^2}{4\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,c^4d}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^5}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^5/(b*x^2+a)^3,x)
```

```
[Out] 1/5*d^5*x^5/b^3-d^5/b^4*x^3*a+5/3*d^4/b^3*x^3*c+6*d^5/b^5*a^2*x-1
5*d^4/b^4*a*c*x+10*d^3/b^3*c^2*x+17/8/b^4/(b*x^2+a)^2*a^3*x^3*d^5
-65/8/b^3/(b*x^2+a)^2*a^2*x^3*c*d^4+45/4/b^2/(b*x^2+a)^2*a*x^3*c^
2*d^3-25/4/b/(b*x^2+a)^2*x^3*c^3*d^2+5/8/(b*x^2+a)^2/a*x^3*c^4*d+
3/8*b/(b*x^2+a)^2/a^2*x^3*c^5+15/8/b^5/(b*x^2+a)^2*a^4*x*d^5-55/8
/b^4/(b*x^2+a)^2*a^3*x*c*d^4+35/4/b^3/(b*x^2+a)^2*a^2*x*c^2*d^3-1
5/4/b^2/(b*x^2+a)^2*a*x*c^3*d^2-5/8/b/(b*x^2+a)^2*x*c^4*d+5/8/(b*
x^2+a)^2/a*x*c^5-63/8/b^5*a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
*d^5+175/8/b^4*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^4-75/4
/b^3*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d^3+15/4/b^2/(a*b)
^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3*d^2+5/8/b/a/(a*b)^(1/2)*arctan
(x*b/(a*b)^(1/2))*c^4*d+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))*c^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219103, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d + 30*a^4*b^3*c^3*d^2
 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (3*b^7*c^
5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 17
5*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^
5*c^4*d + 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*
c*d^4 - 63*a^6*b*d^5)*x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b)
)/(b*x^2 + a)) - 2*(24*a^2*b^4*d^5*x^9 + 8*(25*a^2*b^4*c*d^4 - 9*
a^3*b^3*d^5)*x^7 + 8*(150*a^2*b^4*c^2*d^3 - 175*a^3*b^3*c*d^4 + 6
3*a^4*b^2*d^5)*x^5 + 5*(9*b^6*c^5 + 15*a*b^5*c^4*d - 150*a^2*b^4*
c^3*d^2 + 750*a^3*b^3*c^2*d^3 - 875*a^4*b^2*c*d^4 + 315*a^5*b*d^5
)*x^3 + 15*(5*a*b^5*c^5 - 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 +
150*a^4*b^2*c^2*d^3 - 175*a^5*b*c*d^4 + 63*a^6*d^5)*x)*sqrt(-a*b)
```

```
)/((a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)*sqrt(-a*b)), 1/120*(15
*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d + 30*a^4*b^3*c^3*d^2 - 150*a^5*
b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (3*b^7*c^5 + 5*a*b^6
*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 175*a^4*b^3*c
*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d + 3
0*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*
a^6*b*d^5)*x^2)*arctan(sqrt(a*b)*x/a) + (24*a^2*b^4*d^5*x^9 + 8*(
25*a^2*b^4*c*d^4 - 9*a^3*b^3*d^5)*x^7 + 8*(150*a^2*b^4*c^2*d^3 -
175*a^3*b^3*c*d^4 + 63*a^4*b^2*d^5)*x^5 + 5*(9*b^6*c^5 + 15*a*b^5
*c^4*d - 150*a^2*b^4*c^3*d^2 + 750*a^3*b^3*c^2*d^3 - 875*a^4*b^2*
c*d^4 + 315*a^5*b*d^5)*x^3 + 15*(5*a*b^5*c^5 - 5*a^2*b^4*c^4*d -
30*a^3*b^3*c^3*d^2 + 150*a^4*b^2*c^2*d^3 - 175*a^5*b*c*d^4 + 63*a
^6*d^5)*x)*sqrt(a*b))/((a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)*sq
rt(a*b))]
```

Sympy [A] time = 21.4133, size = 614, normalized size = 3.13

$$\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{1}{a^5b^{11}}}\,(ad-bc)^3\left(63a^2d^2+14abcd+3b^2c^2\right)\log\left(\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+x\right)}{16}$$
$$+\frac{x^3\left(17a^5bd^5-65a^4b^2cd^4+90a^3b^3c^2d^3-50a^2b^4c^3d^2+5ab^5c^4d+3b^6c^5\right)+x\left(15a^6d^5-55a^5bcd^4+70a^4b^2c^2d^3-30a^3b^3c^3\right.}{8a^4b^5+16a^3b^6x^2+8a^2b^7x^4}$$
$$+\frac{d^5x^5}{5b^3}-\frac{x^3\left(3ad^5-5bcd^4\right)}{3b^4}+\frac{x\left(6a^2d^5-15abcd^4+10b^2c^2d^3\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d +
 3*b**2*c**2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d - b*c)**3
*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a*
*4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 -
5*a*b**4*c**4*d - 3*b**5*c**5) + x)/16 - sqrt(-1/(a**5*b**11))*(a
*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(a**3*b
**5*sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*d**2 + 14*a*b*c
*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**4*b*c*d**4 + 150*a**3*b*
*2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d - 3*b**5*
c**5) + x)/16 + (x**3*(17*a**5*b*d**5 - 65*a**4*b**2*c*d**4 + 90*
a**3*b**3*c**2*d**3 - 50*a**2*b**4*c**3*d**2 + 5*a*b**5*c**4*d +
3*b**6*c**5) + x*(15*a**6*d**5 - 55*a**5*b*c*d**4 + 70*a**4*b**2*
c**2*d**3 - 30*a**3*b**3*c**3*d**2 - 5*a**2*b**4*c**4*d + 5*a*b**
5*c**5))/(8*a**4*b**5 + 16*a**3*b**6*x**2 + 8*a**2*b**7*x**4) + d
**5*x**5/(5*b**3) - x**3*(3*a*d**5 - 5*b*c*d**4)/(3*b**4) + x*(6*
a**2*d**5 - 15*a*b*c*d**4 + 10*b**2*c**2*d**3)/b**5
```

GIAC/XCAS [A] time = 0.232694, size = 459, normalized size = 2.34

$$\frac{\left(3\,b^{5}c^{5}+5\,ab^{4}c^{4}d+30\,a^{2}b^{3}c^{3}d^{2}-150\,a^{3}b^{2}c^{2}d^{3}+175\,a^{4}bcd^{4}-63\,a^{5}d^{5}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^{2}b^{5}}+\frac{3\,b^{6}c^{5}x^{3}+5\,ab^{5}c^{4}dx^{3}-50\,a^{2}b^{4}c^{3}d^{2}x^{3}+90\,a^{3}b^{3}c^{2}d^{3}x^{3}-65\,a^{4}b^{2}cd^{4}x^{3}+17\,a^{5}bd^{5}x^{3}+5\,ab^{5}c^{5}x-5\,a^{2}b^{4}c^{4}dx-30\,a^{3}b^{3}c^{3}d}{8\,(bx^{2}+a)^{2}a^{2}b^{5}}+\frac{3\,b^{12}d^{5}x^{5}+25\,b^{12}cd^{4}x^{3}-15\,ab^{11}d^{5}x^{3}+150\,b^{12}c^{2}d^{3}x-225\,ab^{11}cd^{4}x+90\,a^{2}b^{10}d^{5}x}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^5/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2
*c^2*d^3 + 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a^2*b^5) + 1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a
^2*b^4*c^3*d^2*x^3 + 90*a^3*b^3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^
3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x - 5*a^2*b^4*c^4*d*x - 30*a^3
*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^5*b*c*d^4*x + 15*a^6
*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5 + 25*b^12*
c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11*c
*d^4*x + 90*a^2*b^10*d^5*x)/b^15
```

3.36 $\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=160

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4(a+bx^2)}+\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4(a+bx^2)^2}+\frac{d^4x^3}{3b^3}$$

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*
x)/(4*a*b^4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(
8*a^2*b^4*(a + b*x^2)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d +
 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rubi [A] time = 0.416251, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4(a+bx^2)}+\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4(a+bx^2)^2}+\frac{d^4x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*
x)/(4*a*b^4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(
8*a^2*b^4*(a + b*x^2)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d +
 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-d^3\left(3ad-4bc\right)\int\frac{1}{b^4}\,dx+\frac{d^4x^3}{3b^3}+\frac{x\left(ad-bc\right)^4}{4ab^4\left(a+bx^2\right)^2}-\frac{x\left(ad-bc\right)^3\left(13ad+3bc\right)}{8a^2b^4\left(a+bx^2\right)}+\frac{\left(ad-bc\right)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] -d**3*(3*a*d - 4*b*c)*Integral(b**(-4), x) + d**4*x**3/(3*b**3) +
 x*(a*d - b*c)**4/(4*a*b**4*(a + b*x**2)**2) - x*(a*d - b*c)**3*(
13*a*d + 3*b*c)/(8*a**2*b**4*(a + b*x**2)) + (a*d - b*c)**2*(35*a
**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*atan(sqrt(b)*x/sqrt(a))/(8*a
**(5/2)*b**(9/2))
```

Mathematica [A] time = 0.155825, size = 160, normalized size = 1.

$$\frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4\left(a+bx^2\right)}+\frac{(bc-ad)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}+\frac{d^3x(4bc-3ad)}{b^4}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^2\right)^2}+\frac{d^4x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*
x)/(4*a*b^4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(
8*a^2*b^4*(a + b*x^2)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d +
 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Maple [B] time = 0.016, size = 367, normalized size = 2.3

$$\frac{d^4x^3}{3\,b^3}-3\,\frac{ad^4x}{b^4}+4\,\frac{d^3xc}{b^3}-\frac{13\,a^2x^3d^4}{8\,b^3\left(bx^2+a\right)^2}+\frac{9\,ax^3cd^3}{2\,b^2\left(bx^2+a\right)^2}-\frac{15\,x^3c^2d^2}{4\,b\left(bx^2+a\right)^2}+\frac{x^3c^3d}{2\,\left(bx^2+a\right)^2a}+\frac{3\,bx^3c^4}{8\,\left(bx^2+a\right)^2a^2}-\frac{11\,a^3xd^4}{8\,b^4\left(bx^2+a\right)^2}+\frac{7\,a^2cxd^3}{2\,b^3\left(bx^2+a\right)^2}-\frac{9\,ac^2xd^2}{4\,b^2\left(bx^2+a\right)^2}-\frac{xc^3d}{2\,b\left(bx^2+a\right)^2}+\frac{5\,xc^4}{8\,\left(bx^2+a\right)^2a}+\frac{35\,a^2d^4}{8\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{15\,acd^3}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,c^2d^2}{4\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{c^3d}{2\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^4}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^4/(b*x^2+a)^3,x)
```

```
[Out] 1/3*d^4*x^3/b^3-3*d^4/b^4*a*x+4*d^3/b^3*x*c-13/8/b^3/(b*x^2+a)^2*
a^2*x^3*d^4+9/2/b^2/(b*x^2+a)^2*a*x^3*c*d^3-15/4/b/(b*x^2+a)^2*x^
3*c^2*d^2+1/2/(b*x^2+a)^2/a*x^3*c^3*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c
^4-11/8/b^4/(b*x^2+a)^2*a^3*x*d^4+7/2/b^3/(b*x^2+a)^2*a^2*x*c*d^3
-9/4/b^2/(b*x^2+a)^2*a*x*c^2*d^2-1/2/b/(b*x^2+a)^2*x*c^3*d+5/8/(b
*x^2+a)^2/a*x*c^4+35/8/b^4*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*d^4-15/2/b^3*a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d^3+9/4/b^
```

```
2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d^2+1/2/b/a/(a*b)^(1/2)
*arctan(x*b/(a*b)^(1/2))*c^3*d+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*
b)^(1/2))*c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217546, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^3,x, algorithm="fricas")

[Out] [1/48*(3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 -
60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2
*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^
5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 +
 35*a^5*b*d^4)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2
 + a)) + 2*(8*a^2*b^3*d^4*x^7 + 8*(12*a^2*b^3*c*d^3 - 7*a^3*b^2*d
^4)*x^5 + (9*b^5*c^4 + 12*a*b^4*c^3*d - 90*a^2*b^3*c^2*d^2 + 300*
a^3*b^2*c*d^3 - 175*a^4*b*d^4)*x^3 + 3*(5*a*b^4*c^4 - 4*a^2*b^3*c
^3*d - 18*a^3*b^2*c^2*d^2 + 60*a^4*b*c*d^3 - 35*a^5*d^4)*x)*sqrt(
-a*b))/((a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)*sqrt(-a*b)), 1/24
*(3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^
5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*
c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4
 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a
^5*b*d^4)*x^2)*arctan(sqrt(a*b)*x/a) + (8*a^2*b^3*d^4*x^7 + 8*(12
*a^2*b^3*c*d^3 - 7*a^3*b^2*d^4)*x^5 + (9*b^5*c^4 + 12*a*b^4*c^3*d
 - 90*a^2*b^3*c^2*d^2 + 300*a^3*b^2*c*d^3 - 175*a^4*b*d^4)*x^3 +
3*(5*a*b^4*c^4 - 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 60*a^4*b*
c*d^3 - 35*a^5*d^4)*x)*sqrt(a*b))/((a^2*b^6*x^4 + 2*a^3*b^5*x^2 +
 a^4*b^4)*sqrt(a*b))]
```

Sympy [A] time = 13.583, size = 513, normalized size = 3.21

$$-\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^5b^9}}\,(ad-bc)^2\left(35a^2d^2+10abcd+3b^2c^2\right)\log\left(\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16}$$
$$-\frac{x^3\left(13a^4bd^4-36a^3b^2cd^3+30a^2b^3c^2d^2-4ab^4c^3d-3b^5c^4\right)+x\left(11a^5d^4-28a^4bcd^3+18a^3b^2c^2d^2+4a^2b^3c^3d-5ab^4c^4\right)}{8a^4b^4+16a^3b^5x^2+8a^2b^6x^4}$$
$$+\frac{d^4x^3}{3b^3}-\frac{x\left(3ad^4-4bcd^3\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d +
 3*b**2*c**2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*
(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3
*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**
4) + x)/16 + sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 +
10*a*b*c*d + 3*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d
 - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**
4 - 60*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d +
 3*b**4*c**4) + x)/16 - (x**3*(13*a**4*b*d**4 - 36*a**3*b**2*c*d*
*3 + 30*a**2*b**3*c**2*d**2 - 4*a*b**4*c**3*d - 3*b**5*c**4) + x*
(11*a**5*d**4 - 28*a**4*b*c*d**3 + 18*a**3*b**2*c**2*d**2 + 4*a**
2*b**3*c**3*d - 5*a*b**4*c**4))/(8*a**4*b**4 + 16*a**3*b**5*x**2
+ 8*a**2*b**6*x**4) + d**4*x**3/(3*b**3) - x*(3*a*d**4 - 4*b*c*d*
*3)/b**4
```

GIAC/XCAS [A] time = 0.233269, size = 343, normalized size = 2.14

$$\frac{\left(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4}$$
$$+\frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b^2cd^3x^3-13\,a^4bd^4x^3+5\,ab^4c^4x-4\,a^2b^3c^3dx-18\,a^3b^2c^2d^2x+28\,a^4bcd^3x}{8\,(bx^2+a)^2a^2b^4}$$
$$+\frac{b^6d^4x^3+12\,b^6cd^3x-9\,ab^5d^4x}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*
d^3 + 35*a^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8
*(3*b^5*c^4*x^3 + 4*a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36
*a^3*b^2*c*d^3*x^3 - 13*a^4*b*d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3
*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28*a^4*b*c*d^3*x - 11*a^5*d^4*x
)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3 + 12*b^6*c*d^3*x - 9
*a*b^5*d^4*x)/b^9
```

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3} dx$

Optimal. Leaf size=130

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3(a+bx^2)}+\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3(a+bx^2)^2}+\frac{d^3x}{b^3}$$

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c
 - a*d)^2*(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*
d)*(4*a^2*d^2 + (b*c + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(7/2))
```

Rubi [A] time = 0.361539, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3(a+bx^2)}+\frac{3(bc-ad)\left(4a^2d^2+(ad+bc)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3(a+bx^2)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c
 - a*d)^2*(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*
d)*(4*a^2*d^2 + (b*c + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\int \frac{1}{b^3}\,dx-\frac{x\,(ad-bc)^3}{4ab^3\,(a+bx^2)^2}+\frac{3x\,(ad-bc)^2\,(3ad+bc)}{8a^2b^3\,(a+bx^2)}-\frac{3\,(ad-bc)\left(4a^2d^2+(ad+bc)^2\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] d**3*Integral(b**(-3), x) - x*(a*d - b*c)**3/(4*a*b**3*(a + b*x**
2)**2) + 3*x*(a*d - b*c)**2*(3*a*d + b*c)/(8*a**2*b**3*(a + b*x**
2)) - 3*(a*d - b*c)*(4*a**2*d**2 + (a*d + b*c)**2)*atan(sqrt(b)*x
/sqrt(a))/(8*a**(5/2)*b**(7/2))
```

Mathematica [A] time = 0.13165, size = 139, normalized size = 1.07

$$\frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3\left(a+bx^2\right)}+\frac{3\left(-5a^3d^3+3a^2bcd^2+ab^2c^2d+b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^2\right)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c
 - a*d)^2*(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3
+ a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^(7/2))
```

Maple [B] time = 0.015, size = 266, normalized size = 2.1

$$\frac{d^3x}{b^3}+\frac{9\,ax^3d^3}{8\,b^2\left(bx^2+a\right)^2}-\frac{15\,x^3cd^2}{8\,b\left(bx^2+a\right)^2}+\frac{3\,x^3c^2d}{8\,\left(bx^2+a\right)^2a}+\frac{3\,bx^3c^3}{8\,\left(bx^2+a\right)^2a^2}+\frac{7\,a^2xd^3}{8\,b^3\left(bx^2+a\right)^2}-\frac{9\,acxd^2}{8\,b^2\left(bx^2+a\right)^2}-\frac{3\,xc^2d}{8\,b\left(bx^2+a\right)^2}+\frac{5\,xc^3}{8\,\left(bx^2+a\right)^2a}-\frac{15\,ad^3}{8\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9\,cd^2}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^2d}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^3}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^3,x)
```

```
[Out] d^3*x/b^3+9/8/b^2/(b*x^2+a)^2*a*x^3*d^3-15/8/b/(b*x^2+a)^2*x^3*c*
d^2+3/8/(b*x^2+a)^2/a*x^3*c^2*d+3/8*b/(b*x^2+a)^2/a^2*x^3*c^3+7/8
/b^3/(b*x^2+a)^2*a^2*x*d^3-9/8/b^2/(b*x^2+a)^2*a*x*c*d^2-3/8/b/(b
*x^2+a)^2*x*c^2*d+5/8/(b*x^2+a)^2/a*x*c^3-15/8/b^3*a/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*d^3+9/8/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))*c*d^2+3/8/b/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2*d+3/
8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218977, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(a^2b^3c^3+a^3b^2c^2d+3\,a^4bcd^2-5\,a^5d^3+\left(b^5c^3+ab^4c^2d+3\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^4+2\left(ab^4c^3+a^2b^3c^2d+3\,a^3b^2cd^2-5\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^
3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^4
 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*
x^2)*log(-(2*a*b*x - (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(8*
a^2*b^2*d^3*x^5 + (3*b^4*c^3 + 3*a*b^3*c^2*d - 15*a^2*b^2*c*d^2 +
 25*a^3*b*d^3)*x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d
^2 + 15*a^4*d^3)*x)*sqrt(-a*b))/((a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)*sqrt(-a*b)), 1/8*(3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*
b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 -
5*a^3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d
^2 - 5*a^4*b*d^3)*x^2)*arctan(sqrt(a*b)*x/a) + (8*a^2*b^2*d^3*x^5
 + (3*b^4*c^3 + 3*a*b^3*c^2*d - 15*a^2*b^2*c*d^2 + 25*a^3*b*d^3)*
x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 15*a^4*d^3
)*x)*sqrt(a*b))/((a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)*sqrt(a*b
))]
```

Sympy [A] time = 8.40146, size = 422, normalized size = 3.25

$$\frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{a^5b^7}}\,(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16} + \frac{x^3\left(9a^3bd^3-15a^2b^2cd^2+3ab^3c^2d+3b^4c^3\right)+x\left(7a^4d^3-9a^3bcd^2-3a^2b^2c^2d+5ab^3c^3\right)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{d^3x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**
2*c**2)*log(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2
*d**2 + 2*a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 -
3*a*b**2*c**2*d - 3*b**3*c**3) + x)/16 - 3*sqrt(-1/(a**5*b**7))*(
a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*log(3*a**3*b**3*
sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*
c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*
c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*a*
b**3*c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3
*a**2*b**2*c**2*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x
**2 + 8*a**2*b**5*x**4) + d**3*x/b**3
```

GIAC/XCAS [A] time = 0.249606, size = 240, normalized size = 1.85

$$\frac{d^3x}{b^3} + \frac{3\left(b^3c^3 + ab^2c^2d + 3\,a^2bcd^2 - 5\,a^3d^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4c^3x^3 + 3\,ab^3c^2dx^3 - 15\,a^2b^2cd^2x^3 + 9\,a^3bd^3x^3 + 5\,ab^3c^3x - 3\,a^2b^2c^2dx - 9\,a^3bcd^2x + 7\,a^4d^3x}{8\,(bx^2 + a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^
3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3
 + 3*a*b^3*c^2*d*x^3 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5
*a*b^3*c^3*x - 3*a^2*b^2*c^2*d*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)
/((b*x^2 + a)^2*a^2*b^3)
```

3.38 $\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d
*x^2))/(4*a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi [A] time = 0.180542, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8\left(a+bx^2\right)}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d
*x^2))/(4*a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rubi in Sympy [A] time = 23.4706, size = 105, normalized size = 0.91

$$\frac{x\left(-\frac{3d^2}{8b^2}+\frac{3c^2}{8a^2}\right)}{a+bx^2}-\frac{x\left(c+dx^2\right)(ad-bc)}{4ab\left(a+bx^2\right)^2}+\frac{(ad(3ad+bc)+bc(ad+3bc))\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*d**2/(8*b**2) + 3*c**2/(8*a**2))/(a + b*x**2) - x*(c + d*x*
*2)*(a*d - b*c)/(4*a*b*(a + b*x**2)**2) + (a*d*(3*a*d + b*c) + b*
c*(a*d + 3*b*c))*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*b**(5/2))
```

Mathematica [A] time = 0.163337, size = 124, normalized size = 1.07

$$\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}+\frac{x\left(-3a^3d^2-a^2bd\left(2c+5dx^2\right)+ab^2c\left(5c+2dx^2\right)+3b^3c^2x^2\right)}{8a^2b^2\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (x*(-3*a^3*d^2 + 3*b^3*c^2*x^2 + a*b^2*c*(5*c + 2*d*x^2) - a^2*b*
d*(2*c + 5*d*x^2)))/(8*a^2*b^2*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a
*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(5/
2))
```

Maple [A] time = 0.012, size = 147, normalized size = 1.3

$$\frac{1}{\left(bx^2+a\right)^2}\left(-\frac{\left(5\,a^2d^2-2\,abcd-3\,b^2c^2\right)x^3}{8\,a^2b}-\frac{\left(3\,a^2d^2+2\,abcd-5\,b^2c^2\right)x}{8\,ab^2}\right)+\frac{3\,d^2}{8\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{cd}{4\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c^2}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^3,x)
```

```
[Out] (-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*
a*b*c*d-5*b^2*c^2)/a/b^2*x)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))*d^2+1/4/a/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2)
)*c*d+3/8/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.21489, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+\left(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2\right)x^4+2\,\left(3\,ab^3c^2+2\,a^2b^2cd+3\,a^3bd^2\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-a}}{bx^2+a}\right.}{16\,\left(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*
a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d +
 3*a^3*b*d^2)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2
+ a)) + 2*((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b^2
*c^2 - 2*a^2*b*c*d - 3*a^3*d^2)*x)*sqrt(-a*b))/((a^2*b^4*x^4 + 2*
a^3*b^3*x^2 + a^4*b^2)*sqrt(-a*b)), 1/8*((3*a^2*b^2*c^2 + 2*a^3*b
*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4
+ 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*arctan(sqrt(
a*b)*x/a) + ((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b
^2*c^2 - 2*a^2*b*c*d - 3*a^3*d^2)*x)*sqrt(a*b))/((a^2*b^4*x^4 + 2
*a^3*b^3*x^2 + a^4*b^2)*sqrt(a*b))]
```

Sympy [A] time = 4.72889, size = 223, normalized size = 1.92

$$-\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^5}}\left(3a^2d^2+2abcd+3b^2c^2\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16}-\frac{x^3\left(5a^2bd^2-2ab^2cd-3b^3c^2\right)+x\left(3a^3d^2+2a^2bcd-5ab^2c^2\right)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log
(-a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(
3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**
5*b**5)) + x)/16 - (x**3*(5*a**2*b*d**2 - 2*a*b**2*c*d - 3*b**3*c
**2) + x*(3*a**3*d**2 + 2*a**2*b*c*d - 5*a*b**2*c**2))/(8*a**4*b*
*2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

GIAC/XCAS [A] time = 0.232621, size = 170, normalized size = 1.47

$$\frac{\left(3\,b^{2}c^{2}+2\,abcd+3\,a^{2}d^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^{2}b^{2}}+\frac{3\,b^{3}c^{2}x^{3}+2\,ab^{2}cdx^{3}-5\,a^{2}bd^{2}x^{3}+5\,ab^{2}c^{2}x-2\,a^{2}bcdx-3\,a^{3}d^{2}x}{8\,{\left(bx^{2}+a\right)}^{2}a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b
*d^2*x^3 + 5*a*b^2*c^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 +
 a)^2*a^2*b^2)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=92

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*
b*(a + b*x^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2))
```

Rubi [A] time = 0.0850816, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*
b*(a + b*x^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 12.6024, size = 78, normalized size = 0.85

$$-\frac{x\,(ad-bc)}{4ab\,(a+bx^2)^2}+\frac{x\,(ad+3bc)}{8a^2b\,(a+bx^2)}+\frac{(ad+3bc)\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -x*(a*d - b*c)/(4*a*b*(a + b*x**2)**2) + x*(a*d + 3*b*c)/(8*a**2*
b*(a + b*x**2)) + (a*d + 3*b*c)*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/
2)*b**(3/2))
```

Mathematica [A] time = 0.104814, size = 84, normalized size = 0.91

$$\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{x\left(a^2(-d)+ab\left(5c+dx^2\right)+3b^2cx^2\right)}{8a^2b\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] (x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*
x^2)^2) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*
b^(3/2))
```

Maple [A] time = 0.011, size = 89, normalized size = 1.

$$\frac{1}{\left(bx^2+a\right)^2}\left(\frac{(ad+3\,bc)\,x^3}{8\,a^2}-\frac{(ad-5\,bc)\,x}{8\,ab}\right)+\frac{d}{8\,ab}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,c}{8\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^3,x)
```

```
[Out] (1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8/a
/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d+3/8/a^2/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.208923, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(3\,b^3c+ab^2d\right)x^4+3\,a^2bc+a^3d+2\left(3\,ab^2c+a^2bd\right)x^2\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(\left(3\,b^2c+abd\right)x^3+\left(5\,abc-a^2d\right)x\right)\sqrt{\cdots}}{16\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*
c + a^2*b*d)*x^2)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) + 2*((3*b^2*c + a*b*d)*x^3 + (5*a*b*c - a^2*d)*x)*sqrt(-a*b)
)/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(-a*b)), 1/8*(((3*b^
3*c + a*b^2*d)*x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*
x^2)*arctan(sqrt(a*b)*x/a) + ((3*b^2*c + a*b*d)*x^3 + (5*a*b*c -
a^2*d)*x)*sqrt(a*b))/((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*sqrt(
a*b))]
```

Sympy [A] time = 2.7968, size = 150, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad+3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{x^3\left(abd+3b^2c\right)+x\left(-a^2d+5abc\right)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**
3)) + x)/16 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(
-1/(a**5*b**3)) + x)/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d +
 5*a*b*c))/(8*a**4*b + 16*a**3*b**2*x**2 + 8*a**2*b**3*x**4)
```

GIAC/XCAS [A] time = 0.231321, size = 105, normalized size = 1.14

$$\frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}+\frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\,(bx^2+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(
3*b^2*c*x^3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2
*b)
```

3.40 $\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal. Leaf size=161

$$\frac{bx(3bc-7ad)}{8a^2(a+bx^2)(bc-ad)^2}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*
a^2*(b*c - a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d
 + 15*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d
)^3) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)
^3)
```

Rubi [A] time = 0.44586, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{bx(3bc-7ad)}{8a^2(a+bx^2)(bc-ad)^2}+\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}+\frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*
a^2*(b*c - a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d
 + 15*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d
)^3) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)
^3)
```

Rubi in Sympy [A] time = 98.3856, size = 146, normalized size = 0.91

$$\frac{d^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\left(ad-bc\right)^3}-\frac{bx}{4a\left(a+bx^2\right)^2\left(ad-bc\right)}-\frac{bx\left(7ad-3bc\right)}{8a^2\left(a+bx^2\right)\left(ad-bc\right)^2}-\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] d**(5/2)*atan(sqrt(d)*x/sqrt(c))/(sqrt(c)*(a*d - b*c)**3) - b*x/(
4*a*(a + b*x**2)**2*(a*d - b*c)) - b*x*(7*a*d - 3*b*c)/(8*a**2*(a
 + b*x**2)*(a*d - b*c)**2) - sqrt(b)*(15*a**2*d**2 - 10*a*b*c*d +
 3*b**2*c**2)*atan(sqrt(b)*x/sqrt(a))/(8*a**(5/2)*(a*d - b*c)**3)
```

Mathematica [A] time = 0.54819, size = 158, normalized size = 0.98

$$\frac{1}{8}\left(\frac{bx(3bc-7ad)}{a^2\left(a+bx^2\right)(bc-ad)^2}-\frac{\sqrt{b}\left(15a^2d^2-10abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^3}-\frac{8d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}-\frac{2bx}{a\left(a+bx^2\right)^2(ad-bc)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] ((-2*b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x
)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (Sqrt[b]*(3*b^2*c^2 - 10*a*b*
c*d + 15*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) +
 a*d)^3) - (8*d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c
- a*d)^3))/8
```

Maple [B] time = 0.017, size = 309, normalized size = 1.9

$$\frac{d^3}{(ad-bc)^3}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,b^2x^3d^2}{8\,(ad-bc)^3\,(bx^2+a)^2}+\frac{5\,b^3x^3cd}{4\,(ad-bc)^3\,(bx^2+a)^2\,a}-\frac{3\,b^4x^3c^2}{8\,(ad-bc)^3\,(bx^2+a)^2\,a^2}-\frac{9\,abxd^2}{8\,(ad-bc)^3\,(bx^2+a)^2}+\frac{7\,b^2xcd}{4\,(ad-bc)^3\,(bx^2+a)^2}-\frac{5\,b^3xc^2}{8\,(ad-bc)^3\,(bx^2+a)^2\,a}-\frac{15\,d^2b}{8\,(ad-bc)^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{5\,b^2cd}{4\,(ad-bc)^3\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3\,b^3c^2}{8\,(ad-bc)^3\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^3/(d*x^2+c),x)
```

```
[Out] d^3/(a*d-b*c)^3/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))-7/8*b^2/(a*d-
b*c)^3/(b*x^2+a)^2*x^3*d^2+5/4*b^3/(a*d-b*c)^3/(b*x^2+a)^2/a*x^3*
c*d-3/8*b^4/(a*d-b*c)^3/(b*x^2+a)^2/a^2*x^3*c^2-9/8*b/(a*d-b*c)^3
/(b*x^2+a)^2*a*x*d^2+7/4*b^2/(a*d-b*c)^3/(b*x^2+a)^2*x*c*d-5/8*b^
3/(a*d-b*c)^3/(b*x^2+a)^2/a*x*c^2-15/8*b/(a*d-b*c)^3/(a*b)^(1/2)*
arctan(x*b/(a*b)^(1/2))*d^2+5/4*b^2/(a*d-b*c)^3/a/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))*c*d-3/8*b^3/(a*d-b*c)^3/a^2/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.983517, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*
b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d +
 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b
*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-d/c)
*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^3*c^2
 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*
d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*
a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2
*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a
*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2
*x^2 + a^4*d^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - (3*a^2*b^2*
c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*
a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2
)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
 + 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3
 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a
^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2), 1/8*((3*
b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 - 10
```

```
*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*
d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*s
qrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 4*(a^2*b^2*d^2*x^4 + 2*a^3*b
*d^2*x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)
/(d*x^2 + c)) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(
a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^
5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*
(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2
), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b
^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d +
15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*
d^2)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 8*(a^2*b^2*d^2*x^
4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))
) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3
- 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^
3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3
 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.230471, size = 294, normalized size = 1.83

$$-\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{cd}}+\frac{\left(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)\sqrt{ab}}+\frac{3\,b^3cx^3-7\,ab^2dx^3+5\,ab^2cx-9\,a^2bdx}{8\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*
d^2 - a^3*d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^
2*b*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d +
3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*
d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/((a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2)*(b*x^2 + a)^2)
```

3.41 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx$

Optimal. Leaf size=236

$$\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\,(c+dx^2)\,(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\,(a+bx^2)\,(c+dx^2)\,(bc-ad)^2}+\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}-\frac{d^{5/2}(7bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}+\frac{bx}{4a\,(a+bx^2)^2\,(c+dx^2)\,(bc-ad)}$$

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x
^2)) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(
b*c - 3*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (
b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d^(5/2)*(7*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rubi [A] time = 0.71718, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{dx(bc-4ad)(ad+3bc)}{8a^2c\,(c+dx^2)\,(bc-ad)^3}+\frac{3bx(bc-3ad)}{8a^2\,(a+bx^2)\,(c+dx^2)\,(bc-ad)^2}+\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4}-\frac{d^{5/2}(7bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}+\frac{bx}{4a\,(a+bx^2)^2\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x
^2)) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(
b*c - 3*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (
b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d^(5/2)*(7*b*c - a*d)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.775455, size = 197, normalized size = 0.83

$$\frac{1}{8}\left(\frac{b^2x(11ad-3bc)}{a^2\left(a+bx^2\right)(ad-bc)^3}+\frac{b^{3/2}\left(35a^2d^2-14abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4}+\frac{2b^2x}{a\left(a+bx^2\right)^2(bc-ad)^2}+\frac{4d^{5/2}(ad-7bc)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^4}-\frac{4d^3x}{c\left(c+dx^2\right)(bc-ad)^3}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] ((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*
d)*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*
d)^3*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/
2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*
d)^4))/8
```

Maple [A] time = 0.021, size = 403, normalized size = 1.7

$$\frac{d^4xa}{2\,(ad-bc)^4\,c\,(dx^2+c)}-\frac{d^3xb}{2\,(ad-bc)^4\,(dx^2+c)}+\frac{ad^4}{2\,(ad-bc)^4\,c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{7\,d^3b}{2\,(ad-bc)^4}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{11\,b^3x^3d^2}{8\,(ad-bc)^4\,(bx^2+a)^2}-\frac{7\,b^4x^3cd}{4\,(ad-bc)^4\,(bx^2+a)^2\,a}+\frac{3\,b^5x^3c^2}{8\,(ad-bc)^4\,(bx^2+a)^2\,a^2}+\frac{13\,ab^2xd^2}{8\,(ad-bc)^4\,(bx^2+a)^2}-\frac{9\,b^3xcd}{4\,(ad-bc)^4\,(bx^2+a)^2}+\frac{5\,b^4xc^2}{8\,(ad-bc)^4\,(bx^2+a)^2\,a}+\frac{35\,b^2d^2}{8\,(ad-bc)^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{7\,b^3cd}{4\,(ad-bc)^4\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{3\,b^4c^2}{8\,(ad-bc)^4\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^3/(d*x^2+c)^2,x)
```

```
[Out] 1/2*d^4/(a*d-b*c)^4/c*x/(d*x^2+c)*a-1/2*d^3/(a*d-b*c)^4*x/(d*x^2+
c)*b+1/2*d^4/(a*d-b*c)^4/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a-
7/2*d^3/(a*d-b*c)^4/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b+11/8*b^
3/(a*d-b*c)^4/(b*x^2+a)^2*x^3*d^2-7/4*b^4/(a*d-b*c)^4/(b*x^2+a)^2
/a*x^3*c*d+3/8*b^5/(a*d-b*c)^4/(b*x^2+a)^2/a^2*x^3*c^2+13/8*b^2/(
a*d-b*c)^4/(b*x^2+a)^2*a*x*d^2-9/4*b^3/(a*d-b*c)^4/(b*x^2+a)^2*x*
c*d+5/8*b^4/(a*d-b*c)^4/(b*x^2+a)^2/a*x*c^2+35/8*b^2/(a*d-b*c)^4/
(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))*d^2-7/4*b^3/(a*d-b*c)^4/a/(a*
b)^(1/2)*arctan(x*b/(a*b)^(1/2))*c*d+3/8*b^4/(a*d-b*c)^4/a^2/(a*b
)^(1/2)*arctan(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.16856, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^
3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2
 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b
^2*c^3*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 3
5*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2
*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d +
56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*
d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 +
13*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4
*b*c*d^3 - a^5*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c)
 - c)/(d*x^2 + c)) + 2*(5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b
^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b
^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a
^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2
*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*
b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d
```

```
^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*
(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*
d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a
^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^4*b*c^2*d^2 - a^5*c*d^3
+ (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a
^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c
*d^3 - a^5*d^4)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) + (3*a^2
*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 1
4*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*b^4*c^
3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 -
25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt
(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(5*a*b
^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2
- 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*
d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6
+ (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*
c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6
 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^
7*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d
^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*
c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 +
(3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3
*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*
b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*
c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2
)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 2*(7*a^4*b*c^2*d^2 - a^5*
c*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2
+ 13*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a
^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/
c) - c)/(d*x^2 + c)) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b
^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b
^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a
^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2
*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*
b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d
^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*(
(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4
)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^
2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d +
35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c
*d^3)*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a
^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*
c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))
) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d
^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^
4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(d/c)
*arctan(d*x/(c*sqrt(d/c))) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13
*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4
*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^
4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^
5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5
*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 +
2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3
*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.247019, size = 450, normalized size = 1.91

$$-\frac{d^3x}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)}+\frac{\left(3\,b^4c^2-14\,ab^3cd+35\,a^2b^2d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}}-\frac{\left(7\,bcd^3-ad^4\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}}+\frac{3\,b^4cx^3-11\,ab^3dx^3+5\,ab^3cx-13\,a^2b^2dx}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] -1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^
3)*(d*x^2 + c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)
*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^
2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3
- a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*
b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) + 1/8*(3*b^
4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*x)/((a^2*b^
3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^2)
```

3.42 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

Optimal. Leaf size=315

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}-\frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}+\frac{bx(3bc-11ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^2}+\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3
*(c + d*x^2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2
)^2) + (b*(3*b*c - 11*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c
 + d*x^2)^2) + (3*d*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x
)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (3*b^(5/2)*(b^2*c^2 - 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b
*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rubi [A] time = 1.03743, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3dx(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{8a^2c^2\left(c+dx^2\right)(bc-ad)^4}+\frac{dx\left(-2a^2d^2-13abcd+3b^2c^2\right)}{8a^2c\left(c+dx^2\right)^2(bc-ad)^3}-\frac{3d^{5/2}\left(a^2d^2-6abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}+\frac{bx(3bc-11ad)}{8a^2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)^2}+\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5}+\frac{bx}{4a\left(a+bx^2\right)^2\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3
*(c + d*x^2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2
)^2) + (b*(3*b*c - 11*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c
 + d*x^2)^2) + (3*d*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x
)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (3*b^(5/2)*(b^2*c^2 - 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b
*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.71351, size = 233, normalized size = 0.74

$$\frac{1}{8}\left(\frac{x(bc-ad)\left(\frac{3b^4c}{a^2(a+bx^2)}+\frac{b^3(-17ad+2bc-15bdx^2)}{a(a+bx^2)^2}-\frac{d^3(-2ad+17bc+15bdx^2)}{c(c+dx^2)^2}+\frac{3ad^4}{c^2(c+dx^2)}\right)-\frac{3d^{5/2}(a^2d^2-6abcd+21b^2c^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}}}{(bc-ad)^5}-\frac{3b^{5/2}\left(21a^2d^2-6abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(ad-bc)^5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] ((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c
)/(a^2*(a + b*x^2)) + (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c -
 17*a*d - 15*b*d*x^2))/(a*(a + b*x^2)^2) - (d^3*(17*b*c - 2*a*d +
 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b
*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(5/2))/(b*c - a*d)
^5)/8
```

Maple [A] time = 0.024, size = 568, normalized size = 1.8

$$\frac{3d^6x^3a^2}{8(ad-bc)^5(dx^2+c)^2c^2}-\frac{9d^5x^3ab}{4(ad-bc)^5(dx^2+c)^2c}+\frac{15d^4x^3b^2}{8(ad-bc)^5(dx^2+c)^2}+\frac{5d^5xa^2}{8(ad-bc)^5(dx^2+c)^2c}-\frac{11d^4xab}{4(ad-bc)^5(dx^2+c)^2}+\frac{17d^3cxb^2}{8(ad-bc)^5(dx^2+c)^2}+\frac{3d^5a^2}{8(ad-bc)^5c^2}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{9ad^4b}{4(ad-bc)^5c}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+\frac{63d^3b^2}{8(ad-bc)^5}\arctan\left(dx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}-\frac{15b^4x^3d^2}{8(ad-bc)^5(bx^2+a)^2}+\frac{9b^5x^3cd}{4(ad-bc)^5(bx^2+a)^2a}-\frac{3b^6x^3c^2}{8(ad-bc)^5(bx^2+a)^2a^2}-\frac{17b^3axd^2}{8(ad-bc)^5(bx^2+a)^2}+\frac{11b^4xcd}{4(ad-bc)^5(bx^2+a)^2}-\frac{5b^5xc^2}{8(ad-bc)^5(bx^2+a)^2a}-\frac{63d^2b^3}{8(ad-bc)^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{9b^4cd}{4(ad-bc)^5a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{3b^5c^2}{8(ad-bc)^5a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^3/(d*x^2+c)^3,x)
```

```
[Out] 3/8*d^6/(a*d-b*c)^5/(d*x^2+c)^2/c^2*x^3*a^2-9/4*d^5/(a*d-b*c)^5/(
d*x^2+c)^2/c*x^3*a*b+15/8*d^4/(a*d-b*c)^5/(d*x^2+c)^2*x^3*b^2+5/8
*d^5/(a*d-b*c)^5/(d*x^2+c)^2/c*x*a^2-11/4*d^4/(a*d-b*c)^5/(d*x^2+
c)^2*x*a*b+17/8*d^3/(a*d-b*c)^5/(d*x^2+c)^2*c*x*b^2+3/8*d^5/(a*d-
b*c)^5/c^2/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a^2-9/4*d^4/(a*d-b
*c)^5/c/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*a*b+63/8*d^3/(a*d-b*c
)^5/(c*d)^(1/2)*arctan(x*d/(c*d)^(1/2))*b^2-15/8*b^4/(a*d-b*c)^5/
(b*x^2+a)^2*x^3*d^2+9/4*b^5/(a*d-b*c)^5/(b*x^2+a)^2/a*x^3*c*d-3/8
*b^6/(a*d-b*c)^5/(b*x^2+a)^2/a^2*x^3*c^2-17/8*b^3/(a*d-b*c)^5/(b*
x^2+a)^2*a*x*d^2+11/4*b^4/(a*d-b*c)^5/(b*x^2+a)^2*x*c*d-5/8*b^5/(
a*d-b*c)^5/(b*x^2+a)^2/a*x*c^2-63/8*b^3/(a*d-b*c)^5/(a*b)^(1/2)*a
rctan(x*b/(a*b)^(1/2))*d^2+9/4*b^4/(a*d-b*c)^5/a/(a*b)^(1/2)*arct
an(x*b/(a*b)^(1/2))*c*d-3/8*b^5/(a*d-b*c)^5/a^2/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 13.6178, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b
^2*d^6)*x^7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d
^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*
(3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*
d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 - 3*(a^2*b^4*c^6 - 6*a^3*b^3
*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21
*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b
^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d -
 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4
 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4*b
^2*c^3*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b
*x^2 + a)) - 3*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^
4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*
(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2
*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 +
 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^5*c^
6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 +
22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d
 + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^
9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5
*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*
x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 -
5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a^2
*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3
 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c
^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^
2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/16
*(6*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^
6)*x^7 + 2*(6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 +
9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^
6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 +
 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 - 6*(21*a^4*b^2*c^4*d^2 - 6*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 +
a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5
*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^
3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*
(21*a^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*
c*d^5)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c))) - 3*(a^2*b^4*c^6
- 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c
^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2
 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*
b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c
^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2
```

```
 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)) + 2*(5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^
3*b^3*c^4*d^2 - 17*a^4*b^2*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d
^5)*x)/(a^4*b^5*c^9 - 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a
^7*b^2*c^6*d^3 + 5*a^8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2
 - 5*a^3*b^6*c^6*d^3 + 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 +
5*a^6*b^3*c^3*d^6 - a^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a
^3*b^6*c^7*d^2 + 5*a^4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^
2*c^3*d^6 - a^8*b*c^2*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9
*a^4*b^5*c^7*d^2 + 25*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^
7*b^2*c^4*d^5 + a^8*b*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9
 - 4*a^4*b^5*c^8*d + 5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^
8*b*c^4*d^5 - a^9*c^3*d^6)*x^2), 1/16*(6*(b^6*c^4*d^2 - 6*a*b^5*c
^3*d^3 + 6*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x^7 + 2*(6*b^6*c^5*d - 31
*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b
^2*c*d^5 - 6*a^5*b*d^6)*x^5 + 2*(3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a
^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*
x^3 + 6*(a^2*b^4*c^6 - 6*a^3*b^3*c^5*d + 21*a^4*b^2*c^4*d^2 + (b^
6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^
5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*
x^6 + (b^6*c^6 - 2*a*b^5*c^5*d - 2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c
^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4 + 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d
 + 15*a^3*b^3*c^4*d^2 + 21*a^4*b^2*c^3*d^3)*x^2)*sqrt(b/a)*arctan
(b*x/(a*sqrt(b/a))) - 3*(21*a^4*b^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a
^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4 - 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)
*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*d
^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 -
2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*
c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5
*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^4*b^2*c
^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 - 5*a^5*
b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^8*b*c^5
*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3 + 10*a^
4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a^7*b^2*
c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^4*b^5*c
^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2*d^7)*x
^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25*a^5*b^
4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b*c^3*d^
6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d + 5*a^5*b
^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3*d^6)*x
^2), 1/8*(3*(b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 6*a^3*b^3*c*d^5 - a^
4*b^2*d^6)*x^7 + (6*b^6*c^5*d - 31*a*b^5*c^4*d^2 - 9*a^2*b^4*c^3*
d^3 + 9*a^3*b^3*c^2*d^4 + 31*a^4*b^2*c*d^5 - 6*a^5*b*d^6)*x^5 + (
3*b^6*c^6 - 8*a*b^5*c^5*d - 29*a^2*b^4*c^4*d^2 + 29*a^4*b^2*c^2*d
^4 + 8*a^5*b*c*d^5 - 3*a^6*d^6)*x^3 + 3*(a^2*b^4*c^6 - 6*a^3*b^3*
c^5*d + 21*a^4*b^2*c^4*d^2 + (b^6*c^4*d^2 - 6*a*b^5*c^3*d^3 + 21*
a^2*b^4*c^2*d^4)*x^8 + 2*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 15*a^2*b^
4*c^3*d^3 + 21*a^3*b^3*c^2*d^4)*x^6 + (b^6*c^6 - 2*a*b^5*c^5*d -
2*a^2*b^4*c^4*d^2 + 78*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*d^4)*x^4
+ 2*(a*b^5*c^6 - 5*a^2*b^4*c^5*d + 15*a^3*b^3*c^4*d^2 + 21*a^4*b^
2*c^3*d^3)*x^2)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 3*(21*a^4*b
^2*c^4*d^2 - 6*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (21*a^2*b^4*c^2*d^4
- 6*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^8 + 2*(21*a^2*b^4*c^3*d^3 + 15
*a^3*b^3*c^2*d^4 - 5*a^4*b^2*c*d^5 + a^5*b*d^6)*x^6 + (21*a^2*b^4
*c^4*d^2 + 78*a^3*b^3*c^3*d^3 - 2*a^4*b^2*c^2*d^4 - 2*a^5*b*c*d^5
```

```
 + a^6*d^6)*x^4 + 2*(21*a^3*b^3*c^4*d^2 + 15*a^4*b^2*c^3*d^3 - 5*
a^5*b*c^2*d^4 + a^6*c*d^5)*x^2)*sqrt(d/c)*arctan(d*x/(c*sqrt(d/c)
)) + (5*a*b^5*c^6 - 22*a^2*b^4*c^5*d + 17*a^3*b^3*c^4*d^2 - 17*a^
4*b^2*c^3*d^3 + 22*a^5*b*c^2*d^4 - 5*a^6*c*d^5)*x)/(a^4*b^5*c^9 -
 5*a^5*b^4*c^8*d + 10*a^6*b^3*c^7*d^2 - 10*a^7*b^2*c^6*d^3 + 5*a^
8*b*c^5*d^4 - a^9*c^4*d^5 + (a^2*b^7*c^7*d^2 - 5*a^3*b^6*c^6*d^3
+ 10*a^4*b^5*c^5*d^4 - 10*a^5*b^4*c^4*d^5 + 5*a^6*b^3*c^3*d^6 - a
^7*b^2*c^2*d^7)*x^8 + 2*(a^2*b^7*c^8*d - 4*a^3*b^6*c^7*d^2 + 5*a^
4*b^5*c^6*d^3 - 5*a^6*b^3*c^4*d^5 + 4*a^7*b^2*c^3*d^6 - a^8*b*c^2
*d^7)*x^6 + (a^2*b^7*c^9 - a^3*b^6*c^8*d - 9*a^4*b^5*c^7*d^2 + 25
*a^5*b^4*c^6*d^3 - 25*a^6*b^3*c^5*d^4 + 9*a^7*b^2*c^4*d^5 + a^8*b
*c^3*d^6 - a^9*c^2*d^7)*x^4 + 2*(a^3*b^6*c^9 - 4*a^4*b^5*c^8*d +
5*a^5*b^4*c^7*d^2 - 5*a^7*b^2*c^5*d^4 + 4*a^8*b*c^4*d^5 - a^9*c^3
*d^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.494045, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^3*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] Done
```

3.43 $\int \frac{(-1+x^2)^3}{(1+x^2)^4} \, dx$

Optimal. Leaf size=34

$$-\frac{x\left(1-x^2\right)^2}{3\left(x^2+1\right)^3}-\frac{2x}{3\left(x^2+1\right)}$$

```
[Out] -(x*(1 - x^2)^2)/(3*(1 + x^2)^3) - (2*x)/(3*(1 + x^2))
```

Rubi [A] time = 0.0259221, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{x\left(1-x^2\right)^2}{3\left(x^2+1\right)^3}-\frac{2x}{3\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)^3/(1 + x^2)^4,x]

[Out] -(x*(1 - x^2)^2)/(3*(1 + x^2)^3) - (2*x)/(3*(1 + x^2))
```

Rubi in Sympy [A] time = 8.89048, size = 27, normalized size = 0.79

$$-\frac{x\left(-x^2+1\right)^2}{3\left(x^2+1\right)^3}-\frac{2x}{3\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**3/(x**2+1)**4,x)

[Out] -x*(-x**2 + 1)**2/(3*(x**2 + 1)**3) - 2*x/(3*(x**2 + 1))
```

Mathematica [A] time = 0.0118829, size = 24, normalized size = 0.71

$$-\frac{x\left(3x^4+2x^2+3\right)}{3\left(x^2+1\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]

[Out] -(x*(3 + 2*x^2 + 3*x^4))/(3*(1 + x^2)^3)
```

Maple [A] time = 0.01, size = 23, normalized size = 0.7

$$\frac{1}{(x^2+1)^3}\left(-x^5-\frac{2\,x^3}{3}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^3/(x^2+1)^4,x)

[Out] (-x^5-2/3*x^3-x)/(x^2+1)^3
```

Maxima [A] time = 1.3427, size = 45, normalized size = 1.32

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3/(x^2 + 1)^4,x, algorithm="maxima")

[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Fricas [A] time = 0.194033, size = 45, normalized size = 1.32

$$-\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3/(x^2 + 1)^4,x, algorithm="fricas")

[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] time = 0.319225, size = 31, normalized size = 0.91

$$-\frac{3x^5+2x^3+3x}{3x^6+9x^4+9x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**3/(x**2+1)**4,x)

[Out] -(3*x**5 + 2*x**3 + 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

GIAC/XCAS [A] time = 0.234433, size = 27, normalized size = 0.79

$$-\frac{3\left(x+\frac{1}{x}\right)^{2}-4}{3\left(x+\frac{1}{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^3/(x^2 + 1)^4,x, algorithm="giac")

[Out] -1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

3.44 $\int \frac{(-1+x^2)^4}{(1+x^2)^5} \, dx$

Optimal. Leaf size=47

$$\frac{x(1-x^2)^3}{4(x^2+1)^4} + \frac{3x(1-x^2)}{8(x^2+1)^2} + \frac{3}{8}\tan^{-1}(x)$$

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2)
 + (3*ArcTan[x])/8
```

Rubi [A] time = 0.0438572, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x(1-x^2)^3}{4(x^2+1)^4} + \frac{3x(1-x^2)}{8(x^2+1)^2} + \frac{3}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2)
 + (3*ArcTan[x])/8
```

Rubi in Sympy [A] time = 11.2028, size = 39, normalized size = 0.83

$$\frac{x(-x^2+1)^3}{4(x^2+1)^4} + \frac{3x(-x^2+1)}{8(x^2+1)^2} + \frac{3\operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)**4/(x**2+1)**5,x)
```

```
[Out] x*(-x**2 + 1)**3/(4*(x**2 + 1)**4) + 3*x*(-x**2 + 1)/(8*(x**2 + 1
)**2) + 3*atan(x)/8
```

Mathematica [A] time = 0.0208479, size = 41, normalized size = 0.87

$$\frac{-5x^7+3x^5-3x^3+3(x^2+1)^4\tan^{-1}(x)+5x}{8(x^2+1)^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x
^2)^4)
```

Maple [A] time = 0.012, size = 33, normalized size = 0.7

$$\frac{1}{\left(x^2+1\right)^4}\left(-\frac{5\,x^7}{8}+\frac{3\,x^5}{8}-\frac{3\,x^3}{8}+\frac{5\,x}{8}\right)+\frac{3\,\arctan\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)^4/(x^2+1)^5,x)
```

```
[Out] (-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

Maxima [A] time = 1.51301, size = 65, normalized size = 1.38

$$-\frac{5\,x^7-3\,x^5+3\,x^3-5\,x}{8\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)}+\frac{3}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^4/(x^2 + 1)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 +
 1) + 3/8*arctan(x)
```

Fricas [A] time = 0.201242, size = 90, normalized size = 1.91

$$-\frac{5\,x^7-3\,x^5+3\,x^3-3\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)\arctan\left(x\right)-5\,x}{8\left(x^8+4\,x^6+6\,x^4+4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^4/(x^2 + 1)^5,x, algorithm="fricas")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
*arctan(x) - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

Sympy [A] time = 0.449352, size = 46, normalized size = 0.98

$$-\frac{5x^7-3x^5+3x^3-5x}{8x^8+32x^6+48x^4+32x^2+8}+\frac{3\,\mathrm{atan}\,(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)**4/(x**2+1)**5,x)
```

```
[Out] -(5*x**7 - 3*x**5 + 3*x**3 - 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 3
2*x**2 + 8) + 3*atan(x)/8
```

GIAC/XCAS [A] time = 0.232825, size = 73, normalized size = 1.55

$$\frac{3}{32}\,\pi\mathrm{sign}\,(x)-\frac{5\left(x-\frac{1}{x}\right)^3+12\,x-\frac{12}{x}}{8\left(\left(x-\frac{1}{x}\right)^2+4\right)^2}+\frac{3}{16}\,\arctan\left(\frac{x^2-1}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)^4/(x^2 + 1)^5,x, algorithm="giac")
```

```
[Out] 3/32*pi*sign(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2
+ 4)^2 + 3/16*arctan(1/2*(x^2 - 1)/x)
```

3.45 $\int \sqrt{a+bx^2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=231

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(12bc-5ad\right)}{48b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqr
t[a + b*x^2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^
2)*x*(a + b*x^2)^(3/2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*
x^2)^(3/2)*(c + d*x^2))/(48*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*
x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2
- 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rubi [A] time = 0.413403, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{x\sqrt{a+bx^2}\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{a\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)\left(12bc-5ad\right)}{48b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqr
t[a + b*x^2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^
2)*x*(a + b*x^2)^(3/2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*
x^2)^(3/2)*(c + d*x^2))/(48*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*
x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2
- 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(7/2))
```

Rubi in Sympy [A] time = 50.3376, size = 230, normalized size = 1.

$$-\frac{a\left(5a^3d^3-24a^2bcd^2+48ab^2c^2d-64b^3c^3\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{7}{2}}}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^2}{8b}-\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)\left(5ad-12bc\right)}{48b^2}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}-\frac{x\sqrt{a+bx^2}\left(5a^3d^3-24a^2bcd^2+48ab^2c^2d-64b^3c^3\right)}{128b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] -a*(5*a**3*d**3 - 24*a**2*b*c*d**2 + 48*a*b**2*c**2*d - 64*b**3*c
**3)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(128*b**(7/2)) + d*x*(a +
b*x**2)**(3/2)*(c + d*x**2)**2/(8*b) - d*x*(a + b*x**2)**(3/2)*(c
 + d*x**2)*(5*a*d - 12*b*c)/(48*b**2) + d*x*(a + b*x**2)**(3/2)*(
15*a**2*d**2 - 52*a*b*c*d + 72*b**2*c**2)/(192*b**3) - x*sqrt(a +
 b*x**2)*(5*a**3*d**3 - 24*a**2*b*c*d**2 + 48*a*b**2*c**2*d - 64*
b**3*c**3)/(128*b**3)
```

Mathematica [A] time = 0.186951, size = 181, normalized size = 0.78

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^3d^3-2a^2bd^2\left(36c+5dx^2\right)+8ab^2d\left(18c^2+6cdx^2+d^2x^4\right)+48b^3\left(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6\right)\right)-3a\,(}{384b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*
x^2) + 8*a*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 +
 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6)) - 3*a*(-64*b^3*c^3 + 48*a*
b^2*c^2*d - 24*a^2*b*c*d^2 + 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a
+ b*x^2]])/(384*b^(7/2))
```

Maple [A] time = 0.016, size = 310, normalized size = 1.3

$$\frac{c^3x}{2}\sqrt{bx^2+a}+\frac{c^3a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^3x^5}{8\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,ad^3x^3}{48\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2d^3x}{64\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,a^3d^3x}{128\,b^3}\sqrt{bx^2+a}-\frac{5\,d^3a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{cd^2x^3}{2\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,acd^2x}{8\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,a^2cd^2x}{16\,b^2}\sqrt{bx^2+a}+\frac{3\,a^3cd^2}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{3\,c^2dx}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,ac^2dx}{8\,b}\sqrt{bx^2+a}-\frac{3\,c^2da^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/2*c^3*x*(b*x^2+a)^(1/2)+1/2*c^3*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))+1/8*d^3*x^5*(b*x^2+a)^(3/2)/b-5/48*d^3*a/b^2*x^3*(b*x^2+
a)^(3/2)+5/64*d^3*a^2/b^3*x*(b*x^2+a)^(3/2)-5/128*d^3*a^3/b^3*x*(
b*x^2+a)^(1/2)-5/128*d^3*a^4/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)
)+1/2*c*d^2*x^3*(b*x^2+a)^(3/2)/b-3/8*c*d^2*a/b^2*x*(b*x^2+a)^(3/
2)+3/16*c*d^2*a^2/b^2*x*(b*x^2+a)^(1/2)+3/16*c*d^2*a^3/b^(5/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))+3/4*c^2*d*x*(b*x^2+a)^(3/2)/b-3/8*c^2
*d*a/b*x*(b*x^2+a)^(1/2)-3/8*c^2*d*a^2/b^(3/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.412408, size = 1, normalized size = 0.

$$\left[\frac{2\left(48\,b^3d^3x^7+8\left(24\,b^3cd^2+ab^2d^3\right)x^5+2\left(144\,b^3c^2d+24\,ab^2cd^2-5\,a^2bd^3\right)x^3+3\left(64\,b^3c^3+48\,ab^2c^2d-24\,a^2bcd^2+5\,a^3\right.}{768\,b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^3*d^3*x^7 + 8*(24*b^3*c*d^2 + a*b^2*d^3)*x^5 + 2*
(144*b^3*c^2*d + 24*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(64*b^3*c^
3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5*a^3*d^3)*x)*sqrt(b*x^2 +
a)*sqrt(b) - 3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2
- 5*a^4*d^3)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/
b^(7/2), 1/384*((48*b^3*d^3*x^7 + 8*(24*b^3*c*d^2 + a*b^2*d^3)*x^
5 + 2*(144*b^3*c^2*d + 24*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 + 3*(64*
b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5*a^3*d^3)*x)*sqrt(b*
x^2 + a)*sqrt(-b) + 3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b
*c*d^2 - 5*a^4*d^3)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)
*b^3)]
```

Sympy [A] time = 60.359, size = 484, normalized size = 2.1

$$\frac{5a^{\frac{7}{2}}d^3x}{128b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{5}{2}}cd^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}d^3x^3}{384b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{3}{2}}c^2dx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}cd^2x^3}{16b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}d^3x^5}{192b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}c^3x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{9\sqrt{a}c^2dx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}cd^2x^5}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{7\sqrt{a}d^3x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{3a^3cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}-\frac{3a^2c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{3bc^2dx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{bcd^2x^7}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{bd^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] 5*a**(7/2)*d**3*x/(128*b**3*sqrt(1 + b*x**2/a)) - 3*a**(5/2)*c*d*
*2*x/(16*b**2*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*d**3*x**3/(384*b**
2*sqrt(1 + b*x**2/a)) + 3*a**(3/2)*c**2*d*x/(8*b*sqrt(1 + b*x**2/
a)) - a**(3/2)*c*d**2*x**3/(16*b*sqrt(1 + b*x**2/a)) - a**(3/2)*d
**3*x**5/(192*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c**3*x*sqrt(1 + b*x
**2/a)/2 + 9*sqrt(a)*c**2*d*x**3/(8*sqrt(1 + b*x**2/a)) + 5*sqrt(
a)*c*d**2*x**5/(8*sqrt(1 + b*x**2/a)) + 7*sqrt(a)*d**3*x**7/(48*s
qrt(1 + b*x**2/a)) - 5*a**4*d**3*asinh(sqrt(b)*x/sqrt(a))/(128*b*
*(7/2)) + 3*a**3*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) -
3*a**2*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + a*c**3*asin
h(sqrt(b)*x/sqrt(a))/(2*sqrt(b)) + 3*b*c**2*d*x**5/(4*sqrt(a)*sqr
t(1 + b*x**2/a)) + b*c*d**2*x**7/(2*sqrt(a)*sqrt(1 + b*x**2/a)) +
 b*d**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.241171, size = 271, normalized size = 1.17

$$\frac{1}{384}\left(2\left(4\left(6\,d^3x^2+\frac{24\,b^6cd^2+ab^5d^3}{b^6}\right)x^2+\frac{144\,b^6c^2d+24\,ab^5cd^2-5\,a^2b^4d^3}{b^6}\right)x^2+\frac{3\left(64\,b^6c^3+48\,ab^5c^2d-24\,a^2b^4cd^2+5\right.}{b^6}$$
$$-\frac{\left(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (1
44*b^6*c^2*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b
^6*c^3 + 48*a*b^5*c^2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*
sqrt(b*x^2 + a)*x - 1/128*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a
^3*b*c*d^2 - 5*a^4*d^3)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
7/2)
```

3.46 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx$

Optimal. Leaf size=149

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) +
(d*(8*b*c - 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^
2)^(3/2)*(c + d*x^2))/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] time = 0.205224, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) +
(d*(8*b*c - 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^
2)^(3/2)*(c + d*x^2))/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi in Sympy [A] time = 24.8031, size = 143, normalized size = 0.96

$$\frac{a\left(a^2d^2-4abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{5}{2}}}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)}{6b}-\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}(3ad-8bc)}{24b^2}+\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

```
[Out] a*(a**2*d**2 - 4*a*b*c*d + 8*b**2*c**2)*atanh(sqrt(b)*x/sqrt(a +
b*x**2))/(16*b**(5/2)) + d*x*(a + b*x**2)**(3/2)*(c + d*x**2)/(6*
b) - d*x*(a + b*x**2)**(3/2)*(3*a*d - 8*b*c)/(24*b**2) + x*sqrt(a
 + b*x**2)*(a**2*d**2 - 4*a*b*c*d + 8*b**2*c**2)/(16*b**2)
```

Mathematica [A] time = 0.120713, size = 123, normalized size = 0.83

$$\frac{3a\left(a^2d^2-4abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}x\sqrt{a+bx^2}\left(-3a^2d^2+2abd\left(6c+dx^2\right)+8b^2\left(3c^2+3cdx^2+d^2x^4\right)\right)}{48b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a^2*d^2 + 2*a*b*d*(6*c + d*x^2) +
8*b^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4)) + 3*a*(8*b^2*c^2 - 4*a*b*c*d
 + a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(48*b^(5/2))
```

Maple [A] time = 0.012, size = 190, normalized size = 1.3

$$\frac{c^2x}{2}\sqrt{bx^2+a}+\frac{c^2a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^2x^3}{6\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{ad^2x}{8\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{a^2d^2x}{16\,b^2}\sqrt{bx^2+a}$$
$$+\frac{a^3d^2}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{2\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{acdx}{4\,b}\sqrt{bx^2+a}-\frac{a^2cd}{4}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/2*c^2*x*(b*x^2+a)^(1/2)+1/2*c^2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))+1/6*d^2*x^3*(b*x^2+a)^(3/2)/b-1/8*d^2*a/b^2*x*(b*x^2+a)^
(3/2)+1/16*d^2*a^2/b^2*x*(b*x^2+a)^(1/2)+1/16*d^2*a^3/b^(5/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))+1/2*c*d*x*(b*x^2+a)^(3/2)/b-1/4*c*d*a/
b*x*(b*x^2+a)^(1/2)-1/4*c*d*a^2/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264109, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^2d^2x^5+2\left(12\,b^2cd+abd^2\right)x^3+3\left(8\,b^2c^2+4\,abcd-a^2d^2\right)x\right)\sqrt{bx^2+a}\sqrt{b}+3\left(8\,ab^2c^2-4\,a^2bcd+a^3d^2\right)\log\left(-2\,\sqrt{bx\right.}{96\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d^2*x^5 + 2*(12*b^2*c*d + a*b*d^2)*x^3 + 3*(8*b^2
*c^2 + 4*a*b*c*d - a^2*d^2)*x)*sqrt(b*x^2 + a)*sqrt(b) + 3*(8*a*b
^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b
*x^2 + a)*sqrt(b)))/b^(5/2), 1/48*((8*b^2*d^2*x^5 + 2*(12*b^2*c*d
 + a*b*d^2)*x^3 + 3*(8*b^2*c^2 + 4*a*b*c*d - a^2*d^2)*x)*sqrt(b*x
^2 + a)*sqrt(-b) + 3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 33.6193, size = 291, normalized size = 1.95

$$-\frac{a^{\frac{5}{2}}d^2x}{16b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}cdx}{4b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{3}{2}}d^2x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}cdx^3}{4\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}d^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{a^3d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}-\frac{a^2cd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bcdx^5}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{bd^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)
```

```
[Out] -a**(5/2)*d**2*x/(16*b**2*sqrt(1 + b*x**2/a)) + a**(3/2)*c*d*x/(4
*b*sqrt(1 + b*x**2/a)) - a**(3/2)*d**2*x**3/(48*b*sqrt(1 + b*x**2
/a)) + sqrt(a)*c**2*x*sqrt(1 + b*x**2/a)/2 + 3*sqrt(a)*c*d*x**3/(
4*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*d**2*x**5/(24*sqrt(1 + b*x**2/a
)) + a**3*d**2*asinh(sqrt(b)*x/sqrt(a))/(16*b**(5/2)) - a**2*c*d*
asinh(sqrt(b)*x/sqrt(a))/(4*b**(3/2)) + a*c**2*asinh(sqrt(b)*x/sq
rt(a))/(2*sqrt(b)) + b*c*d*x**5/(2*sqrt(a)*sqrt(1 + b*x**2/a)) +
b*d**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.237271, size = 174, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,d^2x^2 + \frac{12\,b^4cd + ab^3d^2}{b^4}\right)x^2 + \frac{3\left(8\,b^4c^2 + 4\,ab^3cd - a^2b^2d^2\right)}{b^4}\right)\sqrt{bx^2 + a}x - \frac{\left(8\,ab^2c^2 - 4\,a^2bcd + a^3d^2\right)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4
*c^2 + 4*a*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(
8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*ln(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(5/2)
```

3.47 $\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx$

Optimal. Leaf size=87

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))
/(4*b) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
8*b^(3/2))
```

Rubi [A] time = 0.0789974, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))
/(4*b) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
8*b^(3/2))
```

Rubi in Sympy [A] time = 9.62465, size = 75, normalized size = 0.86

$$-\frac{a\left(ad-4bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{3}{2}}}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}}{4b}-\frac{x\sqrt{a+bx^2}\left(ad-4bc\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] -a*(a*d - 4*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*b**(3/2)) +
 d*x*(a + b*x**2)**(3/2)/(4*b) - x*sqrt(a + b*x**2)*(a*d - 4*b*c)
/(8*b)
```

Mathematica [A] time = 0.0657133, size = 78, normalized size = 0.9

$$\sqrt{a+bx^2}\left(\frac{x(ad+4bc)}{8b}+\frac{dx^3}{4}\right)-\frac{a(ad-4bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*(((4*b*c + a*d)*x)/(8*b) + (d*x^3)/4) - (a*(-4*b*
c + a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A] time = 0.007, size = 96, normalized size = 1.1

$$\frac{cx}{2}\sqrt{bx^2+a}+\frac{ac}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{dx}{4\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{adx}{8\,b}\sqrt{bx^2+a}-\frac{a^2d}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c),x)
```

```
[Out] 1/2*c*x*(b*x^2+a)^(1/2)+1/2*c*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/4*d*x*(b*x^2+a)^(3/2)/b-1/8*d*a/b*x*(b*x^2+a)^(1/2)-1/8*d*
a^2/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232479, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,bdx^3+(4\,bc+ad)x\right)\sqrt{bx^2+a}\sqrt{b}-\left(4\,abc-a^2d\right)\log\left(2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{16\,b^{\frac{3}{2}}},\frac{\left(2\,bdx^3+(4\,bc+ad)x\right)\sqrt{bx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b*d*x^3 + (4*b*c + a*d)*x)*sqrt(b*x^2 + a)*sqrt(b) -
(4*a*b*c - a^2*d)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(
b)))/b^(3/2), 1/8*((2*b*d*x^3 + (4*b*c + a*d)*x)*sqrt(b*x^2 + a)*
sqrt(-b) + (4*a*b*c - a^2*d)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/
(sqrt(-b)*b)]
```

Sympy [A] time = 17.1212, size = 144, normalized size = 1.66

$$\frac{a^{\frac{3}{2}}dx}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3\sqrt{a}dx^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{bdx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] a**(3/2)*d*x/(8*b*sqrt(1 + b*x**2/a)) + sqrt(a)*c*x*sqrt(1 + b*x*
*2/a)/2 + 3*sqrt(a)*d*x**3/(8*sqrt(1 + b*x**2/a)) - a**2*d*asinh(
sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + a*c*asinh(sqrt(b)*x/sqrt(a))/(2
*sqrt(b)) + b*d*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.232104, size = 95, normalized size = 1.09

$$\frac{1}{8}\sqrt{bx^2+a}\left(2\,dx^2+\frac{4\,b^2c+abd}{b^2}\right)x-\frac{\left(4\,abc-a^2d\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*
a*b*c - a^2*d)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.48 $\int \sqrt{a+bx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*Sqrt[b])
```

Rubi [A] time = 0.0262312, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*Sqrt[b])
```

Rubi in Sympy [A] time = 3.1287, size = 39, normalized size = 0.85

$$\frac{a\,\text{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{x\sqrt{a+bx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2),x)
```

```
[Out] a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*sqrt(b)) + x*sqrt(a + b*x*
*2)/2
```

Mathematica [A] time = 0.0285239, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2
*Sqrt[b])
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{bx^2+a}+\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.220714, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}\sqrt{b}x+a\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{4\sqrt{b}},\frac{\sqrt{bx^2+a}\sqrt{-b}x+a\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*sqrt(b)*x + a*log(-2*sqrt(b*x^2 + a)*b*x
- (2*b*x^2 + a)*sqrt(b)))/sqrt(b), 1/2*(sqrt(b*x^2 + a)*sqrt(-b)*
```

```
x + a*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/sqrt(-b)]
```

Sympy [A] time = 6.34179, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sq
rt(b))
```

GIAC/XCAS [A] time = 0.228143, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{bx^2+a}x-\frac{a\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/sqrt(b)
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2} dx$

Optimal. Leaf size=82

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[
c]*d)
```

Rubi [A] time = 0.133035, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[
c]*d)
```

Rubi in Sympy [A] time = 20.4124, size = 70, normalized size = 0.85

$$\frac{\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}+\frac{\sqrt{ad-bc}\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] sqrt(b)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/d + sqrt(a*d - b*c)*ata
n(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*d)
```

Mathematica [A] time = 0.0689451, size = 84, normalized size = 1.02

$$\frac{\sqrt{ad-bc}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}+\frac{\sqrt{b}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a
 + b*x^2])])/(Sqrt[c]*d) + (Sqrt[b]*Log[b*x + Sqrt[b]*Sqrt[a + b*
x^2]])/d
```

Maple [B] time = 0.055, size = 932, normalized size = 11.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/2*b^(1/2)/d*ln((b*(-c*d)^(1/2)
/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-1/2/(-c*d)^(1/2
)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/
2)/d))*a+1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b*c-1/2/(-c*d)^(1/2)*((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)+1/2*b^(1/2)/d*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^
(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a-1/2/(-c*d)^(1/2)/d
/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2
)/d))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.250814, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2-4\left(ac^2x+\left(2\,bc^2-acd\right)x^3\right)\sqrt{b\ldots}}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 +
 a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2
- a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c
*d*x^2 + c^2)))/d, 1/4*(4*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sq
rt(-b))) + sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*
d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x +
 (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2
*x^4 + 2*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(-1/
2*((2*b*c - a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d
)/c))) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))
/d, 1/2*(2*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) + sqrt
(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^2 + a*c)/(sqrt(b*x^
2 + a)*c*x*sqrt(-(b*c - a*d)/c))))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.260211, size = 150, normalized size = 1.83

$$\frac{\left(b^{\frac{3}{2}}c - a\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2 + abcd}d} - \frac{\sqrt{b}\ln\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] (b^(3/2)*c - a*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a
))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 +
a*b*c*d)*d) - 1/2*sqrt(b)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d
```

3.50 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=82

$$\frac{a\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*
d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.0978889, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{a\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*
d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 17.2215, size = 68, normalized size = 0.83

$$\frac{a\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}\sqrt{ad-bc}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] a*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(2*c**(3/2)*
sqrt(a*d - b*c)) + x*sqrt(a + b*x**2)/(2*c*(c + d*x**2))
```

Mathematica [A] time = 0.151674, size = 82, normalized size = 1.

$$\frac{a\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{ad-bc}}+\frac{x\sqrt{a+bx^2}}{2c\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[-(b*c) + a*d]
)
```

Maple [B] time = 0.039, size = 2521, normalized size = 30.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/4/c/d*b*(-c*
d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/d*b^(3/2)/(a*d-b*c)*ln((b*
(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^
2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/4
/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)
/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a-1/4/d^2*b^2*(-c*d)^(1/2)/(a*
d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2
*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d
)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/4/c*b^(1/2)/(a*
d-b*c)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2))*a+1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/
4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/d*b^(3/2)/(a*d
-b*c)*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c
*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2))-1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln
((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c
)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a+1/4/d^2*b^2*(-c
*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x+(-c*d)^(1/2)/d))-1/4/c*b/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*
b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/4/
c*b^(1/2)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b
^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2))*a+1/4/(-c*d)^(1/2)/c*((x-(-c*d)^(1/2)/d)^
```

```
2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/
c*b^(1/2)/d*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))-1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-
b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2
)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a+1/4/(-c*d)^(1/2)/d/((a*d
-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*
b-1/4/(-c*d)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/c*b^(1/2)/d*ln((-b*(-c*d
)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/4/(-c*
d)^(1/2)/c/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2
*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(
-c*d)^(1/2)/d))*a-1/4/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a
*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(
1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

Fricas [A] time = 0.280321, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{bc^2-acd}\sqrt{bx^2+a}x + \left(adx^2+ac\right)\log\left(\frac{\left(\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2cd\right)x^2\right)\sqrt{bc^2-acd}+4\left(\left(2\,b^2c^3-3\,abc^2d+a^2cd^2\right)x^3+\right.}{d^2x^4+2\,cdx^2+c^2}\right.}{8\left(cdx^2+c^2\right)\sqrt{bc^2-acd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*l
og((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
```

```
^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*
c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a)
)/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((c*d*x^2 + c^2)*sqrt(b*c^2 - a*c
*d)), 1/4*(2*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 +
a*c)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((
b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((c*d*x^2 + c^2)*sqrt(-b*c^2
+ a*c*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{(c + dx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

GIAC/XCAS [A] time = 2.64885, size = 4, normalized size = 0.05

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.51 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=149

$$\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*
c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (
a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b
*x^2])])/(8*c^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.262617, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\left(c+dx^2\right)(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/2}}{4c\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -(d*x*(a + b*x^2)^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*
c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (
a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b
*x^2])])/(8*c^(5/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 31.1512, size = 129, normalized size = 0.87

$$\frac{a\left(3ad-4bc\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{\frac{5}{2}}\left(ad-bc\right)^{\frac{3}{2}}}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}}{4c\left(c+dx^2\right)^2\left(ad-bc\right)}+\frac{x\sqrt{a+bx^2}\left(3ad-4bc\right)}{8c^2\left(c+dx^2\right)\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] a*(3*a*d - 4*b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2
)))/(8*c**(5/2)*(a*d - b*c)**(3/2)) + d*x*(a + b*x**2)**(3/2)/(4*
c*(c + d*x**2)**2*(a*d - b*c)) + x*sqrt(a + b*x**2)*(3*a*d - 4*b*
c)/(8*c**2*(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.197114, size = 128, normalized size = 0.86

$$-\frac{a(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(ad-bc)^{3/2}}-\frac{x\sqrt{a+bx^2}\left(ad\left(5c+3dx^2\right)-2bc\left(2c+dx^2\right)\right)}{8c^2\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -(x*Sqrt[a + b*x^2]*(-2*b*c*(2*c + d*x^2) + a*d*(5*c + 3*d*x^2)))
/(8*c^2*(b*c - a*d)*(c + d*x^2)^2) - (a*(4*b*c - 3*a*d)*ArcTan[(S
qrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(-(b*
c) + a*d)^(3/2))
```

Maple [B] time = 0.037, size = 5101, normalized size = 34.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

Fricas [A] time = 0.356658, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(2\,bcd-3\,ad^2\right)x^3+\left(4\,bc^2-5\,acd\right)x\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}+\left(4\,abc^3-3\,a^2c^2d+\left(4\,abcd^2-3\,a^2d^3\right)x^4+2\left(4\,abc^2d-3\right.\right.}{32\left(bc^5-ac^4d+\left(bc^3d^2-ac^2d^3\right)x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(4*((2*b*c*d - 3*a*d^2)*x^3 + (4*b*c^2 - 5*a*c*d)*x)*sqrt(b
*c^2 - a*c*d)*sqrt(b*x^2 + a) + (4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*log(
(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2
- 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2
*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(
d^2*x^4 + 2*c*d*x^2 + c^2)))/((b*c^5 - a*c^4*d + (b*c^3*d^2 - a*c
^2*d^3)*x^4 + 2*(b*c^4*d - a*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)),
1/16*(2*((2*b*c*d - 3*a*d^2)*x^3 + (4*b*c^2 - 5*a*c*d)*x)*sqrt(-b
*c^2 + a*c*d)*sqrt(b*x^2 + a) + (4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*arct
an(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a
*c*d)*sqrt(b*x^2 + a)*x)))/((b*c^5 - a*c^4*d + (b*c^3*d^2 - a*c^2
*d^3)*x^4 + 2*(b*c^4*d - a*c^3*d^2)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 3.94901, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.52 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx$

Optimal. Leaf size=208

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt
[a + b*x^2])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d
)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d
*x^2)) + (a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*
c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c - a*d)^(
5/2))
```

Rubi [A] time = 0.554456, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{48c^3\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(4bc-5ad)}{24c^2\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt
[a + b*x^2])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d
)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d
*x^2)) + (a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*
c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c - a*d)^(
5/2))
```

Rubi in Sympy [A] time = 82.8335, size = 190, normalized size = 0.91

$$\frac{a\left(5a^2d^2-12abcd+8b^2c^2\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{\frac{7}{2}}\left(ad-bc\right)^{\frac{5}{2}}}+\frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3}+\frac{x\sqrt{a+bx^2}\left(5ad-4bc\right)}{24c^2\left(c+dx^2\right)^2\left(ad-bc\right)}+\frac{x\sqrt{a+bx^2}\left(3ad-4bc\right)\left(5ad-2bc\right)}{48c^3\left(c+dx^2\right)\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
```

```
[Out] a*(5*a**2*d**2 - 12*a*b*c*d + 8*b**2*c**2)*atan(x*sqrt(a*d - b*c)
/(sqrt(c)*sqrt(a + b*x**2)))/(16*c**(7/2)*(a*d - b*c)**(5/2)) + x
*sqrt(a + b*x**2)/(6*c*(c + d*x**2)**3) + x*sqrt(a + b*x**2)*(5*a
*d - 4*b*c)/(24*c**2*(c + d*x**2)**2*(a*d - b*c)) + x*sqrt(a + b*
x**2)*(3*a*d - 4*b*c)*(5*a*d - 2*b*c)/(48*c**3*(c + d*x**2)*(a*d
- b*c)**2)
```

Mathematica [A] time = 0.436169, size = 184, normalized size = 0.88

$$\frac{\frac{\sqrt{c}x\sqrt{a+bx^2}\left(\frac{\left(c+dx^2\right)^2\left(15a^2d^2-26abcd+8b^2c^2\right)}{(bc-ad)^2}+\frac{2c\left(c+dx^2\right)(4bc-5ad)}{bc-ad}+8c^2\right)}{\left(c+dx^2\right)^3}+\frac{3a\left(5a^2d^2-12abcd+8b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{(ad-bc)^{5/2}}}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] ((Sqrt[c]*x*Sqrt[a + b*x^2]*(8*c^2 + (2*c*(4*b*c - 5*a*d)*(c + d*
x^2))/(b*c - a*d) + ((8*b^2*c^2 - 26*a*b*c*d + 15*a^2*d^2)*(c + d
*x^2)^2)/(b*c - a*d)^2))/(c + d*x^2)^3 + (3*a*(8*b^2*c^2 - 12*a*b
*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a +
 b*x^2])])/(-(b*c) + a*d)^(5/2))/(48*c^(7/2))
```

Maple [B] time = 0.047, size = 7922, normalized size = 38.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

Fricas [A] time = 1.25559, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(4*((8*b^2*c^2*d^2 - 26*a*b*c*d^3 + 15*a^2*d^4)*x^5 + 2*(1
2*b^2*c^3*d - 35*a*b*c^2*d^2 + 20*a^2*c*d^3)*x^3 + 3*(8*b^2*c^4 -
 20*a*b*c^3*d + 11*a^2*c^2*d^2)*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2
 + a) + 3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^
2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2
- 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2
*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2
 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c
^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))
/((b^2*c^8 - 2*a*b*c^7*d + a^2*c^6*d^2 + (b^2*c^5*d^3 - 2*a*b*c^4
*d^4 + a^2*c^3*d^5)*x^6 + 3*(b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^
4*d^4)*x^4 + 3*(b^2*c^7*d - 2*a*b*c^6*d^2 + a^2*c^5*d^3)*x^2)*sqr
t(b*c^2 - a*c*d)), 1/96*(2*((8*b^2*c^2*d^2 - 26*a*b*c*d^3 + 15*a^
2*d^4)*x^5 + 2*(12*b^2*c^3*d - 35*a*b*c^2*d^2 + 20*a^2*c*d^3)*x^3
 + 3*(8*b^2*c^4 - 20*a*b*c^3*d + 11*a^2*c^2*d^2)*x)*sqrt(-b*c^2 +
 a*c*d)*sqrt(b*x^2 + a) + 3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3
*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3
*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*
b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*arctan(1/2*sqr
t(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt
(b*x^2 + a)*x)))/((b^2*c^8 - 2*a*b*c^7*d + a^2*c^6*d^2 + (b^2*c^5
*d^3 - 2*a*b*c^4*d^4 + a^2*c^3*d^5)*x^6 + 3*(b^2*c^6*d^2 - 2*a*b*
c^5*d^3 + a^2*c^4*d^4)*x^4 + 3*(b^2*c^7*d - 2*a*b*c^6*d^2 + a^2*c
^5*d^3)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 32.1371, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.53 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=272

$$\frac{3a^2(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3/2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{256b^3}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)(14bc-5ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b
*x^2])/(256*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x*(a + b*x^2)^(3/2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d +
 5*a^2*d^2)*x*(a + b*x^2)^(5/2))/(160*b^3) + (d*(14*b*c - 5*a*d)*
x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2) + (d*x*(a + b*x^2)^(5/2
)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(7/2
))
```

Rubi [A] time = 0.495243, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{3a^2(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}+\frac{dx\left(a+bx^2\right)^{5/2}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}+\frac{x\left(a+bx^2\right)^{3/2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}+\frac{3ax\sqrt{a+bx^2}(4bc-ad)\left(a^2d^2-2abcd+8b^2c^2\right)}{256b^3}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)(14bc-5ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b
*x^2])/(256*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x*(a + b*x^2)^(3/2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d +
 5*a^2*d^2)*x*(a + b*x^2)^(5/2))/(160*b^3) + (d*(14*b*c - 5*a*d)*
x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2) + (d*x*(a + b*x^2)^(5/2
)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(7/2
```

```
))
```

Rubi in Sympy [A] time = 53.8301, size = 269, normalized size = 0.99

$$-\frac{3a^2(ad-4bc)\left(a^2d^2-2abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{7}{2}}}-\frac{3ax\sqrt{a+bx^2}(ad-4bc)\left(a^2d^2-2abcd+8b^2c^2\right)}{256b^3}+\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)^2}{10b}-\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)(5ad-14bc)}{80b^2}+\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}\left(5a^2d^2-20abcd+36b^2c^2\right)}{160b^3}-\frac{x\left(a+bx^2\right)^{\frac{3}{2}}(ad-4bc)\left(a^2d^2-2abcd+8b^2c^2\right)}{128b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] -3*a**2*(a*d - 4*b*c)*(a**2*d**2 - 2*a*b*c*d + 8*b**2*c**2)*atanh
(sqrt(b)*x/sqrt(a + b*x**2))/(256*b**(7/2)) - 3*a*x*sqrt(a + b*x*
*2)*(a*d - 4*b*c)*(a**2*d**2 - 2*a*b*c*d + 8*b**2*c**2)/(256*b**3
) + d*x*(a + b*x**2)**(5/2)*(c + d*x**2)**2/(10*b) - d*x*(a + b*x
**2)**(5/2)*(c + d*x**2)*(5*a*d - 14*b*c)/(80*b**2) + d*x*(a + b*
x**2)**(5/2)*(5*a**2*d**2 - 20*a*b*c*d + 36*b**2*c**2)/(160*b**3)
 - x*(a + b*x**2)**(3/2)*(a*d - 4*b*c)*(a**2*d**2 - 2*a*b*c*d + 8
*b**2*c**2)/(128*b**3)
```

Mathematica [A] time = 0.207767, size = 220, normalized size = 0.81

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(15a^4d^3-10a^3bd^2\left(9c+dx^2\right)+4a^2b^2d\left(60c^2+15cdx^2+2d^2x^4\right)+16ab^3\left(50c^3+70c^2dx^2+45cd^2x^4+11d^3x\right.\right.}{1280b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^
2) + 4*a^2*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(
10*c^3 + 20*c^2*d*x^2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*
c^3 + 70*c^2*d*x^2 + 45*c*d^2*x^4 + 11*d^3*x^6)) - 15*a^2*(-4*b*c
 + a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[
a + b*x^2]])/(1280*b^(7/2))
```

Maple [A] time = 0.018, size = 393, normalized size = 1.4

$$\frac{c^3 x}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,ac^3x}{8}\sqrt{bx^2+a}+\frac{3\,c^3a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^3x^5}{10\,b}\left(bx^2+a\right)^{\frac{5}{2}}$$
$$-\frac{ad^3x^3}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{d^3a^2x}{32\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{a^3d^3x}{128\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,d^3a^4x}{256\,b^3}\sqrt{bx^2+a}$$
$$-\frac{3\,d^3a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{3\,cd^2x^3}{8\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{3\,acd^2x}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}$$
$$+\frac{3\,a^2cd^2x}{64\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{9\,a^3cd^2x}{128\,b^2}\sqrt{bx^2+a}+\frac{9\,cd^2a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$
$$+\frac{c^2dx}{2\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{ac^2dx}{8\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{3\,c^2da^2x}{16\,b}\sqrt{bx^2+a}-\frac{3\,a^3c^2d}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/4*c^3*x*(b*x^2+a)^(3/2)+3/8*c^3*a*x*(b*x^2+a)^(1/2)+3/8*c^3*a^2
/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/10*d^3*x^5*(b*x^2+a)^(5/
2)/b-1/16*d^3*a/b^2*x^3*(b*x^2+a)^(5/2)+1/32*d^3*a^2/b^3*x*(b*x^2
+a)^(5/2)-1/128*d^3*a^3/b^3*x*(b*x^2+a)^(3/2)-3/256*d^3*a^4/b^3*x
*(b*x^2+a)^(1/2)-3/256*d^3*a^5/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))+3/8*c*d^2*x^3*(b*x^2+a)^(5/2)/b-3/16*c*d^2*a/b^2*x*(b*x^2+a)^
(5/2)+3/64*c*d^2*a^2/b^2*x*(b*x^2+a)^(3/2)+9/128*c*d^2*a^3/b^2*x*
(b*x^2+a)^(1/2)+9/128*c*d^2*a^4/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))+1/2*c^2*d*x*(b*x^2+a)^(5/2)/b-1/8*c^2*d*a/b*x*(b*x^2+a)^(3/2
)-3/16*c^2*d*a^2/b*x*(b*x^2+a)^(1/2)-3/16*c^2*d*a^3/b^(3/2)*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.690348, size = 1, normalized size = 0.

$$\left[\frac{2\left(128\,b^4d^3x^9+16\left(30\,b^4cd^2+11\,ab^3d^3\right)x^7+8\left(80\,b^4c^2d+90\,ab^3cd^2+a^2b^2d^3\right)x^5+10\left(32\,b^4c^3+112\,ab^3c^2d+6\,a^2b^2cd^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2*(128*b^4*d^3*x^9 + 16*(30*b^4*c*d^2 + 11*a*b^3*d^3)*x^
7 + 8*(80*b^4*c^2*d + 90*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 10*(32*
b^4*c^3 + 112*a*b^3*c^2*d + 6*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3 + 5*
(160*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 18*a^3*b*c*d^2 + 3*a^4*d^3)*x
)*sqrt(b*x^2 + a)*sqrt(b) - 15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d
 + 6*a^4*b*c*d^2 - a^5*d^3)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2
+ a)*sqrt(b)))/b^(7/2), 1/1280*((128*b^4*d^3*x^9 + 16*(30*b^4*c*d
^2 + 11*a*b^3*d^3)*x^7 + 8*(80*b^4*c^2*d + 90*a*b^3*c*d^2 + a^2*b
^2*d^3)*x^5 + 10*(32*b^4*c^3 + 112*a*b^3*c^2*d + 6*a^2*b^2*c*d^2
- a^3*b*d^3)*x^3 + 5*(160*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 18*a^3*b
*c*d^2 + 3*a^4*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 15*(32*a^2*b^3*
c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 151.033, size = 665, normalized size = 2.44

$$\frac{3a^{\frac{9}{2}}d^3x}{256b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{9a^{\frac{7}{2}}cd^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{7}{2}}d^3x^3}{256b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{5}{2}}c^2dx}{16b\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{\frac{5}{2}}cd^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{5}{2}}d^3x^5}{640b\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}c^3x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a^{\frac{3}{2}}c^3x}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{17a^{\frac{3}{2}}c^2dx^3}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{39a^{\frac{3}{2}}cd^2x^5}{64\sqrt{1+\frac{bx^2}{a}}} + \frac{23a^{\frac{3}{2}}d^3x^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{3\sqrt{a}bc^3x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}bc^2dx^5}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15\sqrt{a}bcd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}} + \frac{19\sqrt{a}bd^3x^9}{80\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^5d^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{7}{2}}} + \frac{9a^4cd^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}} - \frac{3a^3c^2d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{3a^2c^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}} + \frac{b^2c^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2c^2dx^7}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{3b^2cd^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + \frac{b^2d^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] 3*a**(9/2)*d**3*x/(256*b**3*sqrt(1 + b*x**2/a)) - 9*a**(7/2)*c*d*
*2*x/(128*b**2*sqrt(1 + b*x**2/a)) + a**(7/2)*d**3*x**3/(256*b**2
*sqrt(1 + b*x**2/a)) + 3*a**(5/2)*c**2*d*x/(16*b*sqrt(1 + b*x**2/
a)) - 3*a**(5/2)*c*d**2*x**3/(128*b*sqrt(1 + b*x**2/a)) - a**(5/2
)*d**3*x**5/(640*b*sqrt(1 + b*x**2/a)) + a**(3/2)*c**3*x*sqrt(1 +
 b*x**2/a)/2 + a**(3/2)*c**3*x/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/
2)*c**2*d*x**3/(16*sqrt(1 + b*x**2/a)) + 39*a**(3/2)*c*d**2*x**5/
```

```
(64*sqrt(1 + b*x**2/a)) + 23*a**(3/2)*d**3*x**7/(160*sqrt(1 + b*x
**2/a)) + 3*sqrt(a)*b*c**3*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(
a)*b*c**2*d*x**5/(8*sqrt(1 + b*x**2/a)) + 15*sqrt(a)*b*c*d**2*x**
7/(16*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b*d**3*x**9/(80*sqrt(1 + b
*x**2/a)) - 3*a**5*d**3*asinh(sqrt(b)*x/sqrt(a))/(256*b**(7/2)) +
 9*a**4*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) - 3*a**3*c
**2*d*asinh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*a**2*c**3*asinh(
sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c**3*x**5/(4*sqrt(a)*sqrt(1
 + b*x**2/a)) + b**2*c**2*d*x**7/(2*sqrt(a)*sqrt(1 + b*x**2/a)) +
 3*b**2*c*d**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d**3*x*
*11/(10*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.284076, size = 351, normalized size = 1.29

$$\frac{1}{1280}\left(2\left(4\left(2\left(8\,bd^3x^2+\frac{30\,b^9cd^2+11\,ab^8d^3}{b^8}\right)x^2+\frac{80\,b^9c^2d+90\,ab^8cd^2+a^2b^7d^3}{b^8}\right)x^2+\frac{5\left(32\,b^9c^3+112\,ab^8c^2d+6\,a^2b^7cd\right.}{b^8}\right.\right.$$
$$-\frac{3\left(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)
*x^2 + (80*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5
*(32*b^9*c^3 + 112*a*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b
^8)*x^2 + 5*(160*a*b^8*c^3 + 48*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2
+ 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a)*x - 3/256*(32*a^2*b^3*c^3 -
 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*ln(abs(-sqrt(b)*x +
sqrt(b*x^2 + a)))/b^(7/2)
```

3.54 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=196

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{48b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*
b^2) + ((48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2)
)/(192*b^2) + (d*(10*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) +
 (d*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 1
6*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128
*b^(5/2))
```

Rubi [A] time = 0.261809, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{x\left(a+bx^2\right)^{3/2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{dx\left(a+bx^2\right)^{5/2}(10bc-3ad)}{48b^2}+\frac{dx\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*
b^2) + ((48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2)
)/(192*b^2) + (d*(10*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) +
 (d*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 1
6*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128
*b^(5/2))
```

Rubi in Sympy [A] time = 28.5963, size = 192, normalized size = 0.98

$$\frac{a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{5}{2}}}+\frac{ax\sqrt{a+bx^2}\left(3a^2d^2-16abcd+48b^2c^2\right)}{128b^2}+\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}\left(c+dx^2\right)}{8b}-\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}\left(3ad-10bc\right)}{48b^2}+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}\left(3a^2d^2-16abcd+48b^2c^2\right)}{192b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] a**2*(3*a**2*d**2 - 16*a*b*c*d + 48*b**2*c**2)*atanh(sqrt(b)*x/sq
rt(a + b*x**2))/(128*b**(5/2)) + a*x*sqrt(a + b*x**2)*(3*a**2*d**
2 - 16*a*b*c*d + 48*b**2*c**2)/(128*b**2) + d*x*(a + b*x**2)**(5/
2)*(c + d*x**2)/(8*b) - d*x*(a + b*x**2)**(5/2)*(3*a*d - 10*b*c)/
(48*b**2) + x*(a + b*x**2)**(3/2)*(3*a**2*d**2 - 16*a*b*c*d + 48*
b**2*c**2)/(192*b**2)
```

Mathematica [A] time = 0.169046, size = 159, normalized size = 0.81

$$\frac{3a^2\left(3a^2d^2-16abcd+48b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}x\sqrt{a+bx^2}\left(-9a^3d^2+6a^2bd\left(8c+dx^2\right)+8ab^2\left(30c^2+28cdx^2\right.\right.}{384b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*d^2 + 6*a^2*b*d*(8*c + d*x^2)
+ 16*b^3*x^2*(6*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + 8*a*b^2*(30*c^2 +
28*c*d*x^2 + 9*d^2*x^4)) + 3*a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2
*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(384*b^(5/2))
```

Maple [A] time = 0.013, size = 249, normalized size = 1.3

$$\frac{c^2x}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,ac^2x}{8}\sqrt{bx^2+a}+\frac{3\,c^2a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^2x^3}{8\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{ad^2x}{16\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{a^2d^2x}{64\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,d^2a^3x}{128\,b^2}\sqrt{bx^2+a}+\frac{3\,d^2a^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{3\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{acdx}{12\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{cda^2x}{8\,b}\sqrt{bx^2+a}-\frac{cda^3}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/4*c^2*x*(b*x^2+a)^(3/2)+3/8*c^2*a*x*(b*x^2+a)^(1/2)+3/8*c^2*a^2
/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/8*d^2*x^3*(b*x^2+a)^(5/2
)/b-1/16*d^2*a/b^2*x*(b*x^2+a)^(5/2)+1/64*d^2*a^2/b^2*x*(b*x^2+a)
^(3/2)+3/128*d^2*a^3/b^2*x*(b*x^2+a)^(1/2)+3/128*d^2*a^4/b^(5/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/3*c*d*x*(b*x^2+a)^(5/2)/b-1/12*c*
d*a/b*x*(b*x^2+a)^(3/2)-1/8*c*d*a^2/b*x*(b*x^2+a)^(1/2)-1/8*c*d*a
^3/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.348537, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,b^3d^2x^7 + 8\left(16\,b^3cd + 9\,ab^2d^2\right)x^5 + 2\left(48\,b^3c^2 + 112\,ab^2cd + 3\,a^2bd^2\right)x^3 + 3\left(80\,ab^2c^2 + 16\,a^2bcd - 3\,a^3d^2\right)x\right)\sqrt{bx^2 +}}{768\,b^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^3*d^2*x^7 + 8*(16*b^3*c*d + 9*a*b^2*d^2)*x^5 + 2*
(48*b^3*c^2 + 112*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + 3*(80*a*b^2*c^2
+ 16*a^2*b*c*d - 3*a^3*d^2)*x)*sqrt(b*x^2 + a)*sqrt(b) + 3*(48*a^
2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*log(-2*sqrt(b*x^2 + a)*b*x
- (2*b*x^2 + a)*sqrt(b)))/b^(5/2), 1/384*((48*b^3*d^2*x^7 + 8*(16
*b^3*c*d + 9*a*b^2*d^2)*x^5 + 2*(48*b^3*c^2 + 112*a*b^2*c*d + 3*a
^2*b*d^2)*x^3 + 3*(80*a*b^2*c^2 + 16*a^2*b*c*d - 3*a^3*d^2)*x)*sq
rt(b*x^2 + a)*sqrt(-b) + 3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4
*d^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 87.2219, size = 440, normalized size = 2.24

$$-\frac{3a^{\frac{7}{2}}d^2x}{128b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cdx}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}d^2x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{3}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}c^2x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{17a^{\frac{3}{2}}cdx^3}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}d^2x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{3\sqrt{a}bc^2x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{11\sqrt{a}bcdx^5}{12\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}bd^2x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{a^3cd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{3a^2c^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^2c^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2cdx^7}{3\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^2d^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(7/2)*d**2*x/(128*b**2*sqrt(1 + b*x**2/a)) + a**(5/2)*c*d*x
/(8*b*sqrt(1 + b*x**2/a)) - a**(5/2)*d**2*x**3/(128*b*sqrt(1 + b*
x**2/a)) + a**(3/2)*c**2*x*sqrt(1 + b*x**2/a)/2 + a**(3/2)*c**2*x
/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*c*d*x**3/(24*sqrt(1 + b*x**
2/a)) + 13*a**(3/2)*d**2*x**5/(64*sqrt(1 + b*x**2/a)) + 3*sqrt(a)
*b*c**2*x**3/(8*sqrt(1 + b*x**2/a)) + 11*sqrt(a)*b*c*d*x**5/(12*s
qrt(1 + b*x**2/a)) + 5*sqrt(a)*b*d**2*x**7/(16*sqrt(1 + b*x**2/a)
) + 3*a**4*d**2*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) - a**3*c*
d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + 3*a**2*c**2*asinh(sqrt(
b)*x/sqrt(a))/(8*sqrt(b)) + b**2*c**2*x**5/(4*sqrt(a)*sqrt(1 + b*
x**2/a)) + b**2*c*d*x**7/(3*sqrt(a)*sqrt(1 + b*x**2/a)) + b**2*d*
*2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.255164, size = 236, normalized size = 1.2

$$\frac{1}{384}\left(2\left(4\left(6\,bd^2x^2+\frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2+\frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2+\frac{3\left(80\,ab^6c^2+16\,a^2b^5cd-3\,a^3b^4d^2\right)}{b^6}\right)\sqrt{b}$$
$$-\frac{\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 +
(48*b^7*c^2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b
^6*c^2 + 16*a^2*b^5*c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x -
 1/128*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*ln(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.55 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right) \, dx$

Optimal. Leaf size=118

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a
+ b*x^2)^(3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*
b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] time = 0.106461, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}+\frac{x\left(a+bx^2\right)^{3/2}(6bc-ad)}{24b}+\frac{ax\sqrt{a+bx^2}(6bc-ad)}{16b}+\frac{dx\left(a+bx^2\right)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a
+ b*x^2)^(3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*
b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi in Sympy [A] time = 12.2675, size = 102, normalized size = 0.86

$$-\frac{a^2\left(ad-6bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{3}{2}}}-\frac{ax\sqrt{a+bx^2}\left(ad-6bc\right)}{16b}+\frac{dx\left(a+bx^2\right)^{\frac{5}{2}}}{6b}-\frac{x\left(a+bx^2\right)^{\frac{3}{2}}\left(ad-6bc\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] -a**2*(a*d - 6*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*b**(3/2
)) - a*x*sqrt(a + b*x**2)*(a*d - 6*b*c)/(16*b) + d*x*(a + b*x**2)
**(5/2)/(6*b) - x*(a + b*x**2)**(3/2)*(a*d - 6*b*c)/(24*b)
```

Mathematica [A] time = 0.124798, size = 98, normalized size = 0.83

$$\sqrt{a+bx^2}\left(\frac{1}{24}x^3(7ad+6bc)+\frac{ax(ad+10bc)}{16b}+\frac{1}{6}bdx^5\right)-\frac{a^2(ad-6bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a*(10*b*c + a*d)*x)/(16*b) + ((6*b*c + 7*a*d)*x
^3)/24 + (b*d*x^5)/6) - (a^2*(-6*b*c + a*d)*Log[b*x + Sqrt[b]*Sqr
t[a + b*x^2]])/(16*b^(3/2))
```

Maple [A] time = 0.007, size = 131, normalized size = 1.1

$$\frac{cx}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,acx}{8}\sqrt{bx^2+a}+\frac{3\,a^2c}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{dx}{6\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{adx}{24\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{da^2x}{16\,b}\sqrt{bx^2+a}-\frac{a^3d}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c),x)
```

```
[Out] 1/4*c*x*(b*x^2+a)^(3/2)+3/8*c*a*x*(b*x^2+a)^(1/2)+3/8*c*a^2/b^(1/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/6*d*x*(b*x^2+a)^(5/2)/b-1/24*d
*a/b*x*(b*x^2+a)^(3/2)-1/16*d*a^2/b*x*(b*x^2+a)^(1/2)-1/16*d*a^3/
b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.247573, size = 1, normalized size = 0.01

$$\left[\frac{2\,\left(8\,b^{2}dx^{5}+2\,\left(6\,b^{2}c+7\,abd\right)x^{3}+3\,\left(10\,abc+a^{2}d\right)x\right)\sqrt{bx^{2}+a}\sqrt{b}-3\,\left(6\,a^{2}bc-a^{3}d\right)\,\log\left(2\,\sqrt{bx^{2}+a}bx-\left(2\,bx^{2}+a\right)\sqrt{b}\right)}{96\,b^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d*x^5 + 2*(6*b^2*c + 7*a*b*d)*x^3 + 3*(10*a*b*c +
 a^2*d)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(6*a^2*b*c - a^3*d)*log(2*
sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/48*((8*b
^2*d*x^5 + 2*(6*b^2*c + 7*a*b*d)*x^3 + 3*(10*a*b*c + a^2*d)*x)*sq
rt(b*x^2 + a)*sqrt(-b) + 3*(6*a^2*b*c - a^3*d)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 43.6349, size = 253, normalized size = 2.14

$$\frac{a^{\frac{5}{2}}dx}{16b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{a^{\frac{3}{2}}cx\sqrt{1+\frac{bx^{2}}{a}}}{2}+\frac{a^{\frac{3}{2}}cx}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{17a^{\frac{3}{2}}dx^{3}}{48\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3\sqrt{a}bcx^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}+\frac{11\sqrt{a}bdx^{5}}{24\sqrt{1+\frac{bx^{2}}{a}}}$$
$$-\frac{a^{3}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{3a^{2}c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{b^{2}cx^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{b^{2}dx^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] a**(5/2)*d*x/(16*b*sqrt(1 + b*x**2/a)) + a**(3/2)*c*x*sqrt(1 + b*
x**2/a)/2 + a**(3/2)*c*x/(8*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*d*x
**3/(48*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*b*c*x**3/(8*sqrt(1 + b*x*
*2/a)) + 11*sqrt(a)*b*d*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*d*asi
nh(sqrt(b)*x/sqrt(a))/(16*b**(3/2)) + 3*a**2*c*asinh(sqrt(b)*x/sq
rt(a))/(8*sqrt(b)) + b**2*c*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) +
 b**2*d*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.28558, size = 139, normalized size = 1.18

$$\frac{1}{48}\left(2\left(4\,bdx^{2}+\frac{6\,b^{5}c+7\,ab^{4}d}{b^{4}}\right)x^{2}+\frac{3\,\left(10\,ab^{4}c+a^{2}b^{3}d\right)}{b^{4}}\right)\sqrt{bx^{2}+a}x-\frac{\left(6\,a^{2}bc-a^{3}d\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4
*c + a^2*b^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)
*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.56 $\int (a+bx^2)^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] time = 0.0384712, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi in Sympy [A] time = 4.31262, size = 60, normalized size = 0.92

$$\frac{3a^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3ax\sqrt{a+bx^2}}{8}+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 3*a**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*sqrt(b)) + 3*a*x*sqrt
(a + b*x**2)/8 + x*(a + b*x**2)**(3/2)/4
```

Mathematica [A] time = 0.0522244, size = 62, normalized size = 0.95

$$\frac{3a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8\sqrt{b}}+\sqrt{a+bx^2}\left(\frac{5ax}{8}+\frac{bx^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2),x]
```

```
[Out] Sqrt[a + b*x^2]*((5*a*x)/8 + (b*x^3)/4) + (3*a^2*Log[b*x + Sqrt[b
]*Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Maple [A] time = 0.003, size = 51, normalized size = 0.8

$$\frac{x}{4}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,ax}{8}\sqrt{bx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a*x*(b*x^2+a)^(1/2)+3/8*a^2/b^(1/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.222591, size = 1, normalized size = 0.02

$$\left[\frac{3\,a^2\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\,\left(2\,bx^3+5\,ax\right)\sqrt{bx^2+a}\sqrt{b}}{16\,\sqrt{b}},\frac{3\,a^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(2\,bx^3+5\,ax\right)\sqrt{bx^2+a}}{8\,\sqrt{-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))
+ 2*(2*b*x^3 + 5*a*x)*sqrt(b*x^2 + a)*sqrt(b))/sqrt(b), 1/8*(3*a^
```

```
2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*b*x^3 + 5*a*x)*sqrt(b*x
^2 + a)*sqrt(-b))/sqrt(-b)]
```

Sympy [A] time = 10.0578, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**
2/a)/4 + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

GIAC/XCAS [A] time = 0.26475, size = 66, normalized size = 1.02

$$\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x-\frac{3\,a^2\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*ln(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/sqrt(b)
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\, dx$

Optimal. Leaf size=113

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}-\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[
(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rubi [A] time = 0.289931, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}-\frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[
(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rubi in Sympy [A] time = 39.9871, size = 102, normalized size = 0.9

$$\frac{\sqrt{b}\,(3ad-2bc)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2}+\frac{bx\sqrt{a+bx^2}}{2d}+\frac{(ad-bc)^{\frac{3}{2}}\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] sqrt(b)*(3*a*d - 2*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*d**2
) + b*x*sqrt(a + b*x**2)/(2*d) + (a*d - b*c)**(3/2)*atan(x*sqrt(a
*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*d**2)
```

Mathematica [A] time = 0.318346, size = 110, normalized size = 0.97

$$\frac{\sqrt{b}(3ad-2bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}+bdx\sqrt{a+bx^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2] + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] + Sqrt[b]*(-2*b
*c + 3*a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*d^2)
```

Maple [B] time = 0.025, size = 1845, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-
c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/d*
b^(1/2)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-
c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2))*a+1/2/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a-1/2/(-c*d)^(1/2)/
d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+(x-(-c
*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c-1/2/(-c*d)^(1/2)/((a*d
-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*
a^2+1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^
(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2))/(x-(-c*d)^(1/2)/d))*a*b*c-1/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d
)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2
*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b^2*c^2
-1/6/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4*b/d*((x+(-c*d)^(1/2)/d)^2*b
-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/d
*b^(1/2)*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))*a-1/2/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
```

```
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a+1/2/(-c*d)^(1/2
)/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)*b*c-1/2/d^2*b^(3/2)*ln((-b*(-c*d)^(1/2)/d+(x+
(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c+1/2/(-c*d)^(1/2)/((
a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))*a^2-1/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b
*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*
d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2))/(x+(-c*d)^(1/2)/d))*a*b*c+1/2/(-c*d)^(1/2)/d^2/((a*d-b*c
)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d
)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b^2*
c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.411704, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{bx^2+a}abdx-(2bc-3ad)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)-(bc-ad)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8b^2c^2-8abcd+a^2d^2)x^4+a^2c^2+2(\cdots}{\cdots}\right.}{4d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c -
a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*
(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)
*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)
))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x - 2*(2*b*c - 3*a*d)*sqrt(-b)
*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - (b*c - a*d)*sqrt((b*c -
 a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
```

```
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3
)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2
)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c
- a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*
c*x*sqrt(-(b*c - a*d)/c))) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*
d*x - (2*b*c - 3*a*d)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-
b))) - (b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d
)*x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{3}{2}}}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.283003, size = 205, normalized size = 1.81

$$\frac{\sqrt{bx^2+a}bx}{2\,d} + \frac{\left(2\,b^{\frac{3}{2}}c - 3\,a\sqrt{b}d\right)\ln\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{4\,d^2} - \frac{\left(b^{\frac{5}{2}}c^2 - 2\,ab^{\frac{3}{2}}cd + a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b*x/d + 1/4*(2*b^(3/2)*c - 3*a*sqrt(b)*d)*ln(
(sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 - (b^(5/2)*c^2 - 2*a*b^(3/2)
*c*d + a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))
^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*
b*c*d)*d^2)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\, dx$

Optimal. Leaf size=131

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) + (b^(3/2)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*
c + a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(2*c^(3/2)*d^2)
```

Rubi [A] time = 0.248833, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{bc-ad}(ad+2bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}-\frac{x\sqrt{a+bx^2}(bc-ad)}{2cd\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d*(c + d*x^2)) + (b^(3/2)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*
c + a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(2*c^(3/2)*d^2)
```

Rubi in Sympy [A] time = 40.6716, size = 114, normalized size = 0.87

$$\frac{b^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}+\frac{x\sqrt{a+bx^2}\,(ad-bc)}{2cd\,(c+dx^2)}+\frac{\sqrt{ad-bc}\,(ad+2bc)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] b**(3/2)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/d**2 + x*sqrt(a + b*x*
*2)*(a*d - b*c)/(2*c*d*(c + d*x**2)) + sqrt(a*d - b*c)*(a*d + 2*b
*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(2*c**(3/2
)*d**2)
```

Mathematica [A] time = 0.199012, size = 142, normalized size = 1.08

$$\frac{\frac{(a^2d^2+abcd-2b^2c^2)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{ad-bc}}+2b^{3/2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)-\frac{dx\sqrt{a+bx^2}(bc-ad)}{c(c+dx^2)}}{2d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] (-((d*(b*c - a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2))) + ((-2*b^2*
c^2 + a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*S
qrt[a + b*x^2])])/(c^(3/2)*Sqrt[-(b*c) + a*d]) + 2*b^(3/2)*Log[b*
x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*d^2)
```

Maple [B] time = 0.033, size = 4621, normalized size = 35.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b+1/4/(-c*d)^(1/2)/c*((x-(-c*d
)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2)*a-1/4/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*b-1/4/(-c*d)^(1/2)/c*
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(1/2)*a+1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5
/2)+1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)-1/12/(-c*d
)^(1/2)/c*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(3/2)-1/4/d^2*b^(3/2)*ln((-b*(-c*d)^(1/2)/d+(
x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/12/(-c*d)^(1/2)/c
*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(3/2)-1/4/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(
1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+3/4/c/d*b*(-c*d)^(1/2)/(a*d-b
*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(
1/2)/d))*a^2-3/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)
*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-
b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
```

```
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2+3/8/d*b^2/
(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/2/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)/((
a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d
))*a+3/4*c/d^3*b^3*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln(
(2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)
/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/4/c/d*b*(-c*d)^(
1/2)/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a+3/2/d^2*b^2*(-c*d)^(1/2)/(a*d-b*
c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-
c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1
/2)/d))*a-3/4*c/d^3*b^3*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2
)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d
-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/4/c/d*b*(-c
*d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a-1/4/c/d*b*(-c*d)^(1/2)/(a*d
-b*c)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(3/2)-3/8/c*b/(a*d-b*c)*a*((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/2/(
-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(
x-(-c*d)^(1/2)/d))*a*b-1/4/(-c*d)^(1/2)*c/d^2/((a*d-b*c)/d)^(1/2)
*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-
b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b^2+1/4/c/d*b*
(-c*d)^(1/2)/(a*d-b*c)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-3/8/c*b/(a*d-b*c)*a*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2)*x-1/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c
)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*(
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a*b+1/4/(-c*d)^(1/2)*c/d^2/((
a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))*b^2+3/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/4*c/d
^2*b^(5/2)/(a*d-b*c)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)/b
^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2))-1/4/c*b/(a*d-b*c)*((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-3/8/c
*b^(1/2)/(a*d-b*c)*a^2*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d)*b)
/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2))+3/8/d*b^2/(a*d-b*c)*((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+9
/8/d*b^(3/2)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b
)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2))*a-3/4/d^2*b^2*(-c*d)^(1/2)/(a*d-b*c)*(
(x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)-3/4*c/d^2*b^(5/2)/(a*d-b*c)*ln((-b*(-c*d)^(1/2)/d+
```

```
(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^
(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))-1/4/c*b/(a*d-b*c)*
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(3/2)*x-3/8/c*b^(1/2)/(a*d-b*c)*a^2*ln((-b*(-c*d)^(1/2)
/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))+1/8/c*b/d*((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2)*x+3/8/c/d*b^(1/2)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)
/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a-1/4/(-c*d)^(1/2)/c/((a*d-b*c)/d
)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2
*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2+1/8
/c*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)*x+3/8/c/d*b^(1/2)*ln((-b*(-c*d)^(1/2)/d+(x
+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1
/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a+1/4/(-c*d)^(1/2)/c
/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c
*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2
)/d))*a^2+9/8/d*b^(3/2)/(a*d-b*c)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^
(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)
```

Fricas [A] time = 0.365819, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2
)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*
c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a
```

```
^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)
*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(c*d^3*x^2 +
c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 8*(b*c*d*x^
2 + b*c^2)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) - (2*b
*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*
a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a
)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(c*d^3*x^2 +
 c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 +
 a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*
((2*b*c - a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/
c))) - 2*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^
2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(b*x/
(sqrt(b*x^2 + a)*sqrt(-b))) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2
)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^2 + a*c)
/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))))/(c*d^3*x^2 + c^2*d^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{3}{2}}}{(c + dx^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.642145, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.59 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=113

$$\frac{3a^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2
])/(8*c^2*(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt
[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.161113, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{3a^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2
])/(8*c^2*(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt
[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 26.0181, size = 100, normalized size = 0.88

$$\frac{3a^2\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{\frac{5}{2}}\sqrt{ad-bc}}+\frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)}+\frac{x(a+bx^2)^{\frac{3}{2}}}{4c(c+dx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] 3*a**2*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(8*c**(
5/2)*sqrt(a*d - b*c)) + 3*a*x*sqrt(a + b*x**2)/(8*c**2*(c + d*x**
2)) + x*(a + b*x**2)**(3/2)/(4*c*(c + d*x**2)**2)
```

Mathematica [A] time = 0.206014, size = 103, normalized size = 0.91

$$\frac{3a^2\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{ad-bc}}+\frac{\sqrt{a+bx^2}\left(5acx+3adx^3+2bcx^3\right)}{8c^2\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[a + b*x^2]*(5*a*c*x + 2*b*c*x^3 + 3*a*d*x^3))/(8*c^2*(c + d
*x^2)^2) + (3*a^2*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a +
 b*x^2])])/(8*c^(5/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] time = 0.041, size = 9059, normalized size = 80.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

Fricas [A] time = 0.318796, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(2\,bc+3\,ad\right)x^3+5\,acx\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}+3\left(a^2d^2x^4+2\,a^2cdx^2+a^2c^2\right)\log\left(\frac{\left(\left(8\,b^2c^2-8\,abcd+a^2d^2\right)x^4+a^2c^2+2\left(4\,abc^2-3\,a^2c\right.\right.}{}\right.}{32\left(c^2d^2x^4+2\,c^3dx^2+c^4\right)\sqrt{bc^2-acd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(4*((2*b*c + 3*a*d)*x^3 + 5*a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt
(b*x^2 + a) + 3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*log((((8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a
^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d +
a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^2*x
^4 + 2*c*d*x^2 + c^2)))/((c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)*sqrt(b
*c^2 - a*c*d)), 1/16*(2*((2*b*c + 3*a*d)*x^3 + 5*a*c*x)*sqrt(-b*c
^2 + a*c*d)*sqrt(b*x^2 + a) + 3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^
2*c^2)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/
((b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((c^2*d^2*x^4 + 2*c^3*d*x^2
 + c^4)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.48542, size = 4, normalized size = 0.04

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx$

Optimal. Leaf size=199

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3(c+dx^2)(bc-ad)}+\frac{x(a+bx^2)^{3/2}(6bc-5ad)}{24c^2(c+dx^2)^2(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^(5/2))/(6*c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*
c - 5*a*d)*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2
) + (a*(6*b*c - 5*a*d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c
+ d*x^2)) + (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.306788, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3(c+dx^2)(bc-ad)}+\frac{x(a+bx^2)^{3/2}(6bc-5ad)}{24c^2(c+dx^2)^2(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c(c+dx^2)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```


```
[Out] -(d*x*(a + b*x^2)^(5/2))/(6*c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*
c - 5*a*d)*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2
) + (a*(6*b*c - 5*a*d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c
+ d*x^2)) + (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 43.4536, size = 175, normalized size = 0.88

$$\frac{a^2(5ad-6bc)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{\frac{7}{2}}(ad-bc)^{\frac{3}{2}}}+\frac{ax\sqrt{a+bx^2}(5ad-6bc)}{16c^3(c+dx^2)(ad-bc)}+\frac{dx(a+bx^2)^{\frac{5}{2}}}{6c(c+dx^2)^3(ad-bc)}+\frac{x(a+bx^2)^{\frac{3}{2}}(5ad-6bc)}{24c^2(c+dx^2)^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

```
[Out] a**2*(5*a*d - 6*b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x
**2)))/(16*c**(7/2)*(a*d - b*c)**(3/2)) + a*x*sqrt(a + b*x**2)*(5
*a*d - 6*b*c)/(16*c**3*(c + d*x**2)*(a*d - b*c)) + d*x*(a + b*x**
2)**(5/2)/(6*c*(c + d*x**2)**3*(a*d - b*c)) + x*(a + b*x**2)**(3/
2)*(5*a*d - 6*b*c)/(24*c**2*(c + d*x**2)**2*(a*d - b*c))
```

Mathematica [A] time = 0.371867, size = 178, normalized size = 0.89

$$\frac{\frac{\sqrt{c}x\sqrt{a+bx^2}\left(a^2(-d)\left(33c^2+40cdx^2+15d^2x^4\right)+2abc\left(15c^2+11cdx^2+4d^2x^4\right)+4b^2c^2x^2\left(3c+dx^2\right)\right)}{\left(c+dx^2\right)^3(bc-ad)}-\frac{3a^2(6bc-5ad)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{(ad-bc)^{3/2}}}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] ((Sqrt[c]*x*Sqrt[a + b*x^2]*(4*b^2*c^2*x^2*(3*c + d*x^2) + 2*a*b*
c*(15*c^2 + 11*c*d*x^2 + 4*d^2*x^4) - a^2*d*(33*c^2 + 40*c*d*x^2
+ 15*d^2*x^4)))/((b*c - a*d)*(c + d*x^2)^3) - (3*a^2*(6*b*c - 5*a
*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(-(
b*c) + a*d)^(3/2))/(48*c^(7/2))
```

Maple [B] time = 0.059, size = 13766, normalized size = 69.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

Fricas [A] time = 0.564068, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(4*((4*b^2*c^2*d + 8*a*b*c*d^2 - 15*a^2*d^3)*x^5 + 2*(6*b^
2*c^3 + 11*a*b*c^2*d - 20*a^2*c*d^2)*x^3 + 3*(10*a*b*c^3 - 11*a^2
*c^2*d)*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a) + 3*(6*a^2*b*c^4 -
 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3*(6*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^2)*lo
g((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^
2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c
^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))
/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((b*c^7 - a*c^6*d + (b*c^4*d^3 - a
*c^3*d^4)*x^6 + 3*(b*c^5*d^2 - a*c^4*d^3)*x^4 + 3*(b*c^6*d - a*c^
5*d^2)*x^2)*sqrt(b*c^2 - a*c*d)), 1/96*(2*((4*b^2*c^2*d + 8*a*b*c
*d^2 - 15*a^2*d^3)*x^5 + 2*(6*b^2*c^3 + 11*a*b*c^2*d - 20*a^2*c*d
^2)*x^3 + 3*(10*a*b*c^3 - 11*a^2*c^2*d)*x)*sqrt(-b*c^2 + a*c*d)*s
qrt(b*x^2 + a) + 3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 -
5*a^3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2
*b*c^3*d - 5*a^3*c^2*d^2)*x^2)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((
2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((b
*c^7 - a*c^6*d + (b*c^4*d^3 - a*c^3*d^4)*x^6 + 3*(b*c^5*d^2 - a*c
^4*d^3)*x^4 + 3*(b*c^6*d - a*c^5*d^2)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 28.9007, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.61 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx$

Optimal. Leaf size=300

$$\frac{a^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{48c^2d\left(c+dx^2\right)^3}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c*d*(c + d*x^2)^4) + ((2*b*c
+ 7*a*d)*x*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^
2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c -
 a*d)*(c + d*x^2)^2) + ((16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*
c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/(384*c^4*d*(b*c - a*d)^2*
(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTan
h[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/2)*(b
*c - a*d)^(5/2))
```

Rubi [A] time = 0.91508, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{a^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}+\frac{x\sqrt{a+bx^2}\left(-35a^2d^2+24abcd+8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2(bc-ad)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)(bc-ad)^2}+\frac{x\sqrt{a+bx^2}(7ad+2bc)}{48c^2d\left(c+dx^2\right)^3}-\frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\left(c+dx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c*d*(c + d*x^2)^4) + ((2*b*c
+ 7*a*d)*x*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^
2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c -
 a*d)*(c + d*x^2)^2) + ((16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*
c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2])/(384*c^4*d*(b*c - a*d)^2*
(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTan
h[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/2)*(b
*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 158.453, size = 277, normalized size = 0.92

$$\frac{a^2\left(35a^2d^2-80abcd+48b^2c^2\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{\frac{9}{2}}\left(ad-bc\right)^{\frac{5}{2}}}+\frac{x\sqrt{a+bx^2}\left(ad-bc\right)}{8cd\left(c+dx^2\right)^4}+\frac{x\sqrt{a+bx^2}\left(7ad+2bc\right)}{48c^2d\left(c+dx^2\right)^3}+\frac{x\sqrt{a+bx^2}\left(35a^2d^2-24abcd-8b^2c^2\right)}{192c^3d\left(c+dx^2\right)^2\left(ad-bc\right)}+\frac{x\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{384c^4d\left(c+dx^2\right)\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
```

```
[Out] a**2*(35*a**2*d**2 - 80*a*b*c*d + 48*b**2*c**2)*atan(x*sqrt(a*d -
 b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(128*c**(9/2)*(a*d - b*c)**(5/2
)) + x*sqrt(a + b*x**2)*(a*d - b*c)/(8*c*d*(c + d*x**2)**4) + x*s
qrt(a + b*x**2)*(7*a*d + 2*b*c)/(48*c**2*d*(c + d*x**2)**3) + x*s
qrt(a + b*x**2)*(35*a**2*d**2 - 24*a*b*c*d - 8*b**2*c**2)/(192*c*
*3*d*(c + d*x**2)**2*(a*d - b*c)) + x*sqrt(a + b*x**2)*(105*a**3*
d**3 - 170*a**2*b*c*d**2 + 40*a*b**2*c**2*d + 16*b**3*c**3)/(384*
c**4*d*(c + d*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.452451, size = 260, normalized size = 0.87

$$\frac{\frac{3a^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{ad-bc}}-\frac{\sqrt{c}x\sqrt{a+bx^2}\left(-2c\left(c+dx^2\right)^2\left(-35a^2d^2+24abcd+8b^2c^2\right)\left(bc-ad\right)-\left(c+dx^2\right)^3\left(105a^3d^3-170a^2bcd^2+40ab^2c^2\right.}{d\left(c+dx^2\right)^4}}{384c^{9/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (-((Sqrt[c]*x*Sqrt[a + b*x^2]*(48*c^3*(b*c - a*d)^3 - 8*c^2*(b*c
- a*d)^2*(2*b*c + 7*a*d)*(c + d*x^2) - 2*c*(b*c - a*d)*(8*b^2*c^2
 + 24*a*b*c*d - 35*a^2*d^2)*(c + d*x^2)^2 - (16*b^3*c^3 + 40*a*b^
2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*(c + d*x^2)^3))/(d*(c +
d*x^2)^4)) + (3*a^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTan
[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[-(b*c) +
 a*d])/(384*c^(9/2)*(b*c - a*d)^2)
```

Maple [B] time = 0.075, size = 18791, normalized size = 62.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{2}}}{(dx^2 + c)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

Fricas [A] time = 2.4559, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(4*((16*b^3*c^3*d^2 + 40*a*b^2*c^2*d^3 - 170*a^2*b*c*d^4
+ 105*a^3*d^5)*x^7 + (64*b^3*c^4*d + 152*a*b^2*c^3*d^2 - 628*a^2*
b*c^2*d^3 + 385*a^3*c*d^4)*x^5 + (96*b^3*c^5 + 208*a*b^2*c^4*d -
842*a^2*b*c^3*d^2 + 511*a^3*c^2*d^3)*x^3 + 3*(80*a*b^2*c^5 - 176*
a^2*b*c^4*d + 93*a^3*c^3*d^2)*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
 a) + 3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a
^2*b^2*c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2
*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c
^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c
^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2)*log((((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x
^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2
)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*
d*x^2 + c^2)))/((b^2*c^10 - 2*a*b*c^9*d + a^2*c^8*d^2 + (b^2*c^6*
d^4 - 2*a*b*c^5*d^5 + a^2*c^4*d^6)*x^8 + 4*(b^2*c^7*d^3 - 2*a*b*c
^6*d^4 + a^2*c^5*d^5)*x^6 + 6*(b^2*c^8*d^2 - 2*a*b*c^7*d^3 + a^2*
c^6*d^4)*x^4 + 4*(b^2*c^9*d - 2*a*b*c^8*d^2 + a^2*c^7*d^3)*x^2)*s
qrt(b*c^2 - a*c*d)), 1/768*(2*((16*b^3*c^3*d^2 + 40*a*b^2*c^2*d^3
 - 170*a^2*b*c*d^4 + 105*a^3*d^5)*x^7 + (64*b^3*c^4*d + 152*a*b^2
*c^3*d^2 - 628*a^2*b*c^2*d^3 + 385*a^3*c*d^4)*x^5 + (96*b^3*c^5 +
```

```
 208*a*b^2*c^4*d - 842*a^2*b*c^3*d^2 + 511*a^3*c^2*d^3)*x^3 + 3*(
80*a*b^2*c^5 - 176*a^2*b*c^4*d + 93*a^3*c^3*d^2)*x)*sqrt(-b*c^2 +
 a*c*d)*sqrt(b*x^2 + a) + 3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35
*a^4*c^4*d^2 + (48*a^2*b^2*c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)
*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x
^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x
^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2
)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c
^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((b^2*c^10 - 2*a*b*c^9*d + a^2*c
^8*d^2 + (b^2*c^6*d^4 - 2*a*b*c^5*d^5 + a^2*c^4*d^6)*x^8 + 4*(b^2
*c^7*d^3 - 2*a*b*c^6*d^4 + a^2*c^5*d^5)*x^6 + 6*(b^2*c^8*d^2 - 2*
a*b*c^7*d^3 + a^2*c^6*d^4)*x^4 + 4*(b^2*c^9*d - 2*a*b*c^8*d^2 + a
^2*c^7*d^3)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.5878, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5,x, algorithm="giac")

[Out] sage0*x
```

3.62 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=349

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right)^{3/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1536b^3}+\frac{a^2x\sqrt{a+bx^2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1024b^3}+\frac{a^3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)(16bc-5ad)}{120b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)
*x*Sqrt[a + b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*
d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) +
 ((320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*
(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*b^2*c^2 - 68*a*b*c*d + 15
*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*b*c - 5*a*d)*x*
(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^(7/2)
*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36
*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
1024*b^(7/2))
```

Rubi [A] time = 0.56244, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{x\left(a+bx^2\right)^{5/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right)^{3/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1536b^3}+\frac{a^2x\sqrt{a+bx^2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1024b^3}+\frac{a^3\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)(16bc-5ad)}{120b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)
*x*Sqrt[a + b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*
d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) +
 ((320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*
(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*b^2*c^2 - 68*a*b*c*d + 15
*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*b*c - 5*a*d)*x*
(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^(7/2)
*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36
*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
1024*b^(7/2))
```

Rubi in Sympy [A] time = 65.8349, size = 352, normalized size = 1.01

$$-\frac{a^3\left(5a^3d^3-36a^2bcd^2+120ab^2c^2d-320b^3c^3\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{\frac{7}{2}}} - \frac{a^2x\sqrt{a+bx^2}\left(5a^3d^3-36a^2bcd^2+120ab^2c^2d-320b^3c^3\right)}{1024b^3} - \frac{ax\left(a+bx^2\right)^{\frac{3}{2}}\left(5a^3d^3-36a^2bcd^2+120ab^2c^2d-320b^3c^3\right)}{1536b^3} + \frac{dx\left(a+bx^2\right)^{\frac{7}{2}}\left(c+dx^2\right)^2}{12b} - \frac{dx\left(a+bx^2\right)^{\frac{7}{2}}\left(c+dx^2\right)\left(5ad-16bc\right)}{120b^2} + \frac{dx\left(a+bx^2\right)^{\frac{7}{2}}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3} - \frac{x\left(a+bx^2\right)^{\frac{5}{2}}\left(5a^3d^3-36a^2bcd^2+120ab^2c^2d-320b^3c^3\right)}{1920b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

```
[Out] -a**3*(5*a**3*d**3 - 36*a**2*b*c*d**2 + 120*a*b**2*c**2*d - 320*b
**3*c**3)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(1024*b**(7/2)) - a**
2*x*sqrt(a + b*x**2)*(5*a**3*d**3 - 36*a**2*b*c*d**2 + 120*a*b**2
*c**2*d - 320*b**3*c**3)/(1024*b**3) - a*x*(a + b*x**2)**(3/2)*(5
*a**3*d**3 - 36*a**2*b*c*d**2 + 120*a*b**2*c**2*d - 320*b**3*c**3
)/(1536*b**3) + d*x*(a + b*x**2)**(7/2)*(c + d*x**2)**2/(12*b) -
d*x*(a + b*x**2)**(7/2)*(c + d*x**2)*(5*a*d - 16*b*c)/(120*b**2)
+ d*x*(a + b*x**2)**(7/2)*(15*a**2*d**2 - 68*a*b*c*d + 152*b**2*c
**2)/(960*b**3) - x*(a + b*x**2)**(5/2)*(5*a**3*d**3 - 36*a**2*b*
c*d**2 + 120*a*b**2*c**2*d - 320*b**3*c**3)/(1920*b**3)
```

Mathematica [A] time = 0.30782, size = 270, normalized size = 0.77

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(75a^5d^3-10a^4bd^2\left(54c+5dx^2\right)+40a^3b^2d\left(45c^2+9cdx^2+d^2x^4\right)+48a^2b^3\left(220c^3+295c^2dx^2+186cd^2x^4+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d
*x^2) + 40*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4
*(20*c^3 + 45*c^2*d*x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3
*(220*c^3 + 295*c^2*d*x^2 + 186*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^
4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189*c*d^2*x^4 + 50*d^3*x^6)) - 1
5*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 5*a^3*d^
3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

Maple [A] time = 0.018, size = 476, normalized size = 1.4

$$\frac{c^3x}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,ac^3x}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2c^3x}{16}\sqrt{bx^2+a}+\frac{5\,c^3a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$
$$+\frac{d^3x^5}{12\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{ad^3x^3}{24\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{d^3a^2x}{64\,b^3}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{a^3d^3x}{384\,b^3}\left(bx^2+a\right)^{\frac{5}{2}}$$
$$-\frac{5\,d^3a^4x}{1536\,b^3}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,d^3a^5x}{1024\,b^3}\sqrt{bx^2+a}-\frac{5\,d^3a^6}{1024}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}$$
$$+\frac{3\,cd^2x^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{9\,cd^2ax}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{3\,a^2cd^2x}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}$$
$$+\frac{3\,cd^2a^3x}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{9\,cd^2a^4x}{256\,b^2}\sqrt{bx^2+a}+\frac{9\,cd^2a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$
$$+\frac{3\,c^2dx}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{ac^2dx}{16\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,c^2da^2x}{64\,b}\left(bx^2+a\right)^{\frac{3}{2}}$$
$$-\frac{15\,c^2da^3x}{128\,b}\sqrt{bx^2+a}-\frac{15\,c^2da^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x)
```

```
[Out] 1/6*c^3*x*(b*x^2+a)^(5/2)+5/24*c^3*a*x*(b*x^2+a)^(3/2)+5/16*c^3*a
^2*x*(b*x^2+a)^(1/2)+5/16*c^3*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))+1/12*d^3*x^5*(b*x^2+a)^(7/2)/b-1/24*d^3*a/b^2*x^3*(b*x^2+a
)^(7/2)+1/64*d^3*a^2/b^3*x*(b*x^2+a)^(7/2)-1/384*d^3*a^3/b^3*x*(b
*x^2+a)^(5/2)-5/1536*d^3*a^4/b^3*x*(b*x^2+a)^(3/2)-5/1024*d^3*a^5
/b^3*x*(b*x^2+a)^(1/2)-5/1024*d^3*a^6/b^(7/2)*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))+3/10*c*d^2*x^3*(b*x^2+a)^(7/2)/b-9/80*c*d^2*a/b^2*x*(b
*x^2+a)^(7/2)+3/160*c*d^2*a^2/b^2*x*(b*x^2+a)^(5/2)+3/128*c*d^2*a
^3/b^2*x*(b*x^2+a)^(3/2)+9/256*c*d^2*a^4/b^2*x*(b*x^2+a)^(1/2)+9/
```

```
256*c*d^2*a^5/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/8*c^2*d*x*(
b*x^2+a)^(7/2)/b-1/16*c^2*d*a/b*x*(b*x^2+a)^(5/2)-5/64*c^2*d*a^2/
b*x*(b*x^2+a)^(3/2)-15/128*c^2*d*a^3/b*x*(b*x^2+a)^(1/2)-15/128*c
^2*d*a^4/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24011, size = 1, normalized size = 0.

$$\left[\frac{2\left(1280\,b^5d^3x^{11} + 128\left(36\,b^5cd^2 + 25\,ab^4d^3\right)x^9 + 144\left(40\,b^5c^2d + 84\,ab^4cd^2 + 15\,a^2b^3d^3\right)x^7 + 8\left(320\,b^5c^3 + 2040\,ab^4c^2d + 1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/30720*(2*(1280*b^5*d^3*x^11 + 128*(36*b^5*c*d^2 + 25*a*b^4*d^3
)*x^9 + 144*(40*b^5*c^2*d + 84*a*b^4*c*d^2 + 15*a^2*b^3*d^3)*x^7
+ 8*(320*b^5*c^3 + 2040*a*b^4*c^2*d + 1116*a^2*b^3*c*d^2 + 5*a^3*
b^2*d^3)*x^5 + 10*(832*a*b^4*c^3 + 1416*a^2*b^3*c^2*d + 36*a^3*b^
2*c*d^2 - 5*a^4*b*d^3)*x^3 + 15*(704*a^2*b^3*c^3 + 120*a^3*b^2*c^
2*d - 36*a^4*b*c*d^2 + 5*a^5*d^3)*x)*sqrt(b*x^2 + a)*sqrt(b) - 15
*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^
3)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(7/2), 1
/15360*((1280*b^5*d^3*x^11 + 128*(36*b^5*c*d^2 + 25*a*b^4*d^3)*x^
9 + 144*(40*b^5*c^2*d + 84*a*b^4*c*d^2 + 15*a^2*b^3*d^3)*x^7 + 8*
(320*b^5*c^3 + 2040*a*b^4*c^2*d + 1116*a^2*b^3*c*d^2 + 5*a^3*b^2*
d^3)*x^5 + 10*(832*a*b^4*c^3 + 1416*a^2*b^3*c^2*d + 36*a^3*b^2*c*
d^2 - 5*a^4*b*d^3)*x^3 + 15*(704*a^2*b^3*c^3 + 120*a^3*b^2*c^2*d
- 36*a^4*b*c*d^2 + 5*a^5*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 15*(3
20*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.240651, size = 433, normalized size = 1.24

$$\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2d^3x^2 + \frac{36\,b^{12}cd^2 + 25\,ab^{11}d^3}{b^{10}}\right)x^2 + \frac{9\left(40\,b^{12}c^2d + 84\,ab^{11}cd^2 + 15\,a^2b^{10}d^3\right)}{b^{10}}\right)x^2 + \frac{320\,b^{12}c^3 + 2040}{}\right.\right.\right.$$

$$-\frac{\left(320\,a^3b^3c^3 - 120\,a^4b^2c^2d + 36\,a^5bcd^2 - 5\,a^6d^3\right)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*
d^3)/b^10)*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10
*d^3)/b^10)*x^2 + (320*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^
10*c*d^2 + 5*a^3*b^9*d^3)/b^10)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^
2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*a^4*b^8*d^3)/b^10)*x^2 + 15*(
704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4*b^8*c*d^2 + 5*a^5*b
^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*c^3 - 120*a
^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*ln(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(7/2)
```

3.63 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=241

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{256b^2}+\frac{a^3\left(3a^2d^2-20abcd+80b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^2) + (a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(
3/2))/(384*b^2) + ((80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b
*x^2)^(5/2))/(480*b^2) + (3*d*(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/
(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(10*b) + (a^3*(80*
b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(256*b^(5/2))
```

Rubi [A] time = 0.323662, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{x\left(a+bx^2\right)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}+\frac{ax\left(a+bx^2\right)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{256b^2}+\frac{a^3\left(3a^2d^2-20abcd+80b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3dx\left(a+bx^2\right)^{7/2}(4bc-ad)}{80b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^2) + (a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(
3/2))/(384*b^2) + ((80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b
*x^2)^(5/2))/(480*b^2) + (3*d*(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/
(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(10*b) + (a^3*(80*
b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(256*b^(5/2))
```

Rubi in Sympy [A] time = 32.758, size = 238, normalized size = 0.99

$$\frac{a^3\left(3a^2d^2-20abcd+80b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{\frac{5}{2}}}+\frac{a^2x\sqrt{a+bx^2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{256b^2}+\frac{ax\left(a+bx^2\right)^{\frac{3}{2}}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2}+\frac{dx\left(a+bx^2\right)^{\frac{7}{2}}\left(c+dx^2\right)}{10b}-\frac{3dx\left(a+bx^2\right)^{\frac{7}{2}}(ad-4bc)}{80b^2}+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] a**3*(3*a**2*d**2 - 20*a*b*c*d + 80*b**2*c**2)*atanh(sqrt(b)*x/sq
rt(a + b*x**2))/(256*b**(5/2)) + a**2*x*sqrt(a + b*x**2)*(3*a**2*
d**2 - 20*a*b*c*d + 80*b**2*c**2)/(256*b**2) + a*x*(a + b*x**2)**
(3/2)*(3*a**2*d**2 - 20*a*b*c*d + 80*b**2*c**2)/(384*b**2) + d*x*
(a + b*x**2)**(7/2)*(c + d*x**2)/(10*b) - 3*d*x*(a + b*x**2)**(7/
2)*(a*d - 4*b*c)/(80*b**2) + x*(a + b*x**2)**(5/2)*(3*a**2*d**2 -
 20*a*b*c*d + 80*b**2*c**2)/(480*b**2)
```

Mathematica [A] time = 0.211399, size = 191, normalized size = 0.79

$$\frac{15a^3\left(3a^2d^2-20abcd+80b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}x\sqrt{a+bx^2}\left(-45a^4d^2+30a^3bd\left(10c+dx^2\right)+8a^2b^2\left(330c^2+2\right.\right.}{3840b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*d^2 + 30*a^3*b*d*(10*c + d*x^
2) + 64*b^4*x^4*(10*c^2 + 15*c*d*x^2 + 6*d^2*x^4) + 16*a*b^3*x^2*
(130*c^2 + 170*c*d*x^2 + 63*d^2*x^4) + 8*a^2*b^2*(330*c^2 + 295*c
*d*x^2 + 93*d^2*x^4)) + 15*a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d
^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(3840*b^(5/2))
```

Maple [A] time = 0.011, size = 308, normalized size = 1.3

$$\frac{c^2x}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,ac^2x}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2c^2x}{16}\sqrt{bx^2+a}+\frac{5\,c^2a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^2x^3}{10\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{3\,ad^2x}{80\,b^2}\left(bx^2+a\right)^{\frac{7}{2}}+\frac{a^2d^2x}{160\,b^2}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{d^2a^3x}{128\,b^2}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{3\,d^2a^4x}{256\,b^2}\sqrt{bx^2+a}+\frac{3\,d^2a^5}{256}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{4\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{acdx}{24\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,cda^2x}{96\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,cda^3x}{64\,b}\sqrt{bx^2+a}-\frac{5\,cda^4}{64}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x)
```

```
[Out] 1/6*c^2*x*(b*x^2+a)^(5/2)+5/24*c^2*a*x*(b*x^2+a)^(3/2)+5/16*c^2*a
^2*x*(b*x^2+a)^(1/2)+5/16*c^2*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))+1/10*d^2*x^3*(b*x^2+a)^(7/2)/b-3/80*d^2*a/b^2*x*(b*x^2+a)^
(7/2)+1/160*d^2*a^2/b^2*x*(b*x^2+a)^(5/2)+1/128*d^2*a^3/b^2*x*(b*
x^2+a)^(3/2)+3/256*d^2*a^4/b^2*x*(b*x^2+a)^(1/2)+3/256*d^2*a^5/b^
(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1/4*c*d*x*(b*x^2+a)^(7/2)/b-1
/24*c*d*a/b*x*(b*x^2+a)^(5/2)-5/96*c*d*a^2/b*x*(b*x^2+a)^(3/2)-5/
64*c*d*a^3/b*x*(b*x^2+a)^(1/2)-5/64*c*d*a^4/b^(3/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.556019, size = 1, normalized size = 0.

$$\left[\frac{2\left(384\,b^4d^2x^9+48\left(20\,b^4cd+21\,ab^3d^2\right)x^7+8\left(80\,b^4c^2+340\,ab^3cd+93\,a^2b^2d^2\right)x^5+10\left(208\,ab^3c^2+236\,a^2b^2cd+3\,a^3bd^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(2*(384*b^4*d^2*x^9 + 48*(20*b^4*c*d + 21*a*b^3*d^2)*x^7
+ 8*(80*b^4*c^2 + 340*a*b^3*c*d + 93*a^2*b^2*d^2)*x^5 + 10*(208*a
*b^3*c^2 + 236*a^2*b^2*c*d + 3*a^3*b*d^2)*x^3 + 15*(176*a^2*b^2*c
^2 + 20*a^3*b*c*d - 3*a^4*d^2)*x)*sqrt(b*x^2 + a)*sqrt(b) + 15*(8
0*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*log(-2*sqrt(b*x^2 + a)*
b*x - (2*b*x^2 + a)*sqrt(b)))/b^(5/2), 1/3840*((384*b^4*d^2*x^9 +
 48*(20*b^4*c*d + 21*a*b^3*d^2)*x^7 + 8*(80*b^4*c^2 + 340*a*b^3*c
*d + 93*a^2*b^2*d^2)*x^5 + 10*(208*a*b^3*c^2 + 236*a^2*b^2*c*d +
3*a^3*b*d^2)*x^3 + 15*(176*a^2*b^2*c^2 + 20*a^3*b*c*d - 3*a^4*d^2
)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d
 + 3*a^5*d^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 170.682, size = 537, normalized size = 2.23

$$-\frac{3a^{\frac{9}{2}}d^2x}{256b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{7}{2}}cdx}{64b\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{7}{2}}d^2x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}c^2x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}c^2x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}cdx^3}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{129a^{\frac{5}{2}}d^2x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bc^2x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bcdx^5}{96\sqrt{1+\frac{bx^2}{a}}}+\frac{73a^{\frac{3}{2}}bd^2x^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{17\sqrt{a}b^2c^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}b^2cdx^7}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}b^2d^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^5d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}-\frac{5a^4cd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{5a^3c^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^3c^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3cdx^9}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3d^2x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] -3*a**(9/2)*d**2*x/(256*b**2*sqrt(1 + b*x**2/a)) + 5*a**(7/2)*c*d
*x/(64*b*sqrt(1 + b*x**2/a)) - a**(7/2)*d**2*x**3/(256*b*sqrt(1 +
 b*x**2/a)) + a**(5/2)*c**2*x*sqrt(1 + b*x**2/a)/2 + 3*a**(5/2)*c
**2*x/(16*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*c*d*x**3/(192*sqrt(1
 + b*x**2/a)) + 129*a**(5/2)*d**2*x**5/(640*sqrt(1 + b*x**2/a)) +
 35*a**(3/2)*b*c**2*x**3/(48*sqrt(1 + b*x**2/a)) + 127*a**(3/2)*b
*c*d*x**5/(96*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*d**2*x**7/(160*
sqrt(1 + b*x**2/a)) + 17*sqrt(a)*b**2*c**2*x**5/(24*sqrt(1 + b*x*
*2/a)) + 23*sqrt(a)*b**2*c*d*x**7/(24*sqrt(1 + b*x**2/a)) + 29*sq
rt(a)*b**2*d**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*a**5*d**2*asinh(
sqrt(b)*x/sqrt(a))/(256*b**(5/2)) - 5*a**4*c*d*asinh(sqrt(b)*x/sq
rt(a))/(64*b**(3/2)) + 5*a**3*c**2*asinh(sqrt(b)*x/sqrt(a))/(16*s
qrt(b)) + b**3*c**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*c*
d*x**9/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d**2*x**11/(10*sqrt(
a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.366916, size = 298, normalized size = 1.24

$$\frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{20\,b^{10}cd+21\,ab^9d^2}{b^8}\right)x^2+\frac{80\,b^{10}c^2+340\,ab^9cd+93\,a^2b^8d^2}{b^8}\right)x^2+\frac{5\,(208\,ab^9c^2+236\,a^2b^8cd+3\,a}{b^8}\right.\right.$$

$$-\frac{\left(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8
)*x^2 + (80*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 +
 5*(208*a*b^9*c^2 + 236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 1
5*(176*a^2*b^8*c^2 + 20*a^3*b^7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*
x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*ln
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.64 $\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)\, dx$

Optimal. Leaf size=149

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}$$

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a
*d)*x*(a + b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(
5/2))/(48*b) + (d*x*(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*
d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi [A] time = 0.134588, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b}+\frac{x\left(a+bx^2\right)^{5/2}(8bc-ad)}{48b}+\frac{5ax\left(a+bx^2\right)^{3/2}(8bc-ad)}{192b}+\frac{dx\left(a+bx^2\right)^{7/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a
*d)*x*(a + b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(
5/2))/(48*b) + (d*x*(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*
d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rubi in Sympy [A] time = 14.7114, size = 134, normalized size = 0.9

$$-\frac{5a^3\left(ad-8bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{\frac{3}{2}}}-\frac{5a^2x\sqrt{a+bx^2}\left(ad-8bc\right)}{128b}-\frac{5ax\left(a+bx^2\right)^{\frac{3}{2}}\left(ad-8bc\right)}{192b}+\frac{dx\left(a+bx^2\right)^{\frac{7}{2}}}{8b}-\frac{x\left(a+bx^2\right)^{\frac{5}{2}}\left(ad-8bc\right)}{48b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] -5*a**3*(a*d - 8*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(128*b**(
3/2)) - 5*a**2*x*sqrt(a + b*x**2)*(a*d - 8*b*c)/(128*b) - 5*a*x*(
a + b*x**2)**(3/2)*(a*d - 8*b*c)/(192*b) + d*x*(a + b*x**2)**(7/2
)/(8*b) - x*(a + b*x**2)**(5/2)*(a*d - 8*b*c)/(48*b)
```

Mathematica [A] time = 0.132819, size = 121, normalized size = 0.81

$$\sqrt{a+bx^2}\left(\frac{a^2x(5ad+88bc)}{128b}+\frac{1}{48}bx^5(17ad+8bc)+\frac{1}{192}ax^3(59ad+104bc)+\frac{1}{8}b^2dx^7\right)-\frac{5a^3(ad-8bc)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{128b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] Sqrt[a + b*x^2]*((a^2*(88*b*c + 5*a*d)*x)/(128*b) + (a*(104*b*c +
 59*a*d)*x^3)/192 + (b*(8*b*c + 17*a*d)*x^5)/48 + (b^2*d*x^7)/8)
- (5*a^3*(-8*b*c + a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(128*
b^(3/2))
```

Maple [A] time = 0.007, size = 166, normalized size = 1.1

$$\frac{cx}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,acx}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2cx}{16}\sqrt{bx^2+a}+\frac{5\,a^3c}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{dx}{8\,b}\left(bx^2+a\right)^{\frac{7}{2}}-\frac{adx}{48\,b}\left(bx^2+a\right)^{\frac{5}{2}}-\frac{5\,da^2x}{192\,b}\left(bx^2+a\right)^{\frac{3}{2}}-\frac{5\,da^3x}{128\,b}\sqrt{bx^2+a}-\frac{5\,da^4}{128}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)*(d*x^2+c),x)
```

```
[Out] 1/6*c*x*(b*x^2+a)^(5/2)+5/24*c*a*x*(b*x^2+a)^(3/2)+5/16*c*a^2*x*(
b*x^2+a)^(1/2)+5/16*c*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+1
/8*d*x*(b*x^2+a)^(7/2)/b-1/48*d*a/b*x*(b*x^2+a)^(5/2)-5/192*d*a^2
/b*x*(b*x^2+a)^(3/2)-5/128*d*a^3/b*x*(b*x^2+a)^(1/2)-5/128*d*a^4/
b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.325364, size = 1, normalized size = 0.01

$$\left[\frac{2\left(48\,b^3dx^7+8\left(8\,b^3c+17\,ab^2d\right)x^5+2\left(104\,ab^2c+59\,a^2bd\right)x^3+3\left(88\,a^2bc+5\,a^3d\right)x\right)\sqrt{bx^2+a}\sqrt{b}-15\left(8\,a^3bc-a^4d\right)\mathrm{lo}}{768\,b^{\frac{3}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/768*(2*(48*b^3*d*x^7 + 8*(8*b^3*c + 17*a*b^2*d)*x^5 + 2*(104*a
*b^2*c + 59*a^2*b*d)*x^3 + 3*(88*a^2*b*c + 5*a^3*d)*x)*sqrt(b*x^2
 + a)*sqrt(b) - 15*(8*a^3*b*c - a^4*d)*log(2*sqrt(b*x^2 + a)*b*x
- (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/384*((48*b^3*d*x^7 + 8*(8*b^
3*c + 17*a*b^2*d)*x^5 + 2*(104*a*b^2*c + 59*a^2*b*d)*x^3 + 3*(88*
a^2*b*c + 5*a^3*d)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 15*(8*a^3*b*c -
a^4*d)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 88.5209, size = 316, normalized size = 2.12

$$\frac{5a^{\frac{7}{2}}dx}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{a^{\frac{5}{2}}cx\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3a^{\frac{5}{2}}cx}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{133a^{\frac{5}{2}}dx^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}bcx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{127a^{\frac{3}{2}}bdx^5}{192\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{17\sqrt{a}b^2cx^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{23\sqrt{a}b^2dx^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^4d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{5a^3c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{b^3cx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{b^3dx^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] 5*a**(7/2)*d*x/(128*b*sqrt(1 + b*x**2/a)) + a**(5/2)*c*x*sqrt(1 +
 b*x**2/a)/2 + 3*a**(5/2)*c*x/(16*sqrt(1 + b*x**2/a)) + 133*a**(5
/2)*d*x**3/(384*sqrt(1 + b*x**2/a)) + 35*a**(3/2)*b*c*x**3/(48*sq
rt(1 + b*x**2/a)) + 127*a**(3/2)*b*d*x**5/(192*sqrt(1 + b*x**2/a)
) + 17*sqrt(a)*b**2*c*x**5/(24*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b
**2*d*x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*d*asinh(sqrt(b)*x/sqr
t(a))/(128*b**(3/2)) + 5*a**3*c*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt
(b)) + b**3*c*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + b**3*d*x**9/(
```

```
8*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.337106, size = 182, normalized size = 1.22

$$\frac{1}{384} \left(2 \left(4 \left(6 \, b^2 d x^2 + \frac{8 \, b^8 c + 17 \, a b^7 d}{b^6}\right) x^2 + \frac{104 \, a b^7 c + 59 \, a^2 b^6 d}{b^6}\right) x^2 + \frac{3 \left(88 \, a^2 b^6 c + 5 \, a^3 b^5 d\right)}{b^6}\right) \sqrt{b x^2 + a} x - \frac{5 \left(8 \, a^3 b c - a^4 d\right) \ln\left(\left|-\sqrt{b} x + \sqrt{b x^2 + a}\right|\right)}{128 \, b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)*(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104
*a*b^7*c + 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d
)/b^6)*sqrt(b*x^2 + a)*x - 5/128*(8*a^3*b*c - a^4*d)*ln(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.65 $\int \left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=84

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*
(a + b*x^2)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(16*Sqrt[b])
```

Rubi [A] time = 0.052074, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*
(a + b*x^2)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(16*Sqrt[b])
```

Rubi in Sympy [A] time = 5.82938, size = 78, normalized size = 0.93

$$\frac{5a^3\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5a^2x\sqrt{a+bx^2}}{16}+\frac{5ax\left(a+bx^2\right)^{\frac{3}{2}}}{24}+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 5*a**3*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*sqrt(b)) + 5*a**2*x*
sqrt(a + b*x**2)/16 + 5*a*x*(a + b*x**2)**(3/2)/24 + x*(a + b*x**
2)**(5/2)/6
```

Mathematica [A] time = 0.068526, size = 71, normalized size = 0.85

$$\frac{1}{48}\left(\frac{15a^3\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}}+x\sqrt{a+bx^2}\left(33a^2+26abx^2+8b^2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(33*a^2 + 26*a*b*x^2 + 8*b^2*x^4) + (15*a^3*Lo
g[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/Sqrt[b])/48
```

Maple [A] time = 0., size = 66, normalized size = 0.8

$$\frac{x}{6}\left(bx^2+a\right)^{\frac{5}{2}}+\frac{5\,ax}{24}\left(bx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2x}{16}\sqrt{bx^2+a}+\frac{5\,a^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(b*x^2+a)^(5/2)+5/24*a*x*(b*x^2+a)^(3/2)+5/16*a^2*x*(b*x^2+
a)^(1/2)+5/16*a^3/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.23271, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^3\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(8\,b^2x^5+26\,abx^3+33\,a^2x\right)\sqrt{bx^2+a}\sqrt{b}}{96\,\sqrt{b}},\frac{15\,a^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(8\,b^2x^5\right.}{48}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))
 + 2*(8*b^2*x^5 + 26*a*b*x^3 + 33*a^2*x)*sqrt(b*x^2 + a)*sqrt(b))
/sqrt(b), 1/48*(15*a^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^
2*x^5 + 26*a*b*x^3 + 33*a^2*x)*sqrt(b*x^2 + a)*sqrt(-b))/sqrt(-b)
]
```

Sympy [A] time = 14.3182, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 +
 b*x**2/a)/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*a
sinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b))
```

GIAC/XCAS [A] time = 0.285138, size = 85, normalized size = 1.01

$$-\frac{5\,a^3\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2
*(4*b^2*x^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

3.66 $\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx$

Optimal. Leaf size=157

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}-\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}$$

```
[Out] -(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(8*d^2) + (b*x*(a + b*x^2)
^(3/2))/(4*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*
d^3)
```

Rubi [A] time = 0.482411, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}-\frac{bx\sqrt{a+bx^2}(4bc-7ad)}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] -(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(8*d^2) + (b*x*(a + b*x^2)
^(3/2))/(4*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*
d^3)
```

Rubi in Sympy [A] time = 69.3886, size = 146, normalized size = 0.93

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}+\frac{bx\left(a+bx^2\right)^{\frac{3}{2}}}{4d}+\frac{bx\sqrt{a+bx^2}\,(7ad-4bc)}{8d^2}+\frac{(ad-bc)^{\frac{5}{2}}\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] sqrt(b)*(15*a**2*d**2 - 20*a*b*c*d + 8*b**2*c**2)*atanh(sqrt(b)*x
/sqrt(a + b*x**2))/(8*d**3) + b*x*(a + b*x**2)**(3/2)/(4*d) + b*x
*sqrt(a + b*x**2)*(7*a*d - 4*b*c)/(8*d**2) + (a*d - b*c)**(5/2)*a
tan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*d**3)
```

Mathematica [A] time = 0.18899, size = 140, normalized size = 0.89

$$\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+bdx\sqrt{a+bx^2}\left(9ad-4bc+2bdx^2\right)+\frac{8(ad-bc)^{5/2}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}}{8d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) + (8*(-(b*c)
+ a*d)^(5/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/Sqrt[c] + Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log
[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*d^3)
```

Maple [B] time = 0.027, size = 3053, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] -1/6/(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*a-1/2/(-c*d)^(1/2)*((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2)*a^2+1/6/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*a+1/2/(-c*d)^(1/2)*((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2)*a^2+1/8*b/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+15/16/d*b^(1/2)*ln((-b*(
-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2
*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a^2+
1/2/d^3*b^(5/2)*ln((-b*(-c*d)^(1/2)/d+(x+(-c*d)^(1/2)/d)*b)/b^(1/
2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2))*c^2+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((
2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/
d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^3+1/8*b/d*((x-(-c
```

*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(3/2)*x+15/16/d*b^(1/2)*ln((b*(-c*d)^(1/2)/d+(x-(-c*d)^(1/2)/d
)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2))*a^2+1/2/d^3*b^(5/2)*ln((b*(-c*d)^(1
/2)/d+(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(
-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c^2-1/2/(-c*
d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c
*d)^(1/2)/d))*a^3-1/6/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*b*c-1/4/d^2*
b^2*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)*x*c-5/4/d^2*b^(3/2)*ln((-b*(-c*d)^(1/2)/d+(x+
(-c*d)^(1/2)/d)*b)/b^(1/2)+((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/
2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c*a-1/2/(-c*d)^(1/2)/
d^2*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2)*b^2*c^2-5/4/d^2*b^(3/2)*ln((b*(-c*d)^(1/2)/d+
(x-(-c*d)^(1/2)/d)*b)/b^(1/2)+((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^
(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))*c*a+1/2/(-c*d)^(1/
2)/d^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2)*b^2*c^2+3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d)^(
1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((
a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2*b*c-3/
2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))*a*b^2*c^2-3/2/(-c*d)^(1/2)/d/((a*d-b*c)/d
)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2
*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2*b*c
+3/2/(-c*d)^(1/2)/d^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2))/(x+(-c*d)^(1/2)/d))*a*b^2*c^2-1/10/(-c*d)^(1/2)*((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(5/2)+1/10/(-c*d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(5/2)+7/16*b/d*a*((x-(-c*d)^(1/
2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
)*x-1/(-c*d)^(1/2)/d*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(
x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a*b*c+1/2/(-c*d)^(1/2)/d^3/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/
d))*b^3*c^3+1/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*a*b*c-1/2/(-c*d)^(1/
2)/d^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2
b(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d
)^(1/2)/d))*b^3*c^3-1/4/d^2*b^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*c+7/16*b/d*a*((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2)*x+1/6/(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*b*c

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32431, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)
*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*
b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4
 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)
*x)*sqrt(b*x^2 + a))/d^3, 1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d
^2)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) + 2*(b^2*c^2
- 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 -
4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a
*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (2*b^2*d^2*x^3 - (4*b^2*c*
d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, 1/16*(8*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^
2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))) + (8*b^2*c^2
 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x
)*sqrt(b*x^2 + a))/d^3, 1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2
)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) + 4*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a
*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))) + (2*b
^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

GIAC/XCAS [A] time = 0.408494, size = 290, normalized size = 1.85

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,b^2x^2}{d}-\frac{4\,b^4cd^4-9\,ab^3d^5}{b^2d^6}\right)x-\frac{\left(8\,b^{\frac{5}{2}}c^2-20\,ab^{\frac{3}{2}}cd+15\,a^2\sqrt{b}d^2\right)\ln\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,d^3}+\frac{\left(b^{\frac{7}{2}}c^3-3\,ab^{\frac{5}{2}}c^2d+3\,a^2b^{\frac{3}{2}}cd^2-a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*b^2*x^2/d - (4*b^4*c*d^4 - 9*a*b^3*d^5)/(b
^2*d^6))*x - 1/16*(8*b^(5/2)*c^2 - 20*a*b^(3/2)*c*d + 15*a^2*sqrt
(b)*d^2)*ln((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + (b^(7/2)*c^3 -
 3*a*b^(5/2)*c^2*d + 3*a^2*b^(3/2)*c*d^2 - a^3*sqrt(b)*d^3)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2
*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*d^3)
```

3.67 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=175

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}(2bc-ad)}{2cd^2}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a
 + b*x^2)^(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)
*(4*b*c + a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(2*c^(3/2)*d^3)
```

Rubi [A] time = 0.526759, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3}+\frac{(bc-ad)^{3/2}(ad+4bc)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}+\frac{bx\sqrt{a+bx^2}(2bc-ad)}{2cd^2}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a
 + b*x^2)^(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)
*(4*b*c + a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(2*c^(3/2)*d^3)
```

Rubi in Sympy [A] time = 75.1903, size = 155, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\left(5ad-4bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3}-\frac{bx\sqrt{a+bx^2}\left(ad-2bc\right)}{2cd^2}+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}\left(ad-bc\right)}{2cd\left(c+dx^2\right)}+\frac{\left(ad-bc\right)^{\frac{3}{2}}\left(ad+4bc\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] b**(3/2)*(5*a*d - 4*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*d**
3) - b*x*sqrt(a + b*x**2)*(a*d - 2*b*c)/(2*c*d**2) + x*(a + b*x**
2)**(3/2)*(a*d - b*c)/(2*c*d*(c + d*x**2)) + (a*d - b*c)**(3/2)*(
a*d + 4*b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(
2*c**(3/2)*d**3)
```

Mathematica [A] time = 0.279217, size = 144, normalized size = 0.82

$$\frac{b^{3/2}(-(4bc-5ad))\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+dx\sqrt{a+bx^2}\left(\frac{(bc-ad)^2}{c(c+dx^2)}+b^2\right)+\frac{(ad-bc)^{3/2}(ad+4bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(b^2 + (b*c - a*d)^2/(c*(c + d*x^2))) + ((-(
b*c) + a*d)^(3/2)*(4*b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sq
rt[c]*Sqrt[a + b*x^2])])/c^(3/2) - b^(3/2)*(4*b*c - 5*a*d)*Log[b*
x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*d^3)
```

Maple [B] time = 0.037, size = 7345, normalized size = 42.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

Fricas [A] time = 1.00061, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x
^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*
b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 -
a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*
x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(
d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3),
-1/8*(4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^
2)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) + (4*b^2*c^3 -
 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*
x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*
c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 +
 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*
b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 -
a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^2
 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))) + (4*b^2*c^3
- 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2
+ c^2*d^3), -1/4*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a
*b*c*d^2)*x^2)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt(-b))) +
(4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2
 - a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*
x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))) - 2*(b^2*c
*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 +
a))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{2}}}{\left(c + dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)

[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.793526, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2,x, algorithm="giac")

[Out] sage0*x
```

3.68 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(3/2))/(4*c*d*(c + d*x^2)^2) - ((b*c
- a*d)*(4*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2))
 + (b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c
 - a*d]*(8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a
*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*d^3)
```

Rubi [A] time = 0.482747, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\sqrt{bc-ad}\left(3a^2d^2+4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{4cd\left(c+dx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(3/2))/(4*c*d*(c + d*x^2)^2) - ((b*c
- a*d)*(4*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2))
 + (b^(5/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c
 - a*d]*(8*b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a
*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*d^3)
```

Rubi in Sympy [A] time = 73.2336, size = 177, normalized size = 0.91

$$\frac{b^{\frac{5}{2}}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}+\frac{x\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)}{4cd\left(c+dx^2\right)^2}+\frac{x\sqrt{a+bx^2}\,(ad-bc)\,(3ad+4bc)}{8c^2d^2\left(c+dx^2\right)}+\frac{\sqrt{ad-bc}\left(3a^2d^2+4abcd+8b^2c^2\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{\frac{5}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] b**(5/2)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/d**3 + x*(a + b*x**2)*
*(3/2)*(a*d - b*c)/(4*c*d*(c + d*x**2)**2) + x*sqrt(a + b*x**2)*(
a*d - b*c)*(3*a*d + 4*b*c)/(8*c**2*d**2*(c + d*x**2)) + sqrt(a*d
- b*c)*(3*a**2*d**2 + 4*a*b*c*d + 8*b**2*c**2)*atan(x*sqrt(a*d -
b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(8*c**(5/2)*d**3)
```

Mathematica [A] time = 0.343915, size = 184, normalized size = 0.95

$$\frac{\frac{(3a^3d^3+a^2bcd^2+4ab^2c^2d-8b^3c^3)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}\sqrt{ad-bc}}+8b^{5/2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\frac{dx\sqrt{a+bx^2}(ad-bc)\left(ad\left(5c+3dx^2\right)+2bc\left(2c+3dx^2\right)\right)}{c^2\left(c+dx^2\right)^2}}{8d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] ((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + 3*d*x^2) + a*d
*(5*c + 3*d*x^2)))/(c^2*(c + d*x^2)^2) + ((-8*b^3*c^3 + 4*a*b^2*c
^2*d + a^2*b*c*d^2 + 3*a^3*d^3)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sq
rt[c]*Sqrt[a + b*x^2])])/(c^(5/2)*Sqrt[-(b*c) + a*d]) + 8*b^(5/2)
*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*d^3)
```

Maple [B] time = 0.048, size = 14133, normalized size = 72.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

Fricas [A] time = 0.615359, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a
*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d
^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt
((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*
c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c
*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^
2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b
^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^
5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), 1/32*(32*(b^2*c^2*d^2*x^4 + 2*b
^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(b*x/(sqrt(b*x^2 + a)*sqrt
(-b))) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^
2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2
 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*
(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d
)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*
d^3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)
*sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), 1/16*(
(8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*
c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(-1/2*((2*b*c - a*d)*x^2 +
 a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c - a*d)/c))) + 8*(b^2*c^2*d^
2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2
*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x
^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), 1/16*(16*(b^2*c
^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(b*x/(sqrt
(b*x^2 + a)*sqrt(-b))) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2
 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d
 + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(
-1/2*((2*b*c - a*d)*x^2 + a*c)/(sqrt(b*x^2 + a)*c*x*sqrt(-(b*c -
a*d)/c))) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b
^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^
5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.649774, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.69 $\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^4} dx$

Optimal. Leaf size=144

$$\frac{5a^3 \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3
/2))/(24*c^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(
c + d*x^2)) + (5*a^3*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a
+ b*x^2])])/(16*c^(7/2)*Sqrt[b*c - a*d])
```

Rubi [A] time = 0.198217, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{5a^3 \tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5ax\left(a+bx^2\right)^{3/2}}{24c^2\left(c+dx^2\right)^2}+\frac{x\left(a+bx^2\right)^{5/2}}{6c\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3
/2))/(24*c^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(
c + d*x^2)) + (5*a^3*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a
+ b*x^2])])/(16*c^(7/2)*Sqrt[b*c - a*d])
```

Rubi in Sympy [A] time = 35.2803, size = 131, normalized size = 0.91

$$\frac{5a^3 \operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{\frac{7}{2}}\sqrt{ad-bc}}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\left(c+dx^2\right)}+\frac{5ax\left(a+bx^2\right)^{\frac{3}{2}}}{24c^2\left(c+dx^2\right)^2}+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}}{6c\left(c+dx^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] 5*a**3*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(16*c**
(7/2)*sqrt(a*d - b*c)) + 5*a**2*x*sqrt(a + b*x**2)/(16*c**3*(c +
d*x**2)) + 5*a*x*(a + b*x**2)**(3/2)/(24*c**2*(c + d*x**2)**2) +
x*(a + b*x**2)**(5/2)/(6*c*(c + d*x**2)**3)
```

Mathematica [A] time = 0.296115, size = 139, normalized size = 0.97

$$\frac{5a^3\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{ad-bc}}+\frac{\sqrt{a+bx^2}\left(a^2\left(33c^2x+40cdx^3+15d^2x^5\right)+2abcx^3\left(13c+5dx^2\right)+8b^2c^2x^5\right)}{48c^3\left(c+dx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*b^2*c^2*x^5 + 2*a*b*c*x^3*(13*c + 5*d*x^2) +
a^2*(33*c^2*x + 40*c*d*x^3 + 15*d^2*x^5)))/(48*c^3*(c + d*x^2)^3)
 + (5*a^3*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(16*c^(7/2)*Sqrt[-(b*c) + a*d])
```

Maple [B] time = 0.086, size = 21220, normalized size = 147.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

Fricas [A] time = 0.465232, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(8\,b^2c^2+10\,abcd+15\,a^2d^2\right)x^5+33\,a^2c^2x+2\left(13\,abc^2+20\,a^2cd\right)x^3\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}+15\left(a^3d^3x^6+3\,a^3cd^2x^4+3\right.}{192\left(c^3d^3x^6+3\,c^4d^2x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(4*((8*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2)*x^5 + 33*a^2*c^2
*x + 2*(13*a*b*c^2 + 20*a^2*c*d)*x^3)*sqrt(b*c^2 - a*c*d)*sqrt(b*
x^2 + a) + 15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 +
a^3*c^3)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 +
2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^
3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(
b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((c^3*d^3*x^6 + 3*c^4*d
^2*x^4 + 3*c^5*d*x^2 + c^6)*sqrt(b*c^2 - a*c*d)), 1/96*(2*((8*b^2
*c^2 + 10*a*b*c*d + 15*a^2*d^2)*x^5 + 33*a^2*c^2*x + 2*(13*a*b*c^
2 + 20*a^2*c*d)*x^3)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a) + 15*(a
^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*arctan(
1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*
d)*sqrt(b*x^2 + a)*x)))/((c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x
^2 + c^6)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 29.1695, size = 4, normalized size = 0.03

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.70 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx$

Optimal. Leaf size=249

$$\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4(c+dx^2)(bc-ad)}+\frac{5ax(a+bx^2)^{3/2}(8bc-7ad)}{192c^3(c+dx^2)^2(bc-ad)}+\frac{x(a+bx^2)^{5/2}(8bc-7ad)}{48c^2(c+dx^2)^3(bc-ad)}-\frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^(7/2))/(8*c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*
c - 7*a*d)*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3
) + (5*a*(8*b*c - 7*a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d
)*(c + d*x^2)^2) + (5*a^2*(8*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128
*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3*(8*b*c - 7*a*d)*ArcTanh[(S
qrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/2)*(b*c -
 a*d)^(3/2))
```

Rubi [A] time = 0.377254, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}+\frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4(c+dx^2)(bc-ad)}+\frac{5ax(a+bx^2)^{3/2}(8bc-7ad)}{192c^3(c+dx^2)^2(bc-ad)}+\frac{x(a+bx^2)^{5/2}(8bc-7ad)}{48c^2(c+dx^2)^3(bc-ad)}-\frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] -(d*x*(a + b*x^2)^(7/2))/(8*c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*
c - 7*a*d)*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3
) + (5*a*(8*b*c - 7*a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d
)*(c + d*x^2)^2) + (5*a^2*(8*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128
*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3*(8*b*c - 7*a*d)*ArcTanh[(S
qrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(9/2)*(b*c -
 a*d)^(3/2))
```

Rubi in Sympy [A] time = 54.9691, size = 226, normalized size = 0.91

$$\frac{5a^3(7ad-8bc)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{\frac{9}{2}}(ad-bc)^{\frac{3}{2}}}+\frac{5a^2x\sqrt{a+bx^2}(7ad-8bc)}{128c^4(c+dx^2)(ad-bc)}+\frac{5ax\left(a+bx^2\right)^{\frac{3}{2}}(7ad-8bc)}{192c^3(c+dx^2)^2(ad-bc)}+\frac{dx\left(a+bx^2\right)^{\frac{7}{2}}}{8c(c+dx^2)^4(ad-bc)}+\frac{x\left(a+bx^2\right)^{\frac{5}{2}}(7ad-8bc)}{48c^2(c+dx^2)^3(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
```

```
[Out] 5*a**3*(7*a*d - 8*b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b
*x**2)))/(128*c**(9/2)*(a*d - b*c)**(3/2)) + 5*a**2*x*sqrt(a + b*
x**2)*(7*a*d - 8*b*c)/(128*c**4*(c + d*x**2)*(a*d - b*c)) + 5*a*x
*(a + b*x**2)**(3/2)*(7*a*d - 8*b*c)/(192*c**3*(c + d*x**2)**2*(a
*d - b*c)) + d*x*(a + b*x**2)**(7/2)/(8*c*(c + d*x**2)**4*(a*d -
b*c)) + x*(a + b*x**2)**(5/2)*(7*a*d - 8*b*c)/(48*c**2*(c + d*x**
2)**3*(a*d - b*c))
```

Mathematica [A] time = 0.487309, size = 235, normalized size = 0.94

$$\frac{\frac{15a^3(8bc-7ad)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{ad-bc}}-\frac{\sqrt{c}x\sqrt{a+bx^2}\left(a^3d\left(279c^3+511c^2dx^2+385cd^2x^4+105d^3x^6\right)-2a^2bc\left(132c^3+129c^2dx^2+94cd^2x^4+25d^3x^6\right)-8ab^2c^2x^2\left(26c^2+\right.\right.}{(c+dx^2)^4}}{384c^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] (-((Sqrt[c]*x*Sqrt[a + b*x^2]*(-16*b^3*c^3*x^4*(4*c + d*x^2) - 8*
a*b^2*c^2*x^2*(26*c^2 + 11*c*d*x^2 + 3*d^2*x^4) - 2*a^2*b*c*(132*
c^3 + 129*c^2*d*x^2 + 94*c*d^2*x^4 + 25*d^3*x^6) + a^3*d*(279*c^3
 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6)))/(c + d*x^2)^4)
+ (15*a^3*(8*b*c - 7*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*
Sqrt[a + b*x^2])])/Sqrt[-(b*c) + a*d])/(384*c^(9/2)*(b*c - a*d))
```

Maple [B] time = 0.095, size = 28625, normalized size = 115.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{2}}}{\left(dx^2 + c\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

Fricas [A] time = 1.39618, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(4*((16*b^3*c^3*d + 24*a*b^2*c^2*d^2 + 50*a^2*b*c*d^3 - 1
05*a^3*d^4)*x^7 + (64*b^3*c^4 + 88*a*b^2*c^3*d + 188*a^2*b*c^2*d^
2 - 385*a^3*c*d^3)*x^5 + (208*a*b^2*c^4 + 258*a^2*b*c^3*d - 511*a
^3*c^2*d^2)*x^3 + 3*(88*a^2*b*c^4 - 93*a^3*c^3*d)*x)*sqrt(b*c^2 -
 a*c*d)*sqrt(b*x^2 + a) + 15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*
b*c*d^4 - 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6
+ 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*
a^4*c^3*d^2)*x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a
^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*(
(2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)
*x)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((b*c^9 - a*c^
8*d + (b*c^5*d^4 - a*c^4*d^5)*x^8 + 4*(b*c^6*d^3 - a*c^5*d^4)*x^6
 + 6*(b*c^7*d^2 - a*c^6*d^3)*x^4 + 4*(b*c^8*d - a*c^7*d^2)*x^2)*s
qrt(b*c^2 - a*c*d)), 1/768*(2*((16*b^3*c^3*d + 24*a*b^2*c^2*d^2 +
 50*a^2*b*c*d^3 - 105*a^3*d^4)*x^7 + (64*b^3*c^4 + 88*a*b^2*c^3*d
 + 188*a^2*b*c^2*d^2 - 385*a^3*c*d^3)*x^5 + (208*a*b^2*c^4 + 258*
a^2*b*c^3*d - 511*a^3*c^2*d^2)*x^3 + 3*(88*a^2*b*c^4 - 93*a^3*c^3
*d)*x)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a) + 15*(8*a^3*b*c^5 - 7
*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3
 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)*x^4 + 4
*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*arctan(1/2*sqrt(-b*c^2 + a*
c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x
)))/((b*c^9 - a*c^8*d + (b*c^5*d^4 - a*c^4*d^5)*x^8 + 4*(b*c^6*d^
3 - a*c^5*d^4)*x^6 + 6*(b*c^7*d^2 - a*c^6*d^3)*x^4 + 4*(b*c^8*d -
 a*c^7*d^2)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 2.00983, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx$

Optimal. Leaf size=30

$$\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0457435, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(1 + x^2),x]

[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rubi in Sympy [A] time = 10.5398, size = 24, normalized size = 0.8

$$-\operatorname{asin}(x) + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(x**2+1),x)

[Out] -asin(x) + sqrt(2)*atan(sqrt(2)*x/sqrt(-x**2 + 1))
```

Mathematica [A] time = 0.0423309, size = 30, normalized size = 1.

$$\sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/(1 + x^2),x]

[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Maple [A] time = 0.024, size = 33, normalized size = 1.1

$$-\arcsin(x) - \sqrt{2}\arctan\left(\frac{x\sqrt{2}}{x^2-1}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(x^2+1),x)

[Out] -arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(x^2 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

Fricas [A] time = 0.212727, size = 84, normalized size = 2.8

$$\sqrt{2}\arctan\left(\frac{x^2+\sqrt{-x^2+1}-1}{\sqrt{2}\sqrt{-x^2+1}x-\sqrt{2}x}\right) + 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(x^2 + 1),x, algorithm="fricas")

[Out] sqrt(2)*arctan((x^2 + sqrt(-x^2 + 1) - 1)/(sqrt(2)*sqrt(-x^2 + 1)
*x - sqrt(2)*x)) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.265262, size = 128, normalized size = 4.27

$$-\frac{1}{2}\pi\operatorname{sign}(x)+\frac{1}{2}\sqrt{2}\left(\pi\operatorname{sign}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sign(x) + 1/2*sqrt(2)*(pi*sign(x) + 2*arctan(-1/4*sqrt(2)
*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arct
an(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x) - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

```
[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0393877, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$\sinh^{-1}(x) - \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/(-1 + x^2),x]

[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rubi in Sympy [A] time = 8.65158, size = 24, normalized size = 0.89

$$\operatorname{asinh}(x) - \sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(x**2-1),x)

[Out] asinh(x) - sqrt(2)*atanh(sqrt(2)*x/sqrt(x**2 + 1))
```

Mathematica [B] time = 0.0419229, size = 64, normalized size = 2.37

$$\frac{\log\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)-\log\left(\sqrt{2}\sqrt{x^2+1}+x+1\right)+\log(1-x)-\log(x+1)}{\sqrt{2}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] + (Log[1 - x] - Log[1 + x] + Log[1 - x + Sqrt[2]*Sqrt[
1 + x^2]] - Log[1 + x + Sqrt[2]*Sqrt[1 + x^2]])/Sqrt[2]
```

Maple [B] time = 0.017, size = 84, normalized size = 3.1

$$-\frac{1}{2}\sqrt{(1+x)^2-2x}+Arcsinh(x)+\frac{\sqrt{2}}{2}Artanh\left(\frac{(2-2x)\sqrt{2}}{4}\frac{1}{\sqrt{(1+x)^2-2x}}\right)+\frac{1}{2}\sqrt{(-1+x)^2+2x}-\frac{\sqrt{2}}{2}Artanh\left(\frac{(2+2x)\sqrt{2}}{4}\frac{1}{\sqrt{(-1+x)^2+2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(x^2-1),x)
```

```
[Out] -1/2*((1+x)^2-2*x)^(1/2)+arcsinh(x)+1/2*2^(1/2)*arctanh(1/4*(2-2*
x)*2^(1/2)/((1+x)^2-2*x)^(1/2))+1/2*((-1+x)^2+2*x)^(1/2)-1/2*2^(1
/2)*arctanh(1/4*(2+2*x)*2^(1/2)/((-1+x)^2+2*x)^(1/2))
```

Maxima [A] time = 1.52107, size = 80, normalized size = 2.96

$$-\frac{1}{2}\sqrt{2}\,\mathrm{arsinh}\left(\frac{2x}{|2x+2|}-\frac{2}{|2x+2|}\right)-\frac{1}{2}\sqrt{2}\,\mathrm{arsinh}\left(\frac{2x}{|2x-2|}+\frac{2}{|2x-2|}\right)+\mathrm{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 - 1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqr
t(2)*arcsinh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

Fricas [A] time = 0.208779, size = 128, normalized size = 4.74

$$\frac{1}{2}\sqrt{2}\log\left(\frac{2x^4-x^2-2\left(x^3+\sqrt{2}x-x\right)\sqrt{x^2+1}+2\sqrt{2}\left(x^2-1\right)+3}{2x^4-x^2-2\left(x^3-x\right)\sqrt{x^2+1}-1}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 - 1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((2*x^4 - x^2 - 2*(x^3 + sqrt(2)*x - x)*sqrt(x^2 +
 1) + 2*sqrt(2)*(x^2 - 1) + 3)/(2*x^4 - x^2 - 2*(x^3 - x)*sqrt(x^
2 + 1) - 1)) - log(-x + sqrt(x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(x**2-1),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

GIAC/XCAS [A] time = 0.254671, size = 95, normalized size = 3.52

$$-\frac{1}{2}\sqrt{2}\ln\left(\frac{\left|2\left(x-\sqrt{x^2+1}\right)^2-4\sqrt{2}-6\right|}{\left|2\left(x-\sqrt{x^2+1}\right)^2+4\sqrt{2}-6\right|}\right)-\ln\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/(x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*ln(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(
2*(x - sqrt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - ln(-x + sqrt(x^2 + 1)
)
```

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rubi [A] time = 0.043342, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]

[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rubi in Sympy [A] time = 11.2578, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asin}(x)}{2}-\frac{\operatorname{atanh}\left(\frac{x}{\sqrt{-x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(2*x**2-1),x)

[Out] -asin(x)/2 - atanh(x/sqrt(-x**2 + 1))/2
```

Mathematica [A] time = 0.0128124, size = 25, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Maple [B] time = 0.104, size = 187, normalized size = 7.5

$$\frac{\sqrt{2}}{2}\left(\frac{1}{4}\sqrt{-4\left(x-1/2\sqrt{2}\right)^{2}-4\left(x-1/2\sqrt{2}\right)\sqrt{2}+2}-\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}}{4}Artanh\left(\sqrt{2}\left(-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\frac{1}{\sqrt{-4\left(x-1\right.}}\right.\right.$$

$$-\frac{\sqrt{2}}{2}\left(\frac{1}{4}\sqrt{-4\left(x+1/2\sqrt{2}\right)^{2}+4\left(x+1/2\sqrt{2}\right)\sqrt{2}+2}+\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}}{4}Artanh\left(\sqrt{2}\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\frac{1}{\sqrt{-4\left(x+1/\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(2*x^2-1),x)
```

```
[Out] 1/2*2^(1/2)*(1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+
2)^(1/2)-1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh((-(x-1/2*2^(1/
2))*2^(1/2)+1)*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^
(1/2)+2)^(1/2)))-1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*
2^(1/2))*2^(1/2)+2)^(1/2)+1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arcta
nh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/2)/(-4*(x+1/2*2^(1/2))^2+4*(x
+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

Maxima [A] time = 1.49764, size = 143, normalized size = 5.72

$$-\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin(x)-\sqrt{2}\log\left(\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^{2}+1}}{\left|\left(2\sqrt{2}\right)+4x\right|}+\frac{1}{\left|\left(2\sqrt{2}\right)+4x\right|}\right)+\sqrt{2}\log\left(-\frac{1}{4}\sqrt{2}+\frac{\sqrt{2}\sqrt{-x^{2}+1}}{\left|4x-2\sqrt{2}\right|}+\frac{1}{\left|4x-2\sqrt{2}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(2*x^2 - 1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqr
t(2)*sqrt(-x^2 + 1)/abs((2*sqrt(2)) + 4*x) + 1/abs((2*sqrt(2)) +
4*x)) + sqrt(2)*log(-1/4*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x
 - 2*sqrt(2)) + 1/abs(4*x - 2*sqrt(2))))
```

Fricas [A] time = 0.210409, size = 100, normalized size = 4.

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(2*x^2 - 1),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(
x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x
 - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+1)}}{2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(2*x**2-1),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)
```

GIAC/XCAS [A] time = 0.239943, size = 159, normalized size = 6.36

$$-\frac{1}{4}\pi\operatorname{sign}(x)-\frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$
$$-\frac{1}{4}\ln\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)+\frac{1}{4}\ln\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(2*x^2 - 1),x, algorithm="giac")
```

```
[Out] -1/4*pi*sign(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
 1)/(sqrt(-x^2 + 1) - 1)) - 1/4*ln(abs(-x/(sqrt(-x^2 + 1) - 1) +
(sqrt(-x^2 + 1) - 1)/x + 2)) + 1/4*ln(abs(-x/(sqrt(-x^2 + 1) - 1)
 + (sqrt(-x^2 + 1) - 1)/x - 2))
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=169

$$\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{24b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*
b^3) + (5*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2)
 + (d*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*
b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(16*b^(7/2))
```

Rubi [A] time = 0.334012, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{(2bc-ad)\left(5a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}+\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(2bc-ad)}{24b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*
b^3) + (5*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2)
 + (d*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*
b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(16*b^(7/2))
```

Rubi in Sympy [A] time = 42.7041, size = 163, normalized size = 0.96

$$\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2}{6b}-\frac{5dx\sqrt{a+bx^2}\left(c+dx^2\right)(ad-2bc)}{24b^2}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-44abcd+44b^2c^2\right)}{48b^3}-\frac{(ad-2bc)\left(5a^2d^2-8abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**2)*(c + d*x**2)**2/(6*b) - 5*d*x*sqrt(a + b*x**
2)*(c + d*x**2)*(a*d - 2*b*c)/(24*b**2) + d*x*sqrt(a + b*x**2)*(1
5*a**2*d**2 - 44*a*b*c*d + 44*b**2*c**2)/(48*b**3) - (a*d - 2*b*c
)*(5*a**2*d**2 - 8*a*b*c*d + 8*b**2*c**2)*atanh(sqrt(b)*x/sqrt(a
+ b*x**2))/(16*b**(7/2))
```

Mathematica [A] time = 0.156407, size = 140, normalized size = 0.83

$$\frac{\sqrt{b}dx\sqrt{a+bx^2}\left(15a^2d^2-2abd\left(27c+5dx^2\right)+4b^2\left(18c^2+9cdx^2+2d^2x^4\right)\right)+3\left(-5a^3d^3+18a^2bcd^2-24ab^2c^2d+16b^3c^3\right)}{48b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^
2) + 4*b^2*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + 3*(16*b^3*c^3 - 24
*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt
[a + b*x^2]])/(48*b^(7/2))
```

Maple [A] time = 0.017, size = 228, normalized size = 1.4

$$c^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^3x^5}{6\,b}\sqrt{bx^2+a}-\frac{5\,ad^3x^3}{24\,b^2}\sqrt{bx^2+a}+\frac{5\,d^3a^2x}{16\,b^3}\sqrt{bx^2+a}$$
$$-\frac{5\,a^3d^3}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{3\,cd^2x^3}{4\,b}\sqrt{bx^2+a}-\frac{9\,cd^2ax}{8\,b^2}\sqrt{bx^2+a}$$
$$+\frac{9\,a^2cd^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{3\,c^2dx}{2\,b}\sqrt{bx^2+a}-\frac{3\,ac^2d}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^(1/2),x)
```

```
[Out] c^3*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/6*d^3*x^5/b*(b*x^2+a)
^(1/2)-5/24*d^3*a/b^2*x^3*(b*x^2+a)^(1/2)+5/16*d^3*a^2/b^3*x*(b*x
^2+a)^(1/2)-5/16*d^3*a^3/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/
4*c*d^2*x^3/b*(b*x^2+a)^(1/2)-9/8*c*d^2*a/b^2*x*(b*x^2+a)^(1/2)+9
/8*c*d^2*a^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/2*c^2*d*x/b*
(b*x^2+a)^(1/2)-3/2*c^2*d*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270199, size = 1, normalized size = 0.01

$$\left[\frac{2\left(8\,b^{2}d^{3}x^{5}+2\left(18\,b^{2}cd^{2}-5\,abd^{3}\right)x^{3}+3\left(24\,b^{2}c^{2}d-18\,abcd^{2}+5\,a^{2}d^{3}\right)x\right)\sqrt{bx^{2}+a}\sqrt{b}-3\left(16\,b^{3}c^{3}-24\,ab^{2}c^{2}d+18\,a^{2}bc\right.}{96\,b^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*b^2*d^3*x^5 + 2*(18*b^2*c*d^2 - 5*a*b*d^3)*x^3 + 3*(2
4*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x)*sqrt(b*x^2 + a)*sqrt(b
) - 3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*
log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(7/2), 1/48
*((8*b^2*d^3*x^5 + 2*(18*b^2*c*d^2 - 5*a*b*d^3)*x^3 + 3*(24*b^2*c
^2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 3*
(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^3)]
```

Sympy [A] time = 39.8273, size = 400, normalized size = 2.37

$$\frac{5a^{\frac{5}{2}}d^{3}x}{16b^{3}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{9a^{\frac{3}{2}}cd^{2}x}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{5a^{\frac{3}{2}}d^{3}x^{3}}{48b^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3\sqrt{a}c^{2}dx\sqrt{1+\frac{bx^{2}}{a}}}{2b}-\frac{3\sqrt{a}cd^{2}x^{3}}{8b\sqrt{1+\frac{bx^{2}}{a}}}-\frac{\sqrt{a}d^{3}x^{5}}{24b\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5a^{3}d^{3}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{9a^{2}cd^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{3ac^{2}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c^{3}\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{3cd^{2}x^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{d^{3}x^{7}}{6\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*d**3*x/(16*b**3*sqrt(1 + b*x**2/a)) - 9*a**(3/2)*c*d**
2*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*d**3*x**3/(48*b**2*s
qrt(1 + b*x**2/a)) + 3*sqrt(a)*c**2*d*x*sqrt(1 + b*x**2/a)/(2*b)
- 3*sqrt(a)*c*d**2*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt(a)*d**3*x
**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*d**3*asinh(sqrt(b)*x/sqrt(
a))/(16*b**(7/2)) + 9*a**2*c*d**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**
(5/2)) - 3*a*c**2*d*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + c**3*
Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (
sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + 3*
c*d**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + d**3*x**7/(6*sqrt(a)
*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.242198, size = 203, normalized size = 1.2

$$\frac{1}{48}\left(2\left(\frac{4\,d^3x^2}{b}+\frac{18\,b^4cd^2-5\,ab^3d^3}{b^5}\right)x^2+\frac{3\left(24\,b^4c^2d-18\,ab^3cd^2+5\,a^2b^2d^3\right)}{b^5}\right)\sqrt{bx^2+ax}$$
$$-\frac{\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*
(24*b^4*c^2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 +
 a)*x - 1/16*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^
3*d^3)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.75 $\int \frac{\left(c+dx^2\right)^2}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=108

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}$$

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.144331, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}+\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8b^2}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi in Sympy [A] time = 20.4416, size = 102, normalized size = 0.94

$$\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)}{4b}-\frac{3dx\sqrt{a+bx^2}\,(ad-2bc)}{8b^2}+\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**2)*(c + d*x**2)/(4*b) - 3*d*x*sqrt(a + b*x**2)*
(a*d - 2*b*c)/(8*b**2) + (3*a**2*d**2 - 8*a*b*c*d + 8*b**2*c**2)*
atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*b**(5/2))
```

Mathematica [A] time = 0.0895376, size = 91, normalized size = 0.84

$$\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}dx\sqrt{a+bx^2}\left(-3ad+8bc+2bdx^2\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(8*b*c - 3*a*d + 2*b*d*x^2) + (8*b^2
*c^2 - 8*a*b*c*d + 3*a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])
/(8*b^(5/2))
```

Maple [A] time = 0.01, size = 131, normalized size = 1.2

$$c^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{d^2x^3}{4b}\sqrt{bx^2+a}-\frac{3ad^2x}{8b^2}\sqrt{bx^2+a}+\frac{3a^2d^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}+\frac{cdx}{b}\sqrt{bx^2+a}-cda\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^(1/2),x)
```

```
[Out] c^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/4*d^2*x^3/b*(b*x^2+a)
^(1/2)-3/8*d^2*a/b^2*x*(b*x^2+a)^(1/2)+3/8*d^2*a^2/b^(5/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))+c*d*x/b*(b*x^2+a)^(1/2)-c*d*a/b^(3/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.228464, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,bd^2x^3+\left(8\,bcd-3\,ad^2\right)x\right)\sqrt{bx^2+a}\sqrt{b}+\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{16\,b^{\frac{5}{2}}},\frac{\left(2\,bd^2x^3+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b*d^2*x^3 + (8*b*c*d - 3*a*d^2)*x)*sqrt(b*x^2 + a)*sq
rt(b) + (8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(-2*sqrt(b*x^2 + a
)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(5/2), 1/8*((2*b*d^2*x^3 + (8*b
*c*d - 3*a*d^2)*x)*sqrt(b*x^2 + a)*sqrt(-b) + (8*b^2*c^2 - 8*a*b*
c*d + 3*a^2*d^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b^
2)]
```

Sympy [A] time = 20.5179, size = 238, normalized size = 2.2

$$-\frac{3a^{\frac{3}{2}}d^2x}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}cdx\sqrt{1+\frac{bx^2}{a}}}{b}-\frac{\sqrt{a}d^2x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}$$
$$-\frac{acd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+c^2\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{d^2x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*d**2*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*d*x*sq
rt(1 + b*x**2/a)/b - sqrt(a)*d**2*x**3/(8*b*sqrt(1 + b*x**2/a)) +
 3*a**2*d**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*c*d*asinh(
sqrt(b)*x/sqrt(a))/b**(3/2) + c**2*Piecewise((sqrt(-a/b)*asin(x*s
qrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b
/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))
/sqrt(-a), (b > 0) & (a < 0))) + d**2*x**5/(4*sqrt(a)*sqrt(1 + b*
x**2/a))
```

GIAC/XCAS [A] time = 0.239524, size = 122, normalized size = 1.13

$$\frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^2x^2}{b}+\frac{8\,b^2cd-3\,abd^2}{b^3}\right)x-\frac{\left(8\,b^2c^2-8\,abcd+3\,a^2d^2\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x
 - 1/8*(8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ln(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(5/2)
```

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}} \, dx$

Optimal. Leaf size=58

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] time = 0.0541114, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 7.96772, size = 49, normalized size = 0.84

$$\frac{dx\sqrt{a+bx^2}}{2b}-\frac{(ad-2bc)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] d*x*sqrt(a + b*x**2)/(2*b) - (a*d - 2*b*c)*atanh(sqrt(b)*x/sqrt(a
 + b*x**2))/(2*b**(3/2))
```

Mathematica [A] time = 0.0430732, size = 61, normalized size = 1.05

$$\frac{(2bc-ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*Log[b*x + Sqrt[b]*Sq
rt[a + b*x^2]])/(2*b^(3/2))
```

Maple [A] time = 0.006, size = 62, normalized size = 1.1

$$c\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}+\frac{dx}{2\,b}\sqrt{bx^2+a}-\frac{ad}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^(1/2),x)
```

```
[Out] c*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+1/2*d*x*(b*x^2+a)^(1/2)/b
-1/2*d*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219264, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}\sqrt{b}dx-(2\,bc-ad)\log\left(2\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{4\,b^{\frac{3}{2}}},\frac{\sqrt{bx^2+a}\sqrt{-b}dx+(2\,bc-ad)\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*sqrt(b)*d*x - (2*b*c - a*d)*log(2*sqrt(b*
x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/2*(sqrt(b*x^2 +
```

```
 a)*sqrt(-b)*d*x + (2*b*c - a*d)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)))/(sqrt(-b)*b)]
```

Sympy [A] time = 8.5423, size = 126, normalized size = 2.17

$$\frac{\sqrt{a}dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*sqrt(1 + b*x**2/a)/(2*b) - a*d*asinh(sqrt(b)*x/sqrt(a
))/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt
(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (
a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b >
 0) & (a < 0)))
```

GIAC/XCAS [A] time = 0.233724, size = 66, normalized size = 1.14

$$\frac{\sqrt{bx^2+a}dx}{2\,b}-\frac{(2\,bc-ad)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*ln(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(3/2)
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}}\, dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0169649, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi in Sympy [A] time = 2.45359, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2),x)

[Out] atanh(sqrt(b)*x/sqrt(a + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0106199, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0., size = 21, normalized size = 0.8

$$1 \ln\left(x\sqrt{b} + \sqrt{bx^2 + a}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2),x)

[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212995, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(-2\sqrt{bx^2 + a}bx - \left(2bx^2 + a\right)\sqrt{b}\right)}{2\sqrt{b}}, \frac{\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] [1/2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))/sqrt(b),
 arctan(sqrt(-b)*x/sqrt(b*x^2 + a))/sqrt(-b)]
```

Sympy [A] time = 3.61434, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2),x)

[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

GIAC/XCAS [A] time = 0.235535, size = 31, normalized size = 1.24

$$-\frac{\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="giac")

[Out] -ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*S
qrt[b*c - a*d])
```

Rubi [A] time = 0.0580408, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*S
qrt[b*c - a*d])
```

Rubi in Sympy [A] time = 10.4196, size = 42, normalized size = 0.86

$$\frac{\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*sqrt(
a*d - b*c))
```

Mathematica [A] time = 0.0444047, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]
*Sqrt[-(b*c) + a*d])
```

Maple [B] time = 0.023, size = 300, normalized size = 6.1

$$-\frac{1}{2}\ln\left(1\left(2\,\frac{ad-bc}{d}+2\,\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\,\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^{2}b+2\,\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}\right)\left(x-\frac{1}{d}\right.\right.$$
$$+\frac{1}{2}\ln\left(1\left(2\,\frac{ad-bc}{d}-2\,\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+2\,\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-2\,\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}\right)\left(x+\frac{1}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x)
```

```
[Out] -1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/
2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2
))/(x-(-c*d)^(1/2)/d))+1/2/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2
*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d
)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260597, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{\left(\left(8\,b^{2}c^{2}-8\,abcd+a^{2}d^{2}\right)x^{4}+a^{2}c^{2}+2\,\left(4\,abc^{2}-3\,a^{2}cd\right)x^{2}\right)\sqrt{bc^{2}-acd}+4\,\left(\left(2\,b^{2}c^{3}-3\,abc^{2}d+a^{2}cd^{2}\right)x^{3}+\left(abc^{3}-a^{2}c^{2}d\right)x\right)\sqrt{bx^{2}+a}}{d^{2}x^{4}+2\,cdx^{2}+c^{2}}\right)}{4\,\sqrt{bc^{2}-acd}}, \frac{\arctan\left(\frac{\sqrt{-bc^{2}+a \cdots}}{2\,(bc \cdots}\right.}{2\,\sqrt{-b \cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 -
3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^
2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), 1/2*arc
tan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 -
a*c*d)*sqrt(b*x^2 + a)*x))/sqrt(-b*c^2 + a*c*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^{2}}\,(c + dx^{2})}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.241544, size = 95, normalized size = 1.94

$$-\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}d+2\,bc-ad}{2\,\sqrt{-b^{2}c^{2}+abcd}}\right)}{\sqrt{-b^{2}c^{2}+abcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c -
a*d)/sqrt(-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=101

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

```
[Out] -(d*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c -
a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c
^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.140406, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}-\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(2*c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c -
a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c
^(3/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 20.4682, size = 83, normalized size = 0.82

$$\frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(ad-bc)}+\frac{(ad-2bc)\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
```

```
[Out] d*x*sqrt(a + b*x**2)/(2*c*(c + d*x**2)*(a*d - b*c)) + (a*d - 2*b*
c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(2*c**(3/2)
*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.168819, size = 101, normalized size = 1.

$$\frac{dx\sqrt{a+bx^2}}{2c\left(c+dx^2\right)(ad-bc)}-\frac{(2bc-ad)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*c*(-(b*c) + a*d)*(c + d*x^2)) - ((2*b*c
- a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(2*c^(3/2)*(-(b*c) + a*d)^(3/2))
```

Maple [B] time = 0.029, size = 809, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/4/c/d*b*(-c*
d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2)+1/4/c/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln
((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c
)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-1/4/c/(-c*d)^(1/2
)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/
2)/d))+1/4/c/(-c*d)^(1/2)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

Fricas [A] time = 0.36286, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{bc^2-acd}\sqrt{bx^2+a}dx-\left(2bc^2-acd+\left(2bcd-ad^2\right)x^2\right)\log\left(\frac{\left(\left(8b^2c^2-8abcd+a^2d^2\right)x^4+a^2c^2+2\left(4abc^2-3a^2cd\right)x^2\right)\sqrt{bc^2-acd}+4(\cdots}{d^2x^4+2cdx^2+c\cdots}\right.}{8\left(bc^3-ac^2d+(bc^2d-acd^2)x^2\right)\sqrt{bc^2-acd}}\right.$$

$$\left.-\frac{2\sqrt{-bc^2+acd}\sqrt{bx^2+a}dx-\left(2bc^2-acd+\left(2bcd-ad^2\right)x^2\right)\arctan\left(\frac{\sqrt{-bc^2+acd}\left((2bc-ad)x^2+ac\right)}{2(bc^2-acd)\sqrt{bx^2+a}x}\right)}{4\left(bc^3-ac^2d+(bc^2d-acd^2)x^2\right)\sqrt{-bc^2+acd}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="fricas")

[Out] [-1/8*(4*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*d*x - (2*b*c^2 - a*c
*d + (2*b*c*d - a*d^2)*x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^
2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*
c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 -
a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((b*
c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)), -1
/4*(2*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a)*d*x - (2*b*c^2 - a*c*d
 + (2*b*c*d - a*d^2)*x^2)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
 - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((b*c^3
- a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)

[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.249074, size = 327, normalized size = 3.24

$$-\frac{1}{2}\,b^{\frac{3}{2}}\left(\frac{(2\,bc-ad)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-abcd)\sqrt{-b^2c^2+abcd}}+\frac{2\left(2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] -1/2*b^(3/2)*((2*b*c - a*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 +
 a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b
*c*d)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*b*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(
b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b
*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*(b^2*c^2 - a*
b*c*d)))
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=163

$$\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8c^2(c+dx^2)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c(c+dx^2)^2(bc-ad)}$$

```
[Out] -(d*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*
b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) +
 ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)
/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.338882, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{(3a^2d^2-8abcd+8b^2c^2)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}-\frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8c^2(c+dx^2)(bc-ad)^2}-\frac{dx\sqrt{a+bx^2}}{4c(c+dx^2)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] -(d*x*Sqrt[a + b*x^2])/(4*c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*
b*c - a*d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) +
 ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)
/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 55.9347, size = 146, normalized size = 0.9

$$\frac{dx\sqrt{a+bx^2}}{4c(c+dx^2)^2(ad-bc)}+\frac{3dx\sqrt{a+bx^2}(ad-2bc)}{8c^2(c+dx^2)(ad-bc)^2}+\frac{(3a^2d^2-8abcd+8b^2c^2)\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{\frac{5}{2}}(ad-bc)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] d*x*sqrt(a + b*x**2)/(4*c*(c + d*x**2)**2*(a*d - b*c)) + 3*d*x*sq
rt(a + b*x**2)*(a*d - 2*b*c)/(8*c**2*(c + d*x**2)*(a*d - b*c)**2)
 + (3*a**2*d**2 - 8*a*b*c*d + 8*b**2*c**2)*atan(x*sqrt(a*d - b*c)
/(sqrt(c)*sqrt(a + b*x**2)))/(8*c**(5/2)*(a*d - b*c)**(5/2))
```

Mathematica [A] time = 0.276279, size = 143, normalized size = 0.88

$$\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{ad-bc}}+\frac{\sqrt{c}dx\sqrt{a+bx^2}(ad(5c+3dx^2)-2bc(4c+3dx^2))}{(c+dx^2)^2}}{8c^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] ((Sqrt[c]*d*x*Sqrt[a + b*x^2]*(-2*b*c*(4*c + 3*d*x^2) + a*d*(5*c
+ 3*d*x^2)))/(c + d*x^2)^2 + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)
*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[-
(b*c) + a*d])/(8*c^(5/2)*(b*c - a*d)^2)
```

Maple [B] time = 0.039, size = 1815, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x)
```

```
[Out] 3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c^2/d*
b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2))/(x-(-c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/
d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2)+3/16/c^2/d*b*(-c*d)^(1/2)/(a*d-b*c)/((a*d-b*c)
/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/16/
(-c*d)^(1/2)/c^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2
)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)
)/(x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c^2/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*
c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+1/16/(-c*d)^(1/2
)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c*b/(a*d-
b*c)^2/(x-(-c*d)^(1/2)/d)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2)*b^2/(
a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2
```

```
*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(
-c*d)^(1/2)/d))-1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-b*c)/d)^(1/
2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*
d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/16/(-c*d)^
(1/2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)^2*((x+(-c*d)^(1/2)/d)^2*b-2*
b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/c*b/(
a*d-b*c)^2/(x+(-c*d)^(1/2)/d)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^
(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/(-c*d)^(1/2)*b
^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/
d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/
(x+(-c*d)^(1/2)/d))+1/16/(-c*d)^(1/2)/c*b/(a*d-b*c)/((a*d-b*c)/d)
^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*
((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x
+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\,(dx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

Fricas [A] time = 0.616835, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(3\left(2\,bcd^2-ad^3\right)x^3+\left(8\,bc^2d-5\,acd^2\right)x\right)\sqrt{bc^2-acd}\sqrt{bx^2+a}-\left(8\,b^2c^4-8\,abc^3d+3\,a^2c^2d^2+\left(8\,b^2c^2d^2-8\,abcd^3+\right.\right.}{32\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^4d^2\right.\right.}\right.$$

$$-\frac{2\left(3\left(2\,bcd^2-ad^3\right)x^3+\left(8\,bc^2d-5\,acd^2\right)x\right)\sqrt{-bc^2+acd}\sqrt{bx^2+a}-\left(8\,b^2c^4-8\,abc^3d+3\,a^2c^2d^2+\left(8\,b^2c^2d^2-8\,abcd^3+\right.\right.}{16\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)x^4+2\left(b^2c^5d-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(3*(2*b*c*d^2 - a*d^3)*x^3 + (8*b*c^2*d - 5*a*c*d^2)*x)
*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a) - (8*b^2*c^4 - 8*a*b*c^3*d +
 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x^4 +
```

```
2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*log((((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*
d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d + a^2*c
*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^2*x^4 +
2*c*d*x^2 + c^2)))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c
^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*
c^4*d^2 + a^2*c^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)), -1/16*(2*(3*(2*
b*c*d^2 - a*d^3)*x^3 + (8*b*c^2*d - 5*a*c*d^2)*x)*sqrt(-b*c^2 + a
*c*d)*sqrt(b*x^2 + a) - (8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2
+ (8*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d
- 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*arctan(1/2*sqrt(-b*c^2 + a*c*
d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x))
)/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^
3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3
*d^3)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.967293, size = 726, normalized size = 4.45

$$-\frac{1}{8}\, b^{\frac{5}{2}} \left(\frac{\left(8\, b^2 c^2 - 8\, abcd + 3\, a^2 d^2\right) \arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 d + 2\, bc - ad}{2\, \sqrt{-b^2c^2+abcd}}\right)}{\left(b^4 c^4 - 2\, ab^3 c^3 d + a^2 b^2 c^2 d^2\right)\sqrt{-b^2c^2 + abcd}} + \frac{2\left(8\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^6 b^2 c^2 d - 8\left(\sqrt{b}x - \sqrt{bx^2+a}\right)\right.}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3),x, algorithm="giac")
```

```
[Out] -1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2
 + a*b*c*d)) + 2*(8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d^2 + 3*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^3*c^
3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 42*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 +
```

```
a))^4*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d
- 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*a^4*d^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^
4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a
))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

3.81 $\int \frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}+\frac{d\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{dx\sqrt{a+bx^2}\left(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3\right)}{48ab^4}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2(6bc-7ad)}{6ab^2}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[Out] -(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3
)*x*Sqrt[a + b*x^2])/(48*a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 3
5*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c
- 7*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/(6*a*b^2) + ((b*c - a*d
)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2]) + (d*(64*b^3*c^3 - 144*a
*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(16*b^(9/2))
```

Rubi [A] time = 0.632956, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3}+\frac{d\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}-\frac{dx\sqrt{a+bx^2}\left(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3\right)}{48ab^4}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2(6bc-7ad)}{6ab^2}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3
)*x*Sqrt[a + b*x^2])/(48*a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 3
5*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c
- 7*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2)^2)/(6*a*b^2) + ((b*c - a*d
)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2]) + (d*(64*b^3*c^3 - 144*a
*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sq
```

```
rt[a + b*x^2]])/(16*b^(9/2))
```

Rubi in Sympy [A] time = 85.0963, size = 262, normalized size = 1.02

$$-\frac{d\left(35a^3d^3-120a^2bcd^2+144ab^2c^2d-64b^3c^3\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{\frac{9}{2}}}-\frac{x\left(c+dx^2\right)^3(ad-bc)}{ab\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)^2(7ad-6bc)}{6ab^2}-\frac{d^2x\sqrt{a+bx^2}\left(ac\,(7ad-12bc)+x^2\left(35a^2d^2-64abcd+24b^2c^2\right)\right)}{24ab^3}+\frac{dx\sqrt{a+bx^2}\left(105a^3d^3-346a^2bcd^2+352ab^2c^2d-96b^3c^3\right)}{48ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

```
[Out] -d*(35*a**3*d**3 - 120*a**2*b*c*d**2 + 144*a*b**2*c**2*d - 64*b**
3*c**3)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(16*b**(9/2)) - x*(c +
d*x**2)**3*(a*d - b*c)/(a*b*sqrt(a + b*x**2)) + d*x*sqrt(a + b*x*
*2)*(c + d*x**2)**2*(7*a*d - 6*b*c)/(6*a*b**2) - d**2*x*sqrt(a +
b*x**2)*(a*c*(7*a*d - 12*b*c) + x**2*(35*a**2*d**2 - 64*a*b*c*d +
 24*b**2*c**2))/(24*a*b**3) + d*x*sqrt(a + b*x**2)*(105*a**3*d**3
 - 346*a**2*b*c*d**2 + 352*a*b**2*c**2*d - 96*b**3*c**3)/(48*a*b*
*4)
```

Mathematica [A] time = 0.400355, size = 172, normalized size = 0.67

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(3d^2\left(19a^2d^2-56abcd+48b^2c^2\right)+2bd^3x^2(24bc-11ad)+\frac{48(bc-ad)^4}{a(a+bx^2)}+8b^2d^4x^4\right)+3d\left(-35a^3d^3+120a^2bcd^2\right.}{48b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(3*d^2*(48*b^2*c^2 - 56*a*b*c*d + 19*a
^2*d^2) + 2*b*d^3*(24*b*c - 11*a*d)*x^2 + 8*b^2*d^4*x^4 + (48*(b*
c - a*d)^4)/(a*(a + b*x^2))) + 3*d*(64*b^3*c^3 - 144*a*b^2*c^2*d
+ 120*a^2*b*c*d^2 - 35*a^3*d^3)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]
])/(48*b^(9/2))
```

Maple [A] time = 0.024, size = 340, normalized size = 1.3

$$\frac{c^4x}{a}\frac{1}{\sqrt{bx^2+a}}+\frac{d^4x^7}{6\,b}\frac{1}{\sqrt{bx^2+a}}-\frac{7\,ad^4x^5}{24\,b^2}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,d^4a^2x^3}{48\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,a^3d^4x}{16\,b^4}\frac{1}{\sqrt{bx^2+a}}-\frac{35\,a^3d^4}{16}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}+\frac{cd^3x^5}{b}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,cd^3ax^3}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,cd^3a^2x}{2\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,cd^3a^2}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+3\,\frac{c^2d^2x^3}{b\sqrt{bx^2+a}}+9\,\frac{c^2d^2ax}{b^2\sqrt{bx^2+a}}-9\,\frac{c^2d^2a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{5/2}}-4\,\frac{c^3dx}{b\sqrt{bx^2+a}}+4\,\frac{c^3d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^4/(b*x^2+a)^(3/2),x)
```

```
[Out] c^4*x/a/(b*x^2+a)^(1/2)+1/6*d^4*x^7/b/(b*x^2+a)^(1/2)-7/24*d^4*a/
b^2*x^5/(b*x^2+a)^(1/2)+35/48*d^4*a^2/b^3*x^3/(b*x^2+a)^(1/2)+35/
16*d^4*a^3/b^4*x/(b*x^2+a)^(1/2)-35/16*d^4*a^3/b^(9/2)*ln(x*b^(1/
2)+(b*x^2+a)^(1/2))+c*d^3*x^5/b/(b*x^2+a)^(1/2)-5/2*c*d^3*a/b^2*x
^3/(b*x^2+a)^(1/2)-15/2*c*d^3*a^2/b^3*x/(b*x^2+a)^(1/2)+15/2*c*d^
3*a^2/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3*c^2*d^2*x^3/b/(b*x^
2+a)^(1/2)+9*c^2*d^2*a/b^2*x/(b*x^2+a)^(1/2)-9*c^2*d^2*a/b^(5/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))-4*c^3*d*x/b/(b*x^2+a)^(1/2)+4*c^3*d
/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.451806, size = 1, normalized size = 0.

$$\left[\frac{2\left(8\,ab^3d^4x^7+2\left(24\,ab^3cd^3-7\,a^2b^2d^4\right)x^5+\left(144\,ab^3c^2d^2-120\,a^2b^2cd^3+35\,a^3bd^4\right)x^3+3\left(16\,b^4c^4-64\,ab^3c^3d+144\,a^2b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(2*(8*a*b^3*d^4*x^7 + 2*(24*a*b^3*c*d^3 - 7*a^2*b^2*d^4)*x^
5 + (144*a*b^3*c^2*d^2 - 120*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x^3 +
3*(16*b^4*c^4 - 64*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 120*a^3*b*
c*d^3 + 35*a^4*d^4)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(64*a^2*b^3*c^
3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*
b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*b*d^
4)*x^2)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b
^5*x^2 + a^2*b^4)*sqrt(b)), 1/48*((8*a*b^3*d^4*x^7 + 2*(24*a*b^3*
c*d^3 - 7*a^2*b^2*d^4)*x^5 + (144*a*b^3*c^2*d^2 - 120*a^2*b^2*c*d
^3 + 35*a^3*b*d^4)*x^3 + 3*(16*b^4*c^4 - 64*a*b^3*c^3*d + 144*a^2
*b^2*c^2*d^2 - 120*a^3*b*c*d^3 + 35*a^4*d^4)*x)*sqrt(b*x^2 + a)*s
qrt(-b) + 3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c
*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a
^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)))/((a*b^5*x^2 + a^2*b^4)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^{2}\right)^{4}}{\left(a + bx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.248377, size = 317, normalized size = 1.23

$$\frac{\left(\left(2\left(\frac{4\,d^{4}x^{2}}{b} + \frac{24\,ab^{6}cd^{3} - 7\,a^{2}b^{5}d^{4}}{ab^{7}}\right)x^{2} + \frac{144\,ab^{6}c^{2}d^{2} - 120\,a^{2}b^{5}cd^{3} + 35\,a^{3}b^{4}d^{4}}{ab^{7}}\right)x^{2} + \frac{3\left(16\,b^{7}c^{4} - 64\,ab^{6}c^{3}d + 144\,a^{2}b^{5}c^{2}d^{2} - 120\,a^{3}b^{4}cd^{3} + 35\,a^{4}b^{3}d^{4}\right)}{ab^{7}}\right)x}{48\,\sqrt{bx^{2} + a}} - \frac{\left(64\,b^{3}c^{3}d - 144\,ab^{2}c^{2}d^{2} + 120\,a^{2}bcd^{3} - 35\,a^{3}d^{4}\right)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^{2} + a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))
*x^2 + (144*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(
a*b^7))*x^2 + 3*(16*b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^
```

```
2 - 120*a^3*b^4*c*d^3 + 35*a^4*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a
) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 35
*a^3*d^4)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.82 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)(4bc-5ad)}{4ab^2}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[Out] -(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*b^3)
- (d*(4*b*c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + (
(b*c - a*d)*x*(c + d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*
c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(8*b^(7/2))
```

Rubi [A] time = 0.433848, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}-\frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)(4bc-5ad)}{4ab^2}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*b^3)
- (d*(4*b*c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + (
(b*c - a*d)*x*(c + d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*
c^2 - 12*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(8*b^(7/2))
```

Rubi in Sympy [A] time = 44.9421, size = 163, normalized size = 0.96

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{7}{2}}}-\frac{x\left(c+dx^2\right)^2(ad-bc)}{ab\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}\left(ac+x^2(5ad-4bc)\right)}{4ab^2}-\frac{dx\sqrt{a+bx^2}(3ad-2bc)(5ad-8bc)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
```

```
[Out] 3*d*(5*a**2*d**2 - 12*a*b*c*d + 8*b**2*c**2)*atanh(sqrt(b)*x/sqrt
(a + b*x**2))/(8*b**(7/2)) - x*(c + d*x**2)**2*(a*d - b*c)/(a*b*s
qrt(a + b*x**2)) + d**2*x*sqrt(a + b*x**2)*(a*c + x**2*(5*a*d - 4
*b*c))/(4*a*b**2) - d*x*sqrt(a + b*x**2)*(3*a*d - 2*b*c)*(5*a*d -
 8*b*c)/(8*a*b**3)
```

Mathematica [A] time = 0.155896, size = 122, normalized size = 0.72

$$\frac{3d\left(5a^2d^2-12abcd+8b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(d^2(12bc-7ad)+\frac{8(bc-ad)^3}{a(a+bx^2)}+2bd^3x^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(d^2*(12*b*c - 7*a*d) + 2*b*d^3*x^2 + (8*(b*c
- a*d)^3)/(a*(a + b*x^2))))/(8*b^3) + (3*d*(8*b^2*c^2 - 12*a*b*c*
d + 5*a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Maple [A] time = 0.013, size = 219, normalized size = 1.3

$$\frac{c^3x}{a}\frac{1}{\sqrt{bx^2+a}}+\frac{d^3x^5}{4\,b}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,ad^3x^3}{8\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{15\,a^2d^3x}{8\,b^3}\frac{1}{\sqrt{bx^2+a}}+\frac{15\,a^2d^3}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}+\frac{3\,cd^2x^3}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{9\,acd^2x}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{9\,acd^2}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-3\,\frac{c^2dx}{b\sqrt{bx^2+a}}+3\,\frac{c^2d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^(3/2),x)
```

```
[Out] c^3*x/a/(b*x^2+a)^(1/2)+1/4*d^3*x^5/b/(b*x^2+a)^(1/2)-5/8*d^3*a/b
^2*x^3/(b*x^2+a)^(1/2)-15/8*d^3*a^2/b^3*x/(b*x^2+a)^(1/2)+15/8*d^
3*a^2/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+3/2*c*d^2*x^3/b/(b*x^
2+a)^(1/2)+9/2*c*d^2*a/b^2*x/(b*x^2+a)^(1/2)-9/2*c*d^2*a/b^(5/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2))-3*c^2*d*x/b/(b*x^2+a)^(1/2)+3*c^2*d
/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278341, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\,ab^2d^3x^5+\left(12\,ab^2cd^2-5\,a^2bd^3\right)x^3+\left(8\,b^3c^3-24\,ab^2c^2d+36\,a^2bcd^2-15\,a^3d^3\right)x\right)\sqrt{bx^2+a}\sqrt{b}+3\left(8\,a^2b^2c^2d-12\,a^3\right.}{16\left(ab^4x^2+a^2b^3\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*a*b^2*d^3*x^5 + (12*a*b^2*c*d^2 - 5*a^2*b*d^3)*x^3 +
(8*b^3*c^3 - 24*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 15*a^3*d^3)*x)*sqr
t(b*x^2 + a)*sqrt(b) + 3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^
4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*log
(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b^4*x^2 + a
^2*b^3)*sqrt(b)), 1/8*((2*a*b^2*d^3*x^5 + (12*a*b^2*c*d^2 - 5*a^2
*b*d^3)*x^3 + (8*b^3*c^3 - 24*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 15*a
^3*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 3*(8*a^2*b^2*c^2*d - 12*a^3
*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*
b*d^3)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^4*x^2 + a^2
*b^3)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.257793, size = 212, normalized size = 1.25

$$\frac{\left(\left(\frac{2\,d^3x^2}{b}+\frac{12\,ab^4cd^2-5\,a^2b^3d^3}{ab^5}\right)x^2+\frac{8\,b^5c^3-24\,ab^4c^2d+36\,a^2b^3cd^2-15\,a^3b^2d^3}{ab^5}\right)x}{8\,\sqrt{bx^2+a}}-\frac{3\,\left(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2
 + (8*b^5*c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^
3)/(a*b^5))*x/sqrt(b*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 +
 5*a^2*d^3)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.83 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}+\frac{x(bc-ad)^2}{ab^2\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}}{2b^2}$$

```
[Out] ((b*c - a*d)^2*x)/(a*b^2*Sqrt[a + b*x^2]) + (d^2*x*Sqrt[a + b*x^2
])/(2*b^2) + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(5/2))
```

Rubi [A] time = 0.154494, antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}-\frac{dx\sqrt{a+bx^2}(2bc-3ad)}{2ab^2}+\frac{x\left(c+dx^2\right)(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -(d*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(2*a*b^2) + ((b*c - a*d)*x
*(c + d*x^2))/(a*b*Sqrt[a + b*x^2]) + (d*(4*b*c - 3*a*d)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi in Sympy [A] time = 20.4238, size = 95, normalized size = 1.06

$$-\frac{d\left(3ad-4bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{5}{2}}}-\frac{x\left(c+dx^2\right)(ad-bc)}{ab\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\,(3ad-2bc)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] -d*(3*a*d - 4*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(5/2))
 - x*(c + d*x**2)*(a*d - b*c)/(a*b*sqrt(a + b*x**2)) + d*x*sqrt(a
 + b*x**2)*(3*a*d - 2*b*c)/(2*a*b**2)
```

Mathematica [A] time = 0.153965, size = 89, normalized size = 0.99

$$\frac{\sqrt{b}x\sqrt{a+bx^2}\left(\frac{2(bc-ad)^2}{a(a+bx^2)}+d^2\right)+d(4bc-3ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(d^2 + (2*(b*c - a*d)^2)/(a*(a + b*x^2
))) + d*(4*b*c - 3*a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^
(5/2))
```

Maple [A] time = 0.01, size = 123, normalized size = 1.4

$$\frac{c^2x}{a}\frac{1}{\sqrt{bx^2+a}}+\frac{d^2x^3}{2\,b}\frac{1}{\sqrt{bx^2+a}}+\frac{3\,ad^2x}{2\,b^2}\frac{1}{\sqrt{bx^2+a}}-\frac{3\,ad^2}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-2\,\frac{cdx}{b\sqrt{bx^2+a}}+2\,\frac{cd\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^(3/2),x)
```

```
[Out] c^2*x/a/(b*x^2+a)^(1/2)+1/2*d^2*x^3/b/(b*x^2+a)^(1/2)+3/2*d^2*a/b
^2*x/(b*x^2+a)^(1/2)-3/2*d^2*a/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))-2*c*d*x/b/(b*x^2+a)^(1/2)+2*c*d/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229807, size = 1, normalized size = 0.01

$$\left[\frac{2\left(abd^2x^3+\left(2\,b^2c^2-4\,abcd+3\,a^2d^2\right)x\right)\sqrt{bx^2+a}\sqrt{b}-\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^2\right)\log\left(2\,\sqrt{bx^2+a}bx-\left(\right.\right.}{4\left(ab^3x^2+a^2b^2\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a*b*d^2*x^3 + (2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)*sqr
t(b*x^2 + a)*sqrt(b) - (4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d -
3*a^2*b*d^2)*x^2)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(
b)))/((a*b^3*x^2 + a^2*b^2)*sqrt(b)), 1/2*((a*b*d^2*x^3 + (2*b^2*
c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)*sqrt(b*x^2 + a)*sqrt(-b) + (4*a^2
*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)))/((a*b^3*x^2 + a^2*b^2)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.244329, size = 124, normalized size = 1.38

$$\frac{\left(\frac{d^2x^2}{b}+\frac{2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}}-\frac{\left(4\,bcd-3\,ad^2\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))
*x/sqrt(b*x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*ln(abs(-sqrt(b)*x +
sqrt(b*x^2 + a)))/b^(5/2)
```

3.84 $\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/b^(3/2)
```

Rubi [A] time = 0.050372, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/b^(3/2)
```

Rubi in Sympy [A] time = 8.09783, size = 46, normalized size = 0.85

$$\frac{d\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{3}{2}}}-\frac{x\left(ad-bc\right)}{ab\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(3/2) - x*(a*d - b*c)/(a*b
*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0630773, size = 58, normalized size = 1.07

$$\frac{d\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{3/2}}-\frac{x(ad-bc)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -(((-(b*c) + a*d)*x)/(a*b*Sqrt[a + b*x^2])) + (d*Log[b*x + Sqrt[b
]*Sqrt[a + b*x^2]])/b^(3/2)
```

Maple [A] time = 0.007, size = 54, normalized size = 1.

$$\frac{cx}{a}\frac{1}{\sqrt{bx^2+a}}-\frac{dx}{b}\frac{1}{\sqrt{bx^2+a}}+d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^(3/2),x)
```

```
[Out] c*x/a/(b*x^2+a)^(1/2)-d*x/b/(b*x^2+a)^(1/2)+d/b^(3/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.219841, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}(bc-ad)\sqrt{b}x+\left(abdx^2+a^2d\right)\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{2\left(ab^2x^2+a^2b\right)\sqrt{b}},\frac{\sqrt{bx^2+a}(bc-ad)\sqrt{-b}x+\left(abdx^2+a^2d\right.}{\left(ab^2x^2+a^2b\right)\sqrt{-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*sqrt(b*x^2 + a)*(b*c - a*d)*sqrt(b)*x + (a*b*d*x^2 + a^2*
d)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a*b^2*x
```

```
^2 + a^2*b)*sqrt(b)), (sqrt(b*x^2 + a)*(b*c - a*d)*sqrt(-b)*x + (
a*b*d*x^2 + a^2*d)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a*b^2*x^
2 + a^2*b)*sqrt(-b))]
```

Sympy [A] time = 11.1484, size = 60, normalized size = 1.11

$$d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x*
*2/a))) + c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.22948, size = 68, normalized size = 1.26

$$-\frac{d\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{(bc-ad)x}{\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/
(sqrt(b*x^2 + a)*a*b)
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+bx^2}}$$

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.00889905, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 1.27352, size = 12, normalized size = 0.75

$$\frac{x}{a\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2),x)

[Out] x/(a*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.01152, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$\frac{x}{a}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2),x)

[Out] x/a/(b*x^2+a)^(1/2)
```

Maxima [A] time = 1.35486, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-3/2),x, algorithm="maxima")

[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A] time = 0.206676, size = 31, normalized size = 1.94

$$\frac{\sqrt{bx^2+a}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-3/2),x, algorithm="fricas")

[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] time = 1.80627, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.224857, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{bx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

3.86 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx$

Optimal. Leaf size=79

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*
d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.11854, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{bx}{a\sqrt{a+bx^2}(bc-ad)}-\frac{d\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*
d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 18.8374, size = 66, normalized size = 0.84

$$\frac{d\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(ad-bc)^{\frac{3}{2}}}-\frac{bx}{a\sqrt{a+bx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] d*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*(a*
d - b*c)**(3/2)) - b*x/(a*sqrt(a + b*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.159129, size = 79, normalized size = 1.

$$\frac{d\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(ad-bc)^{3/2}}-\frac{bx}{a\sqrt{a+bx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] -((b*x)/(a*(-(b*c) + a*d)*Sqrt[a + b*x^2])) + (d*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(-(b*c) + a*d)
^(3/2))
```

Maple [B] time = 0.025, size = 618, normalized size = 7.8

$$\frac{d}{2ad-2bc}\frac{1}{\sqrt{-cd}}\frac{1}{\sqrt{\left(x-\frac{1}{d}\sqrt{-cd}\right)^{2}b+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}}-\frac{bx}{(2ad-2bc)a}\frac{1}{\sqrt{\left(x-\frac{1}{d}\sqrt{-cd}\right)^{2}b+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}}-\frac{d}{2ad-2bc}\ln\left(1\left(2\frac{ad-bc}{d}+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^{2}b+2\frac{b\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}\right.\right.-\frac{d}{2ad-2bc}\frac{1}{\sqrt{-cd}}\frac{1}{\sqrt{\left(x+\frac{1}{d}\sqrt{-cd}\right)^{2}b-2\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}}-\frac{bx}{(2ad-2bc)a}\frac{1}{\sqrt{\left(x+\frac{1}{d}\sqrt{-cd}\right)^{2}b-2\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}}+\frac{d}{2ad-2bc}\ln\left(1\left(2\frac{ad-bc}{d}-2\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-2\frac{b\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+\frac{ad-bc}{d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x)
```

```
[Out] 1/2/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(
1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x-
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)*x*b-1/2/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*
ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b
*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/2/(-c*d)^(1/2
)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b
+1/2/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)
/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
```

```
b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306097, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{bc^2-acd}\sqrt{bx^2+a}abx-\left(abdx^2+a^2d\right)\log\left(\frac{\left(\left(8b^2c^2-8abcd+a^2d^2\right)x^4+a^2c^2+2\left(4abc^2-3a^2cd\right)x^2\right)\sqrt{bc^2-acd}+4\left(\left(2b^2c^3-3abc^2d+a^2cd^2\right)\right.}{d^2x^4+2cdx^2+c^2}\right.}{4\left(a^2bc-a^3d+(ab^2c-a^2bd)x^2\right)\sqrt{bc^2-acd}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*b*x - (a*b*d*x^2 + a^
2*d)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 -
3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^
2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^2*b*c - a^3*d + (a*b^2*
c - a^2*b*d)*x^2)*sqrt(b*c^2 - a*c*d)), 1/2*(2*sqrt(-b*c^2 + a*c*
d)*sqrt(b*x^2 + a)*b*x - (a*b*d*x^2 + a^2*d)*arctan(1/2*sqrt(-b*c
^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2
 + a)*x)))/((a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-b*c
^2 + a*c*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.231117, size = 143, normalized size = 1.81

$$\frac{\sqrt{b}d\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bc-ad)}+\frac{bx}{(abc-a^2d)\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] sqrt(b)*d*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c -
 a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c -
a*d)) + b*x/((a*b*c - a^2*d)*sqrt(b*x^2 + a))
```

3.87 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=143

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)
/(2*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/
2)*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.323469, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{d(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}+\frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}(bc-ad)^2}-\frac{dx}{2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)
/(2*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/
2)*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 54.3963, size = 121, normalized size = 0.85

$$\frac{dx}{2c\sqrt{a+bx^2}\,(c+dx^2)\,(ad-bc)}+\frac{d\,(ad-4bc)\,\mathrm{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}\,(ad-bc)^{\frac{5}{2}}}+\frac{bx\,(ad+2bc)}{2ac\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*sqrt(a + b*x**2)*(c + d*x**2)*(a*d - b*c)) + d*(a*d - 4*
b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(2*c**(3/
2)*(a*d - b*c)**(5/2)) + b*x*(a*d + 2*b*c)/(2*a*c*sqrt(a + b*x**2
)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.292044, size = 133, normalized size = 0.93

$$\frac{\frac{\sqrt{c}x\left(a^2d^2+abd^2x^2+2b^2c\left(c+dx^2\right)\right)}{a\sqrt{a+bx^2}(c+dx^2)(bc-ad)^2}+\frac{d(ad-4bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{(ad-bc)^{5/2}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((Sqrt[c]*x*(a^2*d^2 + a*b*d^2*x^2 + 2*b^2*c*(c + d*x^2)))/(a*(b*
c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(-4*b*c + a*d)*ArcTa
n[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(-(b*c) + a*
d)^(5/2))/(2*c^(3/2))
```

Maple [B] time = 0.031, size = 1439, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x)
```

```
[Out] 1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-
c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/4/c*b*(-c*d)
^(1/2)/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/4*b^2/(a*d-b*c)^2/a/((x-(-c*
d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d
)^(1/2)*x-3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln
((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c
)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(
1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/4/c/(a*d-b*c)/a
/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2)*x*b+1/4/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d
)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2)-3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2
*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+3/4*b^2/(
a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+3/4/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/
d))+1/4/c/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b+1/4/c/(-c*d)^(1/2)/(a*d
-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)-1/4/c/(-c*d)^(1/2)/(a*d-b*c)*d/((a*d-b*c)
/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)
```

```
+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/4/c
/(-c*d)^(1/2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/4/c/(-c*d)^(1/2)/(a*d
-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-
2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*
d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + a\right)^{\frac{3}{2}}\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

Fricas [A] time = 0.579177, size = 1, normalized size = 0.01

$$\Bigg[\frac{4\left(\left(2\,b^2cd + abd^2\right)x^3 + \left(2\,b^2c^2 + a^2d^2\right)x\right)\sqrt{bc^2 - acd}\sqrt{bx^2 + a} - \Big(4\,a^2bc^2d - a^3cd^2 + \left(4\,ab^2cd^2 - a^2bd^3\right)x^4 + \Big(4\,ab^2c^2d + 3}{8\,\Big(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2 + \Big(ab^3c^3d - 2\,a^2b^2c^2d^2 + a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*((2*b^2*c*d + a*b*d^2)*x^3 + (2*b^2*c^2 + a^2*d^2)*x)*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a) - (4*a^2*b*c^2*d - a^3*c*d^2 + (
4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*
c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*
b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)
*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^
4*c*d^3)*x^2)*sqrt(b*c^2 - a*c*d)), 1/4*(2*((2*b^2*c*d + a*b*d^2)
*x^3 + (2*b^2*c^2 + a^2*d^2)*x)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 +
 a) - (4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^
4 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*arctan(1/2*sqr
t(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt
(b*x^2 + a)*x)))/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a
```

```
*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 -
a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)*sqrt(-b*c^2 + a*c
*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 3.90641, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.88 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\, dx$

Optimal. Leaf size=225

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\sqrt{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*
c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (
d*(4*b*c - a*d)*(2*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c
- a*d)^3*(c + d*x^2)) - (3*d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*
(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.678708, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}+\frac{dx\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{8ac^2\left(c+dx^2\right)(bc-ad)^3}+\frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}\left(c+dx^2\right)(bc-ad)^2}-\frac{dx}{4c\sqrt{a+bx^2}\left(c+dx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*
c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (
d*(4*b*c - a*d)*(2*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c
- a*d)^3*(c + d*x^2)) - (3*d*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Ar
cTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*
(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 115.598, size = 196, normalized size = 0.87

$$\frac{dx}{4c\sqrt{a+bx^2}\left(c+dx^2\right)^2(ad-bc)}+\frac{dx\,(3ad-8bc)}{8c^2\sqrt{a+bx^2}\left(c+dx^2\right)(ad-bc)^2}+\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{\frac{5}{2}}(ad-bc)^{\frac{7}{2}}}+\frac{bx\,(ad-4bc)\,(3ad+2bc)}{8ac^2\sqrt{a+bx^2}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
```

```
[Out] d*x/(4*c*sqrt(a + b*x**2)*(c + d*x**2)**2*(a*d - b*c)) + d*x*(3*a
*d - 8*b*c)/(8*c**2*sqrt(a + b*x**2)*(c + d*x**2)*(a*d - b*c)**2)
 + 3*d*(a**2*d**2 - 4*a*b*c*d + 8*b**2*c**2)*atan(x*sqrt(a*d - b*
c)/(sqrt(c)*sqrt(a + b*x**2)))/(8*c**(5/2)*(a*d - b*c)**(7/2)) +
b*x*(a*d - 4*b*c)*(3*a*d + 2*b*c)/(8*a*c**2*sqrt(a + b*x**2)*(a*d
 - b*c)**3)
```

Mathematica [A] time = 0.794328, size = 181, normalized size = 0.8

$$\frac{1}{8}\left(\frac{3d\left(a^2d^2-4abcd+8b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(ad-bc)^{7/2}}+x\sqrt{a+bx^2}\left(-\frac{8b^3}{a\left(a+bx^2\right)(ad-bc)^3}+\frac{d^2(10bc-3ad)}{c^2\left(c+dx^2\right)(bc-ad)^3}+\frac{2d^2}{c\left(c+dx^2\right)^2(bc-ad)^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((-8*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (
2*d^2)/(c*(b*c - a*d)^2*(c + d*x^2)^2) + (d^2*(10*b*c - 3*a*d))/(
c^2*(b*c - a*d)^3*(c + d*x^2))) + (3*d*(8*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])
/(c^(5/2)*(-(b*c) + a*d)^(7/2)))/8
```

Maple [B] time = 0.04, size = 2919, normalized size = 13.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x)
```

```
[Out] 15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*
(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-15/16*b^3/(a
*d-b*c)^3/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2
/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(1/2)-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)
```

```
^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(1/2)-5/16/c*b/(a*d-b*c)^2/(x+(-c*d)^(1/2)/d)/((x+(-
c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)
/d)^(1/2)-15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-15
/16*b^3/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/16/(-c*d)^(1/2)/c^2/(a*
d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2)-9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x+
(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2)+3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1/1
6/(-c*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/
d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+1
5/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*
d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1
/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/16/(-c*d)^(1/2)/c*d*b/
(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2)-15/16/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/
((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)
/d))+3/16/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/4/c*b^
2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*
c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-
(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^
(1/2)/d))+3/16/c^2/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b-1/4/c*b^2/(a*d
-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2)*x+9/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/(
(a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)
^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/
d))+3/16/c^2/(a*d-b*c)/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)
/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b-3/16/(-c*d)^(1/2)/c^
2/(a*d-b*c)*d/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d
)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(
x-(-c*d)^(1/2)/d))+3/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((a*d-b*c)/d
)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2
*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-5/16/c*
b/(a*d-b*c)^2/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-3/16/(-c*d)^(1/2
)/c*d*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c
*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(
1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2))/(x-(-c*d)^(1/2)/d))+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((
x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2)+3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2)+3/16/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*
```

```
(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)
^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)
/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

Fricas [A] time = 1.30652, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((8*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (1
6*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 5*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 +
 (8*b^3*c^4 + 12*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x)*sqrt(b*c^2 - a*c
*d)*sqrt(b*x^2 + a) - 3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*
c^2*d^3 + (8*a*b^3*c^2*d^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (
16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 + a^4*d^5)*x^4 + (8*a*b^3*c^4*d
+ 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a^4*c*d^4)*x^2)*log(((
(8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 - 3*a*b*c^2*d
 + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(d^
2*x^4 + 2*c*d*x^2 + c^2)))/((a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^
4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 +
3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b
^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4
 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d
^3 - 2*a^5*c^3*d^4)*x^2)*sqrt(b*c^2 - a*c*d)), 1/16*(2*((8*b^3*c^
2*d^2 + 10*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (16*b^3*c^3*d + 12*a*
b^2*c^2*d^2 + 5*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (8*b^3*c^4 + 12*a^
2*b*c^2*d^2 - 5*a^3*c*d^3)*x)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a
) - 3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3
*c^2*d^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 -
 2*a^3*b*c*d^4 + a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d
^2 - 7*a^3*b*c^2*d^3 + 2*a^4*c*d^4)*x^2)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a
```

```
)*x)))/((a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5*c^
4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 -
a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b
^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*
b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*
x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 12.073, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3),x, algorithm="giac")

[Out] sage0*x
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=255

$$\frac{x\left(c+dx^2\right)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abcd+8b^2c^2\right)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{24a^2b^4}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)
*x*Sqrt[a + b*x^2])/(24*a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 3
5*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c -
a*d)*(2*b*c + 7*a*d)*x*(c + d*x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2])
 + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2
*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(8*b^(9/2))
```

Rubi [A] time = 0.614599, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x\left(c+dx^2\right)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}-\frac{dx\sqrt{a+bx^2}\left(c+dx^2\right)\left(-35a^2d^2+24abcd+8b^2c^2\right)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\left(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3\right)}{24a^2b^4}+\frac{x\left(c+dx^2\right)^3(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```


```
[Out] -(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)
*x*Sqrt[a + b*x^2])/(24*a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 3
5*a^2*d^2)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c -
a*d)*(2*b*c + 7*a*d)*x*(c + d*x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2])
 + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2
*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(8*b^(9/2))
```

Rubi in Sympy [A] time = 91.4008, size = 260, normalized size = 1.02

$$\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{9}{2}}}-\frac{x\left(c+dx^2\right)^3(ad-bc)}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x\left(c+dx^2\right)^2(ad-bc)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}\left(ac(7ad-4bc)+x^2\left(35a^2d^2-24abcd-8b^2c^2\right)\right)}{12a^2b^3}-\frac{dx\sqrt{a+bx^2}\left(105a^3d^3-226a^2bcd^2+80ab^2c^2d+32b^3c^3\right)}{24a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] d**2*(35*a**2*d**2 - 80*a*b*c*d + 48*b**2*c**2)*atanh(sqrt(b)*x/s
qrt(a + b*x**2))/(8*b**(9/2)) - x*(c + d*x**2)**3*(a*d - b*c)/(3*
a*b*(a + b*x**2)**(3/2)) - x*(c + d*x**2)**2*(a*d - b*c)*(7*a*d +
 2*b*c)/(3*a**2*b**2*sqrt(a + b*x**2)) + d**2*x*sqrt(a + b*x**2)*
(a*c*(7*a*d - 4*b*c) + x**2*(35*a**2*d**2 - 24*a*b*c*d - 8*b**2*c
**2))/(12*a**2*b**3) - d*x*sqrt(a + b*x**2)*(105*a**3*d**3 - 226*
a**2*b*c*d**2 + 80*a*b**2*c**2*d + 32*b**3*c**3)/(24*a**2*b**4)
```

Mathematica [A] time = 0.277545, size = 157, normalized size = 0.62

$$\frac{x\sqrt{a+bx^2}\left(\frac{16(bc-ad)^3(5ad+bc)}{a^2(a+bx^2)}+3d^3(16bc-11ad)+\frac{8(bc-ad)^4}{a(a+bx^2)^2}+6bd^4x^2\right)}{24b^4}+\frac{d^2\left(35a^2d^2-80abcd+48b^2c^2\right)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(3*d^3*(16*b*c - 11*a*d) + 6*b*d^4*x^2 + (8*(b
*c - a*d)^4)/(a*(a + b*x^2)^2) + (16*(b*c - a*d)^3*(b*c + 5*a*d))
/(a^2*(a + b*x^2))))/(24*b^4) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 3
5*a^2*d^2)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Maple [A] time = 0.024, size = 351, normalized size = 1.4

$$\frac{c^4x}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,c^4x}{3\,a^2}\frac{1}{\sqrt{bx^2+a}}+\frac{d^4x^7}{4\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{7\,ad^4x^5}{8\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$
$$-\frac{35\,a^2d^4x^3}{24\,b^3}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{35\,a^2d^4x}{8\,b^4}\frac{1}{\sqrt{bx^2+a}}+\frac{35\,a^2d^4}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{9}{2}}$$
$$+2\,\frac{cd^3x^5}{b\left(bx^2+a\right)^{3/2}}+\frac{10\,acd^3x^3}{3\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+10\,\frac{acd^3x}{b^3\sqrt{bx^2+a}}$$
$$-10\,\frac{acd^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{7/2}}-2\,\frac{c^2d^2x^3}{b\left(bx^2+a\right)^{3/2}}-6\,\frac{c^2d^2x}{b^2\sqrt{bx^2+a}}$$
$$+6\,\frac{c^2d^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{5/2}}-\frac{4\,c^3dx}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{4\,c^3dx}{3\,ab}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^4/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*c^4*x/a/(b*x^2+a)^(3/2)+2/3*c^4/a^2*x/(b*x^2+a)^(1/2)+1/4*d^4
*x^7/b/(b*x^2+a)^(3/2)-7/8*d^4*a/b^2*x^5/(b*x^2+a)^(3/2)-35/24*d^
4*a^2/b^3*x^3/(b*x^2+a)^(3/2)-35/8*d^4*a^2/b^4*x/(b*x^2+a)^(1/2)+
35/8*d^4*a^2/b^(9/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+2*c*d^3*x^5/b/
(b*x^2+a)^(3/2)+10/3*c*d^3*a/b^2*x^3/(b*x^2+a)^(3/2)+10*c*d^3*a/b
^3*x/(b*x^2+a)^(1/2)-10*c*d^3*a/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))-2*c^2*d^2*x^3/b/(b*x^2+a)^(3/2)-6*c^2*d^2/b^2*x/(b*x^2+a)^(1
/2)+6*c^2*d^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-4/3*c^3*d/b*x
/(b*x^2+a)^(3/2)+4/3*c^3*d/a/b*x/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.561206, size = 1, normalized size = 0.

$$\left[\frac{2\left(6\,a^2b^3d^4x^7+3\left(16\,a^2b^3cd^3-7\,a^3b^2d^4\right)x^5+4\left(4\,b^5c^4+8\,ab^4c^3d-48\,a^2b^3c^2d^2+80\,a^3b^2cd^3-35\,a^4bd^4\right)x^3+3\left(8\,ab^4c^4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(2*(6*a^2*b^3*d^4*x^7 + 3*(16*a^2*b^3*c*d^3 - 7*a^3*b^2*d^4
)*x^5 + 4*(4*b^5*c^4 + 8*a*b^4*c^3*d - 48*a^2*b^3*c^2*d^2 + 80*a^
3*b^2*c*d^3 - 35*a^4*b*d^4)*x^3 + 3*(8*a*b^4*c^4 - 48*a^3*b^2*c^2
*d^2 + 80*a^4*b*c*d^3 - 35*a^5*d^4)*x)*sqrt(b*x^2 + a)*sqrt(b) +
3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4
*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3
*c^2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*log(-2*sqrt(b*x^
2 + a)*b*x - (2*b*x^2 + a)*sqrt(b)))/((a^2*b^6*x^4 + 2*a^3*b^5*x^
2 + a^4*b^4)*sqrt(b)), 1/24*((6*a^2*b^3*d^4*x^7 + 3*(16*a^2*b^3*c
*d^3 - 7*a^3*b^2*d^4)*x^5 + 4*(4*b^5*c^4 + 8*a*b^4*c^3*d - 48*a^2
*b^3*c^2*d^2 + 80*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^3 + 3*(8*a*b^4*
c^4 - 48*a^3*b^2*c^2*d^2 + 80*a^4*b*c*d^3 - 35*a^5*d^4)*x)*sqrt(b
*x^2 + a)*sqrt(-b) + 3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*
a^6*d^4 + (48*a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4
)*x^4 + 2*(48*a^3*b^3*c^2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*
x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a^2*b^6*x^4 + 2*a^3*b^
5*x^2 + a^4*b^4)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^4}{\left(a + bx^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.234234, size = 320, normalized size = 1.25

$$\frac{\left(\left(3\left(\frac{2\,d^4x^2}{b} + \frac{16\,a^2b^6cd^3 - 7\,a^3b^5d^4}{a^2b^7}\right)x^2 + \frac{4\left(4\,b^8c^4 + 8\,ab^7c^3d - 48\,a^2b^6c^2d^2 + 80\,a^3b^5cd^3 - 35\,a^4b^4d^4\right)}{a^2b^7}\right)x^2 + \frac{3\left(8\,ab^7c^4 - 48\,a^3b^5c^2d^2 + 80\,a^4b^4cd^3 - 35\,a^5b}{a^2b^7}}{24\left(bx^2 + a\right)^{\frac{3}{2}}} - \frac{\left(48\,b^2c^2d^2 - 80\,abcd^3 + 35\,a^2d^4\right)\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^4/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b
^7))*x^2 + 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80
*a^3*b^5*c*d^3 - 35*a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4
- 48*a^3*b^5*c^2*d^2 + 80*a^4*b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^
7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^2*c^2*d^2 - 80*a*b*c*d^3 + 35
*a^2*d^4)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.90 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=172

$$\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] -(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(6*a^
2*b^3) + ((b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*S
qrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2
)^(3/2)) + (d^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(7/2))
```

Rubi [A] time = 0.368917, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{x\left(c+dx^2\right)(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}-\frac{dx\sqrt{a+bx^2}\left(-15a^2d^2+8abcd+4b^2c^2\right)}{6a^2b^3}+\frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}+\frac{x\left(c+dx^2\right)^2(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] -(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(6*a^
2*b^3) + ((b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*S
qrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2
)^(3/2)) + (d^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 50.9247, size = 162, normalized size = 0.94

$$-\frac{d^2\left(5ad-6bc\right)\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{7}{2}}}-\frac{x\left(c+dx^2\right)^2\left(ad-bc\right)}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x\left(c+dx^2\right)\left(ad-bc\right)\left(5ad+2bc\right)}{3a^2b^2\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\left(15a^2d^2-8abcd-4b^2c^2\right)}{6a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

```
[Out] -d**2*(5*a*d - 6*b*c)*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(7/
2)) - x*(c + d*x**2)**2*(a*d - b*c)/(3*a*b*(a + b*x**2)**(3/2)) -
 x*(c + d*x**2)*(a*d - b*c)*(5*a*d + 2*b*c)/(3*a**2*b**2*sqrt(a +
 b*x**2)) + d*x*sqrt(a + b*x**2)*(15*a**2*d**2 - 8*a*b*c*d - 4*b*
*2*c**2)/(6*a**2*b**3)
```

Mathematica [A] time = 0.156789, size = 125, normalized size = 0.73

$$\frac{x\left(3a^2d^3\left(a+bx^2\right)^2+2\left(a+bx^2\right)(bc-ad)^2(7ad+2bc)+2a(bc-ad)^3\right)}{6a^2b^3\left(a+bx^2\right)^{3/2}}+\frac{d^2(6bc-5ad)\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(2*a*(b*c - a*d)^3 + 2*(b*c - a*d)^2*(2*b*c + 7*a*d)*(a + b*x^
2) + 3*a^2*d^3*(a + b*x^2)^2))/(6*a^2*b^3*(a + b*x^2)^(3/2)) + (d
^2*(6*b*c - 5*a*d)*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]])/(2*b^(7/2)
)
```

Maple [A] time = 0.012, size = 228, normalized size = 1.3

$$\frac{c^3x}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,c^3x}{3\,a^2}\frac{1}{\sqrt{bx^2+a}}+\frac{d^3x^5}{2\,b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,ad^3x^3}{6\,b^2}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{5\,ad^3x}{2\,b^3}\frac{1}{\sqrt{bx^2+a}}-\frac{5\,ad^3}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{7}{2}}-\frac{cd^2x^3}{b}\left(bx^2+a\right)^{-\frac{3}{2}}-3\,\frac{cd^2x}{b^2\sqrt{bx^2+a}}+3\,\frac{cd^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{5/2}}-\frac{c^2dx}{b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{c^2dx}{ab}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^3/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*c^3*x/a/(b*x^2+a)^(3/2)+2/3*c^3/a^2*x/(b*x^2+a)^(1/2)+1/2*d^3
*x^5/b/(b*x^2+a)^(3/2)+5/6*d^3*a/b^2*x^3/(b*x^2+a)^(3/2)+5/2*d^3*
a/b^3*x/(b*x^2+a)^(1/2)-5/2*d^3*a/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^
```

```
(1/2))-c*d^2*x^3/b/(b*x^2+a)^(3/2)-3*c*d^2/b^2*x/(b*x^2+a)^(1/2)+
3*c*d^2/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-c^2*d/b*x/(b*x^2+a)
^(3/2)+c^2*d/a/b*x/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302988, size = 1, normalized size = 0.01

$$\left[\frac{2\left(3\,a^2b^2d^3x^5+2\left(2\,b^4c^3+3\,ab^3c^2d-12\,a^2b^2cd^2+10\,a^3bd^3\right)x^3+3\left(2\,ab^3c^3-6\,a^3bcd^2+5\,a^4d^3\right)x\right)\sqrt{bx^2+a}\sqrt{b}-3\left(6\,a^4b \cdots\right.}{12\left(a^2b^5x^4+2\,a^3b^4x^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(3*a^2*b^2*d^3*x^5 + 2*(2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a
^2*b^2*c*d^2 + 10*a^3*b*d^3)*x^3 + 3*(2*a*b^3*c^3 - 6*a^3*b*c*d^2
 + 5*a^4*d^3)*x)*sqrt(b*x^2 + a)*sqrt(b) - 3*(6*a^4*b*c*d^2 - 5*a
^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^4 + 2*(6*a^3*b^2*c*d
^2 - 5*a^4*b*d^3)*x^2)*log(2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*
sqrt(b)))/((a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3)*sqrt(b)), 1/6*
((3*a^2*b^2*d^3*x^5 + 2*(2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c
*d^2 + 10*a^3*b*d^3)*x^3 + 3*(2*a*b^3*c^3 - 6*a^3*b*c*d^2 + 5*a^4
*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 3*(6*a^4*b*c*d^2 - 5*a^5*d^3
+ (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*
a^4*b*d^3)*x^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a^2*b^5*x^4
 + 2*a^3*b^4*x^2 + a^4*b^3)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.233084, size = 213, normalized size = 1.24

$$\frac{\left(\left(\frac{3\,d^3x^2}{b}+\frac{2\left(2\,b^6c^3+3\,ab^5c^2d-12\,a^2b^4cd^2+10\,a^3b^3d^3\right)}{a^2b^5}\right)x^2+\frac{3\left(2\,ab^5c^3-6\,a^3b^3cd^2+5\,a^4b^2d^3\right)}{a^2b^5}\right)x}{6\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{\left(6\,bcd^2-5\,ad^3\right)\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d
^2 + 10*a^3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*
c*d^2 + 5*a^4*b^2*d^3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*
c*d^2 - 5*a*d^3)*ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.91 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}} \, dx$

Optimal. Leaf size=105

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b
*c - a*d)*x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi [A] time = 0.133348, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}+\frac{x\left(c+dx^2\right)(bc-ad)}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b
*c - a*d)*x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rubi in Sympy [A] time = 23.8377, size = 94, normalized size = 0.9

$$\frac{d^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{\frac{5}{2}}}-\frac{x\left(c+dx^2\right)(ad-bc)}{3ab\left(a+bx^2\right)^{\frac{3}{2}}}-\frac{x(ad-bc)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] d**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/b**(5/2) - x*(c + d*x**2)*
(a*d - b*c)/(3*a*b*(a + b*x**2)**(3/2)) - x*(a*d - b*c)*(3*a*d +
2*b*c)/(3*a**2*b**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.219513, size = 97, normalized size = 0.92

$$\frac{x\left(2\left(a+bx^2\right)\left(-2a^2d^2+abcd+b^2c^2\right)+a(bc-ad)^2\right)}{3a^2b^2\left(a+bx^2\right)^{3/2}}+\frac{d^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(a*(b*c - a*d)^2 + 2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(a + b*x^
2)))/(3*a^2*b^2*(a + b*x^2)^(3/2)) + (d^2*Log[b*x + Sqrt[b]*Sqrt[
a + b*x^2]])/b^(5/2)
```

Maple [A] time = 0.01, size = 136, normalized size = 1.3

$$\frac{c^2x}{3\,a}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,c^2x}{3\,a^2}\frac{1}{\sqrt{bx^2+a}}-\frac{d^2x^3}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}-\frac{d^2x}{b^2}\frac{1}{\sqrt{bx^2+a}}+d^2\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}-\frac{2\,cdx}{3\,b}\left(bx^2+a\right)^{-\frac{3}{2}}+\frac{2\,cdx}{3\,ab}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^2/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*c^2*x/a/(b*x^2+a)^(3/2)+2/3*c^2/a^2*x/(b*x^2+a)^(1/2)-1/3*d^2
*x^3/b/(b*x^2+a)^(3/2)-d^2/b^2*x/(b*x^2+a)^(1/2)+d^2/b^(5/2)*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))-2/3*c*d/b*x/(b*x^2+a)^(3/2)+2/3*c*d/a/b
*x/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236081, size = 1, normalized size = 0.01

$$\left[\frac{2\left(2\left(b^3c^2 + ab^2cd - 2\,a^2bd^2\right)x^3 + 3\left(ab^2c^2 - a^3d^2\right)x\right)\sqrt{bx^2+a}\sqrt{b} + 3\left(a^2b^2d^2x^4 + 2\,a^3bd^2x^2 + a^4d^2\right)\log\left(-2\,\sqrt{bx^2+a}\,abx\right.}{6\left(a^2b^4x^4 + 2\,a^3b^3x^2 + a^4b^2\right)\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*(b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3 + 3*(a*b^2*c^2
 - a^3*d^2)*x)*sqrt(b*x^2 + a)*sqrt(b) + 3*(a^2*b^2*d^2*x^4 + 2*a
^3*b*d^2*x^2 + a^4*d^2)*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a
)*sqrt(b)))/((a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sqrt(b)), 1/
3*((2*(b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3 + 3*(a*b^2*c^2 - a^
3*d^2)*x)*sqrt(b*x^2 + a)*sqrt(-b) + 3*(a^2*b^2*d^2*x^4 + 2*a^3*b
*d^2*x^2 + a^4*d^2)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/((a^2*b^4
*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sqrt(-b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^2\right)^2}{\left(a + bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.232037, size = 139, normalized size = 1.32

$$\frac{x\left(\frac{2\left(b^4c^2 + ab^3cd - 2\,a^2b^2d^2\right)x^2}{a^2b^3} + \frac{3\left(ab^3c^2 - a^3bd^2\right)}{a^2b^3}\right)}{3\left(bx^2+a\right)^{\frac{3}{2}}} - \frac{d^2\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*
(a*b^3*c^2 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*ln(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.92 $\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}}$$

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2
)^(3/2))
```

Rubi [A] time = 0.036464, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2
)^(3/2))
```

Rubi in Sympy [A] time = 6.51894, size = 41, normalized size = 0.87

$$\frac{x(c+dx^2)}{3a(a+bx^2)^{\frac{3}{2}}}+\frac{2cx}{3a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] x*(c + d*x**2)/(3*a*(a + b*x**2)**(3/2)) + 2*c*x/(3*a**2*sqrt(a +
 b*x**2))
```

Mathematica [A] time = 0.0429686, size = 37, normalized size = 0.79

$$\frac{x(3ac+adx^2+2bcx^2)}{3a^2(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 34, normalized size = 0.7

$$\frac{x\left(adx^2+2\,cx^2b+3\,ac\right)}{3\,a^2}\left(bx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(a*d*x^2+2*b*c*x^2+3*a*c)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 1.34891, size = 92, normalized size = 1.96

$$\frac{2\,cx}{3\,\sqrt{bx^2+a}a^2}+\frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{dx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{dx}{3\,\sqrt{bx^2+a}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1
/3*d*x/((b*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

Fricas [A] time = 0.213794, size = 73, normalized size = 1.55

$$\frac{\left((2\,bc+ad)x^3+3\,acx\right)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 +
2*a^3*b*x^2 + a^4)
```

Sympy [A] time = 36.3887, size = 144, normalized size = 3.06

$$c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(
1 + b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(
5/2)*b*x**2*sqrt(1 + b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*
x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.225743, size = 54, normalized size = 1.15

$$\frac{x\left(\frac{3\,c}{a}+\frac{\left(2\,b^2c+abd\right)x^2}{a^2b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.93 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\,(a+bx^2)^{3/2}}$$

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rubi [A] time = 0.0195958, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rubi in Sympy [A] time = 2.09013, size = 32, normalized size = 0.82

$$\frac{x}{3a\,(a+bx^2)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] x/(3*a*(a + b*x**2)**(3/2)) + 2*x/(3*a**2*sqrt(a + b*x**2))
```

Mathematica [A] time = 0.0162062, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2bx^2\right)}{3a^2\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 26, normalized size = 0.7

$$\frac{x\left(2\,bx^{2}+3\,a\right)}{3\,a^{2}}\left(bx^{2}+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 1.34542, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{bx^{2}+a}a^{2}}+\frac{x}{3\,(bx^{2}+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 0.212356, size = 63, normalized size = 1.62

$$\frac{\left(2\,bx^{3}+3\,ax\right)\sqrt{bx^{2}+a}}{3\,(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2
+ a^4)
```

Sympy [A] time = 2.77091, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2
)*b*x**2*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.226225, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

3.94 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx$

Optimal. Leaf size=122

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)
/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rubi [A] time = 0.345895, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}(bc-ad)^2}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)
/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rubi in Sympy [A] time = 55.144, size = 107, normalized size = 0.88

$$\frac{d^2\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\,(ad-bc)^{\frac{5}{2}}}-\frac{bx}{3a\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)}-\frac{bx\,(5ad-2bc)}{3a^2\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] d**2*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(sqrt(c)*
(a*d - b*c)**(5/2)) - b*x/(3*a*(a + b*x**2)**(3/2)*(a*d - b*c)) -
 b*x*(5*a*d - 2*b*c)/(3*a**2*sqrt(a + b*x**2)*(a*d - b*c)**2)
```

Mathematica [A] time = 0.360047, size = 112, normalized size = 0.92

$$\frac{bx\left(-6a^2d+ab\left(3c-5dx^2\right)+2b^2cx^2\right)}{3a^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x*(-6*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 5*d*x^2)))/(3*a^2*(b*c
- a*d)^2*(a + b*x^2)^(3/2)) + (d^2*ArcTan[(Sqrt[-(b*c) + a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2))
```

Maple [B] time = 0.027, size = 1070, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x)
```

```
[Out] 1/6/(-c*d)^(1/2)/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(
1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(3/2)*x-1/3*b/(a*d-b*c)/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(
-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/2/(-c*d)^
(1/2)*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*
(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/2*d/(a*d-b*c)^2/a/((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2)*x*b-1/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)
*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-
b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-1/6/(-c*d)^(1/
2)/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-1/6*b/(a*d-b*c)/a/((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*
x-1/3*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*
(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/2/(-c*d)^(1/2)*d^2/(a*d
-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)-1/2*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2
*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b+1
/2/(-c*d)^(1/2)*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*
((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.546142, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(2\,b^3c - 5\,ab^2d\right)x^3 + 3\left(ab^2c - 2\,a^2bd\right)x\right)\sqrt{bc^2 - acd}\sqrt{bx^2 + a} + 3\left(a^2b^2d^2x^4 + 2\,a^3bd^2x^2 + a^4d^2\right)\log\left(\frac{((8\,b^2c^2 - 8\,abcd + a^2c}{}\right.}{12\left(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + (a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)x^4 + 2\,(a^3b^3c^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*((2*b^3*c - 5*a*b^2*d)*x^3 + 3*(a*b^2*c - 2*a^2*b*d)*x)*
sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a) + 3*(a^2*b^2*d^2*x^4 + 2*a^3*
b*d^2*x^2 + a^4*d^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4
+ a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) +
4*((2*b^2*c^3 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2
*d)*x)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^4*b^2*c
^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b
^2*d^2)*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x^2)*sq
rt(b*c^2 - a*c*d)), 1/6*(2*((2*b^3*c - 5*a*b^2*d)*x^3 + 3*(a*b^2*
c - 2*a^2*b*d)*x)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a) + 3*(a^2*b
^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*arctan(1/2*sqrt(-b*c^2 +
a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c*d)*sqrt(b*x^2 + a)
*x)))/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^
3*b^3*c*d + a^4*b^2*d^2)*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a
^5*b*d^2)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bx^2\right)^{\frac{5}{2}}\left(c + dx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.229209, size = 432, normalized size = 3.54

$$-\frac{\sqrt{b}d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c^2+abcd}}+\frac{\left(\frac{\left(2\,b^6c^3-9\,ab^5c^2d+12\,a^2b^4cd^2-5\,a^3b^3d^3\right)x^2}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}+\frac{3\left(ab^5c^3-4\,a^2b^4c^2d+5\,a^3b^3cd^2-2\,a^4b^2d^3\right)}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}\right)x}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*
c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*sqrt(-b^2*c^2 + a*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d +
12*a^2*b^4*c*d^2 - 5*a^3*b^3*d^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^
3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) + 3*(a*b^5
*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*a^4*b^2*d^3)/(a^2*b^
5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a
^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

3.95 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=202

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3ad+2bc)}{6ac\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (
b*(4*b^2*c^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*
Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c +
d*x^2)) + (d^2*(6*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]
*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(7/2))
```

Rubi [A] time = 0.619936, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{bx\left(-3a^2d^2-16abcd+4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3}+\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(bc-ad)}+\frac{bx(3ad+2bc)}{6ac\left(a+bx^2\right)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```


```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (
b*(4*b^2*c^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*
Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c +
d*x^2)) + (d^2*(6*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]
*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(7/2))
```

Rubi in Sympy [A] time = 116.464, size = 178, normalized size = 0.88

$$\frac{dx}{2c\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)(ad-bc)}+\frac{d^2\left(ad-6bc\right)\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{\frac{3}{2}}\left(ad-bc\right)^{\frac{7}{2}}}+\frac{bx\left(3ad+2bc\right)}{6ac\left(a+bx^2\right)^{\frac{3}{2}}\left(ad-bc\right)^2}+\frac{bx\left(3a^2d^2+16abcd-4b^2c^2\right)}{6a^2c\sqrt{a+bx^2}\left(ad-bc\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] d*x/(2*c*(a + b*x**2)**(3/2)*(c + d*x**2)*(a*d - b*c)) + d**2*(a*
d - 6*b*c)*atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2)))/(2*
c**(3/2)*(a*d - b*c)**(7/2)) + b*x*(3*a*d + 2*b*c)/(6*a*c*(a + b*
x**2)**(3/2)*(a*d - b*c)**2) + b*x*(3*a**2*d**2 + 16*a*b*c*d - 4*
b**2*c**2)/(6*a**2*c*sqrt(a + b*x**2)*(a*d - b*c)**3)
```

Mathematica [A] time = 1.02774, size = 170, normalized size = 0.84

$$\frac{1}{6}\left(x\sqrt{a+bx^2}\left(\frac{4b^2(4ad-bc)}{a^2\left(a+bx^2\right)(ad-bc)^3}+\frac{2b^2}{a\left(a+bx^2\right)^2(bc-ad)^2}-\frac{3d^3}{c\left(c+dx^2\right)(bc-ad)^3}\right)+\frac{3d^2(ad-6bc)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}(ad-bc)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((2*b^2)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (4*
b^2*(-(b*c) + 4*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (3*d^3
)/(c*(b*c - a*d)^3*(c + d*x^2))) + (3*d^2*(-6*b*c + a*d)*ArcTan[(
Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(c^(3/2)*(-(b*c
) + a*d)^(7/2)))/6
```

Maple [B] time = 0.036, size = 2371, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x)
```

```
[Out] -1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-5/4/c*d*b*(-c*d)
^(1/2)/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*
d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1
/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))-1/4/c*d/(a*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b
-1/4/c*d/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x*b+5/4/c*d*b*(-c*d)^(1/2)
/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1/
```

```
2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)
^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x
+(-c*d)^(1/2)/d))-1/4/(-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((x+(-c*d)^(
1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1
/2)-5/12/c*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b
*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+5/12*b^2/(a
*d-b*c)^2/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+5/6*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)*x+1/12/(-c*d)^(1/2)/c/(a*d-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*
b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4/(-c*d)
^(1/2)/c*d^2/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)+5/12/c*b*(-c*d)^(1/2)/(a
*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(3/2)+5/12*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)
/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*
x+5/6*b^2/(a*d-b*c)^2/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2
)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/(-c*d)^(1/2)/c*d^
2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*b*(-c*d)^(1
/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(
x+(-c*d)^(1/2)/d))-5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)+5/4*d*b^2/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(
1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/4/c*b/(a*d-b*c)/
a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(
a*d-b*c)/d)^(3/2)*x+1/2/c*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b
-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-1/4/(
-c*d)^(1/2)/c*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)
/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+5/4/c*d*b*(-c*d)^(1/2)/(a*d-b*
c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(1/2)+5/4*d*b^2/(a*d-b*c)^3/a/((x-(-c*d)^(1/2)/d)^
2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+1/
4/c*b/(a*d-b*c)/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x+1/2/c*b/(a*d-b*c)/a^2/((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2)*x+1/4/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)
^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/4
/c/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

Fricas [A] time = 1.22512, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*((4*b^4*c^2*d - 16*a*b^3*c*d^2 - 3*a^2*b^2*d^3)*x^5 + 2*
(2*b^4*c^3 - 5*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 +
 3*(2*a*b^3*c^3 - 6*a^2*b^2*c^2*d - a^4*d^3)*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a) + 3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*
c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3
- 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^
4)*x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3
 - 3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b
*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^4*b^3*c^5 - 3*a^5*b^
2*c^4*d + 3*a^6*b*c^3*d^2 - a^7*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*
b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2*c*d^4)*x^6 + (a^2*b^5*c
^5 - a^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5*a^5*b^2*c^2*d^3 - 2*a^
6*b*c*d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d + 3*a^5*b^2*c^3
*d^2 + a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)), 1/12
*(2*((4*b^4*c^2*d - 16*a*b^3*c*d^2 - 3*a^2*b^2*d^3)*x^5 + 2*(2*b^
4*c^3 - 5*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 + 3*(2
*a*b^3*c^3 - 6*a^2*b^2*c^2*d - a^4*d^3)*x)*sqrt(-b*c^2 + a*c*d)*s
qrt(b*x^2 + a) + 3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^
3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*
a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x
^2)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b
*c^2 - a*c*d)*sqrt(b*x^2 + a)*x)))/((a^4*b^3*c^5 - 3*a^5*b^2*c^4*
d + 3*a^6*b*c^3*d^2 - a^7*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*b^4*c^
3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2*c*d^4)*x^6 + (a^2*b^5*c^5 - a
^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5*a^5*b^2*c^2*d^3 - 2*a^6*b*c*
d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d + 3*a^5*b^2*c^3*d^2 +
 a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 4.94363, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.96 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx$

Optimal. Leaf size=313

$$\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}+\frac{dx\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{24a^2c^2\,(c+dx^2)\,(bc-ad)^4}-\frac{dx}{4c\,(a+bx^2)^{3/2}\,(c+dx^2)^2\,(bc-ad)}+\frac{bx(3ad+4bc)}{12ac\,(a+bx^2)^{3/2}\,(c+dx^2)\,(bc-ad)^2}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*
b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^
2)) + (b*(8*b^2*c^2 - 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c -
 a*d)^3*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*
c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sqrt[a + b*x^2])/(24*a^2*c^
2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2 - 16*a*b*c*d + 3*
a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rubi [A] time = 1.08367, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{bx\left(-3a^2d^2-40abcd+8b^2c^2\right)}{12a^2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)^3}+\frac{d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tanh^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}+\frac{dx\sqrt{a+bx^2}\left(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3\right)}{24a^2c^2\,(c+dx^2)\,(bc-ad)^4}-\frac{dx}{4c\,(a+bx^2)^{3/2}\,(c+dx^2)^2\,(bc-ad)}+\frac{bx(3ad+4bc)}{12ac\,(a+bx^2)^{3/2}\,(c+dx^2)\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*
b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^
2)) + (b*(8*b^2*c^2 - 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c -
 a*d)^3*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*
c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*Sqrt[a + b*x^2])/(24*a^2*c^
2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2 - 16*a*b*c*d + 3*
a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.22183, size = 221, normalized size = 0.71

$$\frac{1}{24}\left(x\sqrt{a+bx^2}\left(\frac{8b^3(2bc-11ad)}{a^2\left(a+bx^2\right)(bc-ad)^4}-\frac{8b^3}{a\left(a+bx^2\right)^2(ad-bc)^3}+\frac{3d^3(3ad-14bc)}{c^2\left(c+dx^2\right)(bc-ad)^4}-\frac{6d^3}{c\left(c+dx^2\right)^2(bc-ad)^3}\right)+\frac{3d^2\left(3a^2d^2-16abcd+48b^2c^2\right)\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{5/2}(ad-bc)^{9/2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((-8*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)^2) +
 (8*b^3*(2*b*c - 11*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^2)) - (6*d^
3)/(c*(b*c - a*d)^3*(c + d*x^2)^2) + (3*d^3*(-14*b*c + 3*a*d))/(c
^2*(b*c - a*d)^4*(c + d*x^2))) + (3*d^2*(48*b^2*c^2 - 16*a*b*c*d
+ 3*a^2*d^2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^
2])])/(c^(5/2)*(-(b*c) + a*d)^(9/2)))/24
```

Maple [B] time = 0.046, size = 4495, normalized size = 14.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x)
```

```
[Out] -7/16/c*b/(a*d-b*c)^2/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+
2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/16/(-c
*d)^(1/2)/c/(a*d-b*c)/(x-(-c*d)^(1/2)/d)^2/((x-(-c*d)^(1/2)/d)^2*
b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-3/16/(
-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*
```

c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*
((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*
d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-35/16*d*b^3/(a*d-b*c)^4/a/((
x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-
b*c)/d)^(1/2)*x-35/16/(-c*d)^(1/2)*d^2*b^2/(a*d-b*c)^4/((a*d-b*c)
/d)^(1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)
+2*((a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d
*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+5/48/
(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+5/16/(-c*d)^(1/2)/
c*d^2*b/(a*d-b*c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-15/16/c^2*d*b*(-c*d)^(1/2)/(a
*d-b*c)^3/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+(a*d-b*c)/d)^(1/2)+3/16/c^2*b/(a*d-b*c)/a/((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*
x+3/8/c^2*b/(a*d-b*c)/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2
)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/8/c*b^2/(a*d-b*c)^2
/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+
(a*d-b*c)/d)^(3/2)*x-3/4/c*b^2/(a*d-b*c)^2/a^2/((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+
3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((a*d-b*c)/d)^(1/2)*ln((2*(
a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^
(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/
d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+3/16/c^2*b/(a*d-b*c)/a
/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(3/2)*x+3/8/c^2*b/(a*d-b*c)/a^2/((x-(-c*d)^(1/2)/d)^2*
b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-3/8/
c*b^2/(a*d-b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x
-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-3/4/c*b^2/(a*d-b*c)^2/a^2/(
(x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d
-b*c)/d)^(1/2)*x+15/16/c^2*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((x-(-c*d
)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)
^(1/2)-35/16*d*b^3/(a*d-b*c)^4/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*
d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(
1/2)*d^2*b^2/(a*d-b*c)^4/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-2*
b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(-c
*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2))/(x+(-c*d)^(1/2)/d))-5/48/(-c*d)^(1/2)/c*d*b/(a*d-b*c)^2
/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a
*d-b*c)/d)^(3/2)-5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((x+(-c*d)
^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^
(1/2)+5/16/(-c*d)^(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*l
n((2*(a*d-b*c)/d-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*
c)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x+(-c*d)^(1/2)/d))-3/16/c^2*d/(a*d-
b*c)^2/a/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(1/2)*x*b-3/16/c^2*d/(a*d-b*c)^2/a/((x+(-c*d)^
(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(
1/2)*x*b+5/8/c*d*b^2/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-
c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+15/16/c^2*d*
b*(-c*d)^(1/2)/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d-
2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x+(
-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+5/8/c*d*b^2/(a*d-b*c)^3/a/((x-(-c
*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/
d)^(1/2)*x-15/16/c^2*d*b*(-c*d)^(1/2)/(a*d-b*c)^3/((a*d-b*c)/d)^(

```
1/2)*ln((2*(a*d-b*c)/d+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((
a*d-b*c)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))-5/16/(-c*d
)^(1/2)/c*d^2*b/(a*d-b*c)^3/((a*d-b*c)/d)^(1/2)*ln((2*(a*d-b*c)/d
+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*((a*d-b*c)/d)^(1/2)*((x-
(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*
c)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+35/48/(-c*d)^(1/2)*d*b^2/(a*d-b*
c)^3/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+(a*d-b*c)/d)^(3/2)-35/48*b^3/(a*d-b*c)^3/a/((x-(-c*d)^(1/2)/d)^
2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-35
/24*b^3/(a*d-b*c)^3/a^2/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/
d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x+35/16/(-c*d)^(1/2)*d^2*
b^2/(a*d-b*c)^4/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-1/16/(-c*d)^(1/2)/c/(a*d-b*c)/(x+
(-c*d)^(1/2)/d)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-7/16/c*b/(a*d-b*c)^2/(x+(-c*d)^
(1/2)/d)/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/
2)/d)+(a*d-b*c)/d)^(3/2)-35/48/(-c*d)^(1/2)*d*b^2/(a*d-b*c)^3/((x
+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(3/2)-35/48*b^3/(a*d-b*c)^3/a/((x+(-c*d)^(1/2)/d)^2*b-2*b*
(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)*x-35/24*b^3/
(a*d-b*c)^3/a^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)*x-35/16/(-c*d)^(1/2)*d^2*b^2/(a*d
-b*c)^4/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(1/2)+3/16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((x
-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b
*c)/d)^(1/2)-1/16/(-c*d)^(1/2)/c^2/(a*d-b*c)*d/((x+(-c*d)^(1/2)/d
)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)-3/
16/(-c*d)^(1/2)/c^2*d^2/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(1/2)-5/16/c^2*b*(-
c*d)^(1/2)/(a*d-b*c)^2/((x+(-c*d)^(1/2)/d)^2*b-2*b*(-c*d)^(1/2)/d
*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+1/16/(-c*d)^(1/2)/c^2/(a*d
-b*c)*d/((x-(-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+(a*d-b*c)/d)^(3/2)+5/16/c^2*b*(-c*d)^(1/2)/(a*d-b*c)^2/((x-(
-c*d)^(1/2)/d)^2*b+2*b*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c
)/d)^(3/2)+3/16/c^2/(a*d-b*c)/(x+(-c*d)^(1/2)/d)/((x+(-c*d)^(1/2)
/d)^2*b-2*b*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)+
3/16/c^2/(a*d-b*c)/(x-(-c*d)^(1/2)/d)/((x-(-c*d)^(1/2)/d)^2*b+2*b
*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+(a*d-b*c)/d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

Fricas [A] time = 3.90419, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3),x, algorithm="fricas")
```

```
[Out] [1/96*(4*((16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 - 42*a^2*b^3*c*d^4 +
 9*a^3*b^2*d^5)*x^7 + (32*b^5*c^4*d - 152*a*b^4*c^3*d^2 - 144*a^2
*b^3*c^2*d^3 - 69*a^3*b^2*c*d^4 + 18*a^4*b*d^5)*x^5 + (16*b^5*c^5
 - 40*a*b^4*c^4*d - 192*a^2*b^3*c^3*d^2 - 96*a^3*b^2*c^2*d^3 - 12
*a^4*b*c*d^4 + 9*a^5*d^5)*x^3 + 3*(8*a*b^4*c^5 - 32*a^2*b^3*c^4*d
 - 16*a^4*b*c^2*d^3 + 5*a^5*c*d^4)*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*
x^2 + a) + 3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d
^4 + (48*a^2*b^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8
+ 2*(48*a^2*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 +
 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*d^3 - 1
3*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^
3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*
x^2)*log((((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*c^2 - a*c*d) + 4*((2*b^2*c^3 -
3*a*b*c^2*d + a^2*c*d^2)*x^3 + (a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^
2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a^4*b^4*c^8 - 4*a^5*b^3*c
^7*d + 6*a^6*b^2*c^6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b
^6*c^6*d^2 - 4*a^3*b^5*c^5*d^3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^
3*d^5 + a^6*b^2*c^2*d^6)*x^8 + 2*(a^2*b^6*c^7*d - 3*a^3*b^5*c^6*d
^2 + 2*a^4*b^4*c^5*d^3 + 2*a^5*b^3*c^4*d^4 - 3*a^6*b^2*c^3*d^5 +
a^7*b*c^2*d^6)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4*c^6*d^2 + 16*a^5*b^
3*c^5*d^3 - 9*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(a^3*b^5*c^8
 - 3*a^4*b^4*c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*a^
7*b*c^4*d^4 + a^8*c^3*d^5)*x^2)*sqrt(b*c^2 - a*c*d)), 1/48*(2*((1
6*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 - 42*a^2*b^3*c*d^4 + 9*a^3*b^2*d
^5)*x^7 + (32*b^5*c^4*d - 152*a*b^4*c^3*d^2 - 144*a^2*b^3*c^2*d^3
 - 69*a^3*b^2*c*d^4 + 18*a^4*b*d^5)*x^5 + (16*b^5*c^5 - 40*a*b^4*
c^4*d - 192*a^2*b^3*c^3*d^2 - 96*a^3*b^2*c^2*d^3 - 12*a^4*b*c*d^4
 + 9*a^5*d^5)*x^3 + 3*(8*a*b^4*c^5 - 32*a^2*b^3*c^4*d - 16*a^4*b*
c^2*d^3 + 5*a^5*c*d^4)*x)*sqrt(-b*c^2 + a*c*d)*sqrt(b*x^2 + a) +
3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^
2*b^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2
*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^
6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c
^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 +
 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*x^2)*arctan
(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)/((b*c^2 - a*c
*d)*sqrt(b*x^2 + a)*x)))/((a^4*b^4*c^8 - 4*a^5*b^3*c^7*d + 6*a^6*
b^2*c^6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b^6*c^6*d^2 -
4*a^3*b^5*c^5*d^3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^3*d^5 + a^6*b
^2*c^2*d^6)*x^8 + 2*(a^2*b^6*c^7*d - 3*a^3*b^5*c^6*d^2 + 2*a^4*b^
4*c^5*d^3 + 2*a^5*b^3*c^4*d^4 - 3*a^6*b^2*c^3*d^5 + a^7*b*c^2*d^6
)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4*c^6*d^2 + 16*a^5*b^3*c^5*d^3 - 9
*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(a^3*b^5*c^8 - 3*a^4*b^4*
c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*a^7*b*c^4*d^4 +
```

```
a^8*c^3*d^5)*x^2)*sqrt(-b*c^2 + a*c*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [A] time = 1.16482, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3),x, algorithm="giac")

[Out] sage0*x
```

3.97 $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx$

Optimal. Leaf size=224

$$\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}(bc-ad)}+\frac{8a^2x\left(a+bx^2\right)(9bc-8ad)}{315c^4\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{2ax\left(a+bx^2\right)^2(9bc-8ad)}{105c^3\left(c+dx^2\right)^{5/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^3(9bc-8ad)}{63c^2\left(c+dx^2\right)^{7/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^4)/(9*c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*
c - 8*a*d)*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)
) + (2*a*(9*b*c - 8*a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c
 + d*x^2)^(5/2)) + (8*a^2*(9*b*c - 8*a*d)*x*(a + b*x^2))/(315*c^4
*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*a^3*(9*b*c - 8*a*d)*x)/(315
*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.279336, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}(bc-ad)}+\frac{8a^2x\left(a+bx^2\right)(9bc-8ad)}{315c^4\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{2ax\left(a+bx^2\right)^2(9bc-8ad)}{105c^3\left(c+dx^2\right)^{5/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^3(9bc-8ad)}{63c^2\left(c+dx^2\right)^{7/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] -(d*x*(a + b*x^2)^4)/(9*c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*
c - 8*a*d)*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)
) + (2*a*(9*b*c - 8*a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c
 + d*x^2)^(5/2)) + (8*a^2*(9*b*c - 8*a*d)*x*(a + b*x^2))/(315*c^4
*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*a^3*(9*b*c - 8*a*d)*x)/(315
*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 41.4738, size = 204, normalized size = 0.91

$$\frac{16a^3x\,(8ad-9bc)}{315c^5\sqrt{c+dx^2}\,(ad-bc)}+\frac{8a^2x\left(a+bx^2\right)(8ad-9bc)}{315c^4\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{2ax\left(a+bx^2\right)^2(8ad-9bc)}{105c^3\left(c+dx^2\right)^{\frac{5}{2}}(ad-bc)}+\frac{dx\left(a+bx^2\right)^4}{9c\left(c+dx^2\right)^{\frac{9}{2}}(ad-bc)}+\frac{x\left(a+bx^2\right)^3(8ad-9bc)}{63c^2\left(c+dx^2\right)^{\frac{7}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] 16*a**3*x*(8*a*d - 9*b*c)/(315*c**5*sqrt(c + d*x**2)*(a*d - b*c))
 + 8*a**2*x*(a + b*x**2)*(8*a*d - 9*b*c)/(315*c**4*(c + d*x**2)**
(3/2)*(a*d - b*c)) + 2*a*x*(a + b*x**2)**2*(8*a*d - 9*b*c)/(105*c
**3*(c + d*x**2)**(5/2)*(a*d - b*c)) + d*x*(a + b*x**2)**4/(9*c*(
c + d*x**2)**(9/2)*(a*d - b*c)) + x*(a + b*x**2)**3*(8*a*d - 9*b*
c)/(63*c**2*(c + d*x**2)**(7/2)*(a*d - b*c))
```

Mathematica [A] time = 0.157624, size = 163, normalized size = 0.73

$$\frac{a^3\left(315c^4x+840c^3dx^3+1008c^2d^2x^5+576cd^3x^7+128d^4x^9\right)+3a^2bcx^3\left(105c^3+126c^2dx^2+72cd^2x^4+16d^3x^6\right)+3ab^2c^2x^5}{315c^5\left(c+dx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d
*x^2 + 8*d^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c
*d^2*x^4 + 16*d^3*x^6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^
2*d^2*x^5 + 576*c*d^3*x^7 + 128*d^4*x^9))/(315*c^5*(c + d*x^2)^(9
/2))
```

Maple [A] time = 0.011, size = 190, normalized size = 0.9

$$\frac{x\left(128\,a^3d^4x^8+48\,a^2bcd^3x^8+24\,ab^2c^2d^2x^8+10\,b^3c^3dx^8+576\,a^3cd^3x^6+216\,a^2bc^2d^2x^6+108\,ab^2c^3dx^6+45\,b^3c^4x^6+1008\right.}{315\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3/(d*x^2+c)^(11/2),x)
```

```
[Out] 1/315*x*(128*a^3*d^4*x^8+48*a^2*b*c*d^3*x^8+24*a*b^2*c^2*d^2*x^8+
10*b^3*c^3*d*x^8+576*a^3*c*d^3*x^6+216*a^2*b*c^2*d^2*x^6+108*a*b^
2*c^3*d*x^6+45*b^3*c^4*x^6+1008*a^3*c^2*d^2*x^4+378*a^2*b*c^3*d*x
^4+189*a*b^2*c^4*x^4+840*a^3*c^3*d*x^2+315*a^2*b*c^4*x^2+315*a^3*
c^4)/(d*x^2+c)^(9/2)/c^5
```

Maxima [A] time = 1.36496, size = 628, normalized size = 2.8

$$-\frac{b^3x^5}{4\,(dx^2+c)^{\frac{9}{2}}d}-\frac{5\,b^3cx^3}{24\,(dx^2+c)^{\frac{9}{2}}d^2}-\frac{ab^2x^3}{2\,(dx^2+c)^{\frac{9}{2}}d}+\frac{128\,a^3x}{315\,\sqrt{dx^2+c}c^5}+\frac{64\,a^3x}{315\,(dx^2+c)^{\frac{3}{2}}c^4}+\frac{16\,a^3x}{105\,(dx^2+c)^{\frac{5}{2}}c^3}+\frac{8\,a^3x}{63\,(dx^2+c)^{\frac{7}{2}}c^2}+\frac{a^3x}{9\,(dx^2+c)^{\frac{9}{2}}c}+\frac{b^3x}{84\,(dx^2+c)^{\frac{5}{2}}d^3}+\frac{2\,b^3x}{63\,\sqrt{dx^2+c}c^2d^3}+\frac{b^3x}{63\,(dx^2+c)^{\frac{3}{2}}cd^3}+\frac{5\,b^3cx}{504\,(dx^2+c)^{\frac{7}{2}}d^3}-\frac{5\,b^3c^2x}{72\,(dx^2+c)^{\frac{9}{2}}d^3}+\frac{ab^2x}{42\,(dx^2+c)^{\frac{7}{2}}d^2}+\frac{8\,ab^2x}{105\,\sqrt{dx^2+c}c^3d^2}+\frac{4\,ab^2x}{105\,(dx^2+c)^{\frac{3}{2}}c^2d^2}+\frac{ab^2x}{35\,(dx^2+c)^{\frac{5}{2}}cd^2}-\frac{ab^2cx}{6\,(dx^2+c)^{\frac{9}{2}}d^2}-\frac{a^2bx}{3\,(dx^2+c)^{\frac{9}{2}}d}+\frac{16\,a^2bx}{105\,\sqrt{dx^2+c}c^4d}+\frac{8\,a^2bx}{105\,(dx^2+c)^{\frac{3}{2}}c^3d}+\frac{2\,a^2bx}{35\,(dx^2+c)^{\frac{5}{2}}c^2d}+\frac{a^2bx}{21\,(dx^2+c)^{\frac{7}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^
(9/2)*d^2) - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/
(sqrt(d*x^2 + c)*c^5) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16
/105*a^3*x/((d*x^2 + c)^(5/2)*c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2
)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2)*c) + 1/84*b^3*x/((d*x^2 + c
)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^2*d^3) + 1/63*b^3*x/
((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)^(7/2)*d^3)
 - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^2
+ c)^(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105
*a*b^2*x/((d*x^2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^
(5/2)*c*d^2) - 1/6*a*b^2*c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*
x/((d*x^2 + c)^(9/2)*d) + 16/105*a^2*b*x/(sqrt(d*x^2 + c)*c^4*d)
+ 8/105*a^2*b*x/((d*x^2 + c)^(3/2)*c^3*d) + 2/35*a^2*b*x/((d*x^2
+ c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 + c)^(7/2)*c*d)
```

Fricas [A] time = 0.80248, size = 309, normalized size = 1.38

$$\frac{\left(2\left(5\,b^3c^3d+12\,ab^2c^2d^2+24\,a^2bcd^3+64\,a^3d^4\right)x^9+315\,a^3c^4x+9\left(5\,b^3c^4+12\,ab^2c^3d+24\,a^2bc^2d^2+64\,a^3cd^3\right)x^7+63\left(3\,ab\right.\right.}{315\left(c^5d^5x^{10}+5\,c^6d^4x^8+10\,c^7d^3x^6+10\,c^8d^2x^4+5\,c^9dx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^
3*d^4)*x^9 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a
^2*b*c^2*d^2 + 64*a^3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*
d + 16*a^3*c^2*d^2)*x^5 + 105*(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sq
rt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*d^4*x^8 + 10*c^7*d^3*x^6 + 10
```

```
*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.23366, size = 294, normalized size = 1.31

$$\frac{\left(\left(\left(x^2\left(\frac{2\left(5\,b^3c^3d^5+12\,ab^2c^2d^6+24\,a^2bcd^7+64\,a^3d^8\right)x^2}{c^5d^4}+\frac{9\left(5\,b^3c^4d^4+12\,ab^2c^3d^5+24\,a^2bc^2d^6+64\,a^3cd^7\right)}{c^5d^4}\right)+\frac{63\left(3\,ab^2c^4d^4+6\,a^2bc^3d^5+16\,a^3c^2d^6\right)}{c^5d^4}\right)x^2+}{315\left(dx^2+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/(d*x^2 + c)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^
7 + 64*a^3*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d
^5 + 24*a^2*b*c^2*d^6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^
4*d^4 + 6*a^2*b*c^3*d^5 + 16*a^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3
*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5*d^4))*x^2 + 315*a^3/c)*x/(d*
x^2 + c)^(9/2)
```

3.98 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\, dx$

Optimal. Leaf size=174

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}(bc-ad)}+\frac{4ax\left(a+bx^2\right)(7bc-6ad)}{105c^3\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^2(7bc-6ad)}{35c^2\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^3}{7c\left(c+dx^2\right)^{7/2}(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^2)^3)/(7*c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*
c - 6*a*d)*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)
) + (4*a*(7*b*c - 6*a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c +
 d*x^2)^(3/2)) + (8*a^2*(7*b*c - 6*a*d)*x)/(105*c^4*(b*c - a*d)*S
qrt[c + d*x^2])
```

Rubi [A] time = 0.200514, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}(bc-ad)}+\frac{4ax\left(a+bx^2\right)(7bc-6ad)}{105c^3\left(c+dx^2\right)^{3/2}(bc-ad)}+\frac{x\left(a+bx^2\right)^2(7bc-6ad)}{35c^2\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^3}{7c\left(c+dx^2\right)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] -(d*x*(a + b*x^2)^3)/(7*c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*
c - 6*a*d)*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)
) + (4*a*(7*b*c - 6*a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c +
 d*x^2)^(3/2)) + (8*a^2*(7*b*c - 6*a*d)*x)/(105*c^4*(b*c - a*d)*S
qrt[c + d*x^2])
```

Rubi in Sympy [A] time = 30.0561, size = 156, normalized size = 0.9

$$\frac{8a^2x\,(6ad-7bc)}{105c^4\sqrt{c+dx^2}\,(ad-bc)}+\frac{4ax\left(a+bx^2\right)(6ad-7bc)}{105c^3\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{dx\left(a+bx^2\right)^3}{7c\left(c+dx^2\right)^{\frac{7}{2}}(ad-bc)}+\frac{x\left(a+bx^2\right)^2(6ad-7bc)}{35c^2\left(c+dx^2\right)^{\frac{5}{2}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] 8*a**2*x*(6*a*d - 7*b*c)/(105*c**4*sqrt(c + d*x**2)*(a*d - b*c))
+ 4*a*x*(a + b*x**2)*(6*a*d - 7*b*c)/(105*c**3*(c + d*x**2)**(3/2
)*(a*d - b*c)) + d*x*(a + b*x**2)**3/(7*c*(c + d*x**2)**(7/2)*(a*
d - b*c)) + x*(a + b*x**2)**2*(6*a*d - 7*b*c)/(35*c**2*(c + d*x**
```

```
2)**(5/2)*(a*d - b*c))
```

Mathematica [A] time = 0.106191, size = 107, normalized size = 0.61

$$\frac{3a^2\left(35c^3x+70c^2dx^3+56cd^2x^5+16d^3x^7\right)+2abcx^3\left(35c^2+28cdx^2+8d^2x^4\right)+3b^2c^2x^5\left(7c+2dx^2\right)}{105c^4\left(c+dx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] (3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2
 + 8*d^2*x^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 1
6*d^3*x^7))/(105*c^4*(c + d*x^2)^(7/2))
```

Maple [A] time = 0.01, size = 115, normalized size = 0.7

$$\frac{x\left(48\,a^2d^3x^6+16\,abcd^2x^6+6\,b^2c^2dx^6+168\,a^2cd^2x^4+56\,abc^2dx^4+21\,b^2c^3x^4+210\,a^2c^2dx^2+70\,abc^3x^2+105\,a^2c^3\right)}{105\,c^4}\left(dx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)^(9/2),x)
```

```
[Out] 1/105*x*(48*a^2*d^3*x^6+16*a*b*c*d^2*x^6+6*b^2*c^2*d*x^6+168*a^2*
c*d^2*x^4+56*a*b*c^2*d*x^4+21*b^2*c^3*x^4+210*a^2*c^2*d*x^2+70*a*
b*c^3*x^2+105*a^2*c^3)/(d*x^2+c)^(7/2)/c^4
```

Maxima [A] time = 1.36794, size = 336, normalized size = 1.93

$$-\frac{b^2x^3}{4\,(dx^2+c)^{\frac{7}{2}}d}+\frac{16\,a^2x}{35\,\sqrt{dx^2+c}c^4}+\frac{8\,a^2x}{35\,(dx^2+c)^{\frac{3}{2}}c^3}+\frac{6\,a^2x}{35\,(dx^2+c)^{\frac{5}{2}}c^2}+\frac{a^2x}{7\,(dx^2+c)^{\frac{7}{2}}c}$$
$$+\frac{3\,b^2x}{140\,(dx^2+c)^{\frac{5}{2}}d^2}+\frac{2\,b^2x}{35\,\sqrt{dx^2+c}c^2d^2}+\frac{b^2x}{35\,(dx^2+c)^{\frac{3}{2}}cd^2}-\frac{3\,b^2cx}{28\,(dx^2+c)^{\frac{7}{2}}d^2}$$
$$-\frac{2\,abx}{7\,(dx^2+c)^{\frac{7}{2}}d}+\frac{16\,abx}{105\,\sqrt{dx^2+c}c^3d}+\frac{8\,abx}{105\,(dx^2+c)^{\frac{3}{2}}c^2d}+\frac{2\,abx}{35\,(dx^2+c)^{\frac{5}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)
*c^4) + 8/35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 +
 c)^(5/2)*c^2) + 1/7*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((
d*x^2 + c)^(5/2)*d^2) + 2/35*b^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/
35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) - 3/28*b^2*c*x/((d*x^2 + c)^(7
/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) + 16/105*a*b*x/(sqrt(d
*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*d) + 2/35*a
*b*x/((d*x^2 + c)^(5/2)*c*d)
```

Fricas [A] time = 0.3322, size = 204, normalized size = 1.17

$$\frac{\left(2\left(3\,b^2c^2d + 8\,abcd^2 + 24\,a^2d^3\right)x^7 + 105\,a^2c^3x + 7\left(3\,b^2c^3 + 8\,abc^2d + 24\,a^2cd^2\right)x^5 + 70\left(abc^3 + 3\,a^2c^2d\right)x^3\right)\sqrt{dx^2 + c}}{105\left(c^4d^4x^8 + 4\,c^5d^3x^6 + 6\,c^6d^2x^4 + 4\,c^7dx^2 + c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c
^3*x + 7*(3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c
^3 + 3*a^2*c^2*d)*x^3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x
^6 + 6*c^6*d^2*x^4 + 4*c^7*d*x^2 + c^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.231906, size = 186, normalized size = 1.07

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,b^2c^2d^4+8\,abcd^5+24\,a^2d^6\right)x^2}{c^4d^3} + \frac{7\left(3\,b^2c^3d^3+8\,abc^2d^4+24\,a^2cd^5\right)}{c^4d^3}\right) + \frac{70\left(abc^3d^3+3\,a^2c^2d^4\right)}{c^4d^3}\right)x^2 + \frac{105\,a^2}{c}\right)x}{105\left(dx^2 + c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^2/(d*x^2 + c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^
4*d^3) + 7*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^
3)) + 70*(a*b*c^3*d^3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c
)*x/(d*x^2 + c)^(7/2)
```

3.99 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}} dx$

Optimal. Leaf size=91

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\,(c+dx^2)^{3/2}}-\frac{x(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

```
[Out] -((b*c - a*d)*x)/(5*c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(1
5*c^2*d*(c + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c
 + d*x^2])
```

Rubi [A] time = 0.0816296, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\,(c+dx^2)^{3/2}}-\frac{x(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] -((b*c - a*d)*x)/(5*c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(1
5*c^2*d*(c + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c
 + d*x^2])
```

Rubi in Sympy [A] time = 10.6007, size = 78, normalized size = 0.86

$$\frac{x\,(ad-bc)}{5cd\,(c+dx^2)^{\frac{5}{2}}}+\frac{x\,(4ad+bc)}{15c^2d\,(c+dx^2)^{\frac{3}{2}}}+\frac{2x\,(4ad+bc)}{15c^3d\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] x*(a*d - b*c)/(5*c*d*(c + d*x**2)**(5/2)) + x*(4*a*d + b*c)/(15*c
**2*d*(c + d*x**2)**(3/2)) + 2*x*(4*a*d + b*c)/(15*c**3*d*sqrt(c
+ d*x**2))
```

Mathematica [A] time = 0.0599623, size = 59, normalized size = 0.65

$$\frac{a\left(15c^2x+20cdx^3+8d^2x^5\right)+bcx^3\left(5c+2dx^2\right)}{15c^3\,(c+dx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] (b*c*x^3*(5*c + 2*d*x^2) + a*(15*c^2*x + 20*c*d*x^3 + 8*d^2*x^5))
/(15*c^3*(c + d*x^2)^(5/2))
```

Maple [A] time = 0.006, size = 57, normalized size = 0.6

$$\frac{x\left(8\,ad^2x^4+2\,bcdx^4+20\,acdx^2+5\,bc^2x^2+15\,c^2a\right)}{15\,c^3}\left(dx^2+c\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] 1/15*x*(8*a*d^2*x^4+2*b*c*d*x^4+20*a*c*d*x^2+5*b*c^2*x^2+15*a*c^2
)/(d*x^2+c)^(5/2)/c^3
```

Maxima [A] time = 1.3498, size = 139, normalized size = 1.53

$$\frac{8\,ax}{15\,\sqrt{dx^2+c}c^3}+\frac{4\,ax}{15\,(dx^2+c)^{\frac{3}{2}}c^2}+\frac{ax}{5\,(dx^2+c)^{\frac{5}{2}}c}-\frac{bx}{5\,(dx^2+c)^{\frac{5}{2}}d}+\frac{2\,bx}{15\,\sqrt{dx^2+c}c^2d}+\frac{bx}{15\,(dx^2+c)^{\frac{3}{2}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2)
 + 1/5*a*x/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d)
+ 2/15*b*x/(sqrt(d*x^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*
c*d)
```

Fricas [A] time = 0.228327, size = 117, normalized size = 1.29

$$\frac{\left(2\left(bcd+4\,ad^2\right)x^5+15\,ac^2x+5\left(bc^2+4\,acd\right)x^3\right)\sqrt{dx^2+c}}{15\,(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*
x^3)*sqrt(d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 +
 c^6)
```

Sympy [A] time = 127.945, size = 566, normalized size = 6.22

$$a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{35c^4dx^3}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{28c^3d^2x^5}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}+\frac{8c^2d^3x^7}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}\right)+b\left(\frac{5cx^3}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^5}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x*
*2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c)
 + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(1
5*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x
**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)
*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2*x**5/(15*c**(17/2)*
sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*
c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*s
qrt(1 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x*
*2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**
2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**
2/c))) + b*(5*c*x**3/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*c**(7/2
)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x*
*2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*c**(7/2)*d
*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x**2/
c)))
```

GIAC/XCAS [A] time = 0.229023, size = 97, normalized size = 1.07

$$\frac{\left(x^2\left(\frac{2\left(bcd^3+4\,ad^4\right)x^2}{c^3d^2}+\frac{5\left(bc^2d^2+4\,acd^3\right)}{c^3d^2}\right)+\frac{15\,a}{c}\right)x}{15\left(dx^2+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(d*x^2 + c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4
*a*c*d^3)/(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

3.100 $\int \frac{1}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\,(c+dx^2)^{3/2}}$$

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0191651, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(-5/2),x]

[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 2.02585, size = 32, normalized size = 0.82

$$\frac{x}{3c\,(c+dx^2)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**2+c)**(5/2),x)

[Out] x/(3*c*(c + d*x**2)**(3/2)) + 2*x/(3*c**2*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.0216859, size = 29, normalized size = 0.74

$$\frac{x\left(3c+2dx^2\right)}{3c^2\,(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(-5/2),x]
```

```
[Out] (x*(3*c + 2*d*x^2))/(3*c^2*(c + d*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 26, normalized size = 0.7

$$\frac{x\left(2\,dx^{2}+3\,c\right)}{3\,c^{2}}\left(dx^{2}+c\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*x*(2*d*x^2+3*c)/(d*x^2+c)^(3/2)/c^2
```

Maxima [A] time = 1.35008, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{dx^{2}+c}c^{2}}+\frac{x}{3\,(dx^{2}+c)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

Fricas [A] time = 0.210507, size = 63, normalized size = 1.62

$$\frac{\left(2\,dx^{3}+3\,cx\right)\sqrt{dx^{2}+c}}{3\,(c^{2}d^{2}x^{4}+2\,c^{3}dx^{2}+c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(-5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2
+ c^4)
```

Sympy [A] time = 2.73728, size = 95, normalized size = 2.44

$$\frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^{2}}{c}}+3c^{\frac{5}{2}}dx^{2}\sqrt{1+\frac{dx^{2}}{c}}}+\frac{2dx^{3}}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^{2}}{c}}+3c^{\frac{5}{2}}dx^{2}\sqrt{1+\frac{dx^{2}}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**2+c)**(5/2),x)
```

```
[Out] 3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 +
 d*x**2/c)) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2
)*d*x**2*sqrt(1 + d*x**2/c))
```

GIAC/XCAS [A] time = 0.22776, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,dx^2}{c^2} + \frac{3}{c}\right)}{3\,(dx^2 + c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.121559, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 20.6043, size = 66, normalized size = 0.84

$$\frac{dx}{c\sqrt{c+dx^2}\,(ad-bc)}-\frac{b\,\text{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] d*x/(c*sqrt(c + d*x**2)*(a*d - b*c)) - b*atanh(x*sqrt(a*d - b*c)/
(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.151802, size = 78, normalized size = 0.99

$$\frac{dx}{c\sqrt{c+dx^2}(ad-bc)}+\frac{b\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (b*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Maple [B] time = 0.054, size = 628, normalized size = 8.

$$-\frac{b}{2ad-2bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{dx}{(2ad-2bc)\,c}\frac{1}{\sqrt{\left(x-\frac{1}{b}\sqrt{-ab}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{b}{2ad-2bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}\right.\right.$$
$$+\frac{b}{2ad-2bc}\frac{1}{\sqrt{-ab}}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$+\frac{dx}{(2ad-2bc)\,c}\frac{1}{\sqrt{\left(x+\frac{1}{b}\sqrt{-ab}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}}$$
$$-\frac{b}{2ad-2bc}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b
)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*c)/c
/((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2)
)-(a*d-b*c)/b)^(1/2)*x*d+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-b*c)
/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/
2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(
1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/
2)))+1/2/(-a*b)^(1/2)/(a*d-b*c)*b/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(
-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/2/(a*d-b*
c)/c/((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))-(a*d-b*c)/b)^(1/2)*x*d-1/2/(-a*b)^(1/2)/(a*d-b*c)*b/(-(a*d-
b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)
^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*
b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)
```

```
^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309897, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{-abc+a^2d}\sqrt{dx^2+c}dx+\left(bcdx^2+bc^2\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}-4\left(\left(ab^2c^2-3\,a^2bcd+2\right.\right.}{b^2x^4+2\,abx^2+a^2}\right.}{4\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)\sqrt{-abc+a^2d}}\right.$$

$$\left.-\frac{2\sqrt{abc-a^2d}\sqrt{dx^2+c}dx-\left(bcdx^2+bc^2\right)\arctan\left(\frac{(bc-2\,ad)x^2-ac}{2\sqrt{abc-a^2d}\sqrt{dx^2+c}x}\right)}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)\sqrt{abc-a^2d}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*d*x + (b*c*d*x^2 +
b*c^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) - 4*((a*b^2*c^2
 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d
*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((b*c^3 - a*c^2*d + (b*c
^2*d - a*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)), -1/2*(2*sqrt(a*b*c -
a^2*d)*sqrt(d*x^2 + c)*d*x - (b*c*d*x^2 + b*c^2)*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((
b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.230092, size = 146, normalized size = 1.85

$$-\frac{b\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)}-\frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c +
2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a
*d)) - d*x/((b*c^2 - a*c*d)*sqrt(d*x^2 + c))
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.14441, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(
3/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 21.8175, size = 83, normalized size = 0.83

$$-\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(ad-bc)}+\frac{(2ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{\frac{3}{2}}\left(ad-bc\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*x*sqrt(c + d*x**2)/(2*a*(a + b*x**2)*(a*d - b*c)) + (2*a*d - b
*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(2*a**(3/
2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.176456, size = 100, normalized size = 1.

$$\frac{\frac{\sqrt{a}bx\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] ((Sqrt[a]*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + ((b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
b*c - a*d)^(3/2))/(2*a^(3/2))
```

Maple [B] time = 0.041, size = 823, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/4/a/(a*d-b*c)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^(1/2))^2*d+2
*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)+1/4/b/a
*d*(-a*b)^(1/2)/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b
+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*(
(x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-
(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/4/a/(a*d-b*c)/(x+1/b*
(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b
*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2)-1/4/b/a*d*(-a*b)^(1/2)/(a*d-b*c
)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1
/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d
-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1
/b*(-a*b)^(1/2)))-1/4/a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*
(a*d-b*c)/b+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)
/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a
*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4/a/(-a*b)
^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b
*(x+1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2)
)^2*d-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))
/(x+1/b*(-a*b)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Fricas [A] time = 0.355736, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{-abc+a^2d}\sqrt{dx^2+c}\,bx+\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}+4}{b^2x^4+2\,abx^2+a}\right.}{8\left(a^2bc-a^3d+(ab^2c-a^2bd)x^2\right)\sqrt{-abc+a^2d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c)*b*x + (a*b*c - 2*a^2
*d + (b^2*c - 2*a*b*d)*x^2)*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a
^2*d) + 4*((a*b^2*c^2 - 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2
 - a^3*c*d)*x)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a
^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-a*b*c + a^2*d)),
1/4*(2*sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*b*x + (a*b*c - 2*a^2*d
 + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(s
qrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a^2*b*c - a^3*d + (a*b^
2*c - a^2*b*d)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 0.232772, size = 304, normalized size = 3.04

$$-\frac{1}{2}\,d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 \cdots\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc+4\left(\sqrt{d}x-\sqrt{dx^2 \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d
^2)^(3/2) + 2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*d - b*c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2)))
```

3.103 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=149

$$\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{c+dx^2}(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2
)) + (b*x*(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c
*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.229334, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{x\sqrt{c+dx^2}(3bc-4ad)}{8a^2\left(a+bx^2\right)(bc-ad)}+\frac{bx\left(c+dx^2\right)^{3/2}}{4a\left(a+bx^2\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2
)) + (b*x*(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c
*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x
^2])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 32.7139, size = 129, normalized size = 0.87

$$-\frac{bx\left(c+dx^2\right)^{\frac{3}{2}}}{4a\left(a+bx^2\right)^2(ad-bc)}+\frac{x\sqrt{c+dx^2}\,(4ad-3bc)}{8a^2\left(a+bx^2\right)(ad-bc)}+\frac{c\,(4ad-3bc)\,\mathrm{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{\frac{5}{2}}\,(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] -b*x*(c + d*x**2)**(3/2)/(4*a*(a + b*x**2)**2*(a*d - b*c)) + x*sq
rt(c + d*x**2)*(4*a*d - 3*b*c)/(8*a**2*(a + b*x**2)*(a*d - b*c))
+ c*(4*a*d - 3*b*c)*atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x
**2)))/(8*a**(5/2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.262205, size = 130, normalized size = 0.87

$$\frac{\frac{\sqrt{a}x\sqrt{c+dx^2}\left(-4a^2d+ab\left(5c-2dx^2\right)+3b^2cx^2\right)}{\left(a+bx^2\right)^2(bc-ad)}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*x*Sqrt[c + d*x^2]*(-4*a^2*d + 3*b^2*c*x^2 + a*b*(5*c -
2*d*x^2)))/((b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/
2))/(8*a^(5/2))
```

Maple [B] time = 0.05, size = 5177, normalized size = 34.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\left(bx^2 + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

Fricas [A] time = 0.404444, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(3\,b^2c-2\,abd\right)x^3+\left(5\,abc-4\,a^2d\right)x\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}+\left(3\,a^2bc^2-4\,a^3cd+\left(3\,b^3c^2-4\,ab^2cd\right)x^4+2\left(3\,ab^2c^2-4\right.\right.}{32\left(a^4bc-a^5d+(a^2b^3c-a^3b^2d)x^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(4*((3*b^2*c - 2*a*b*d)*x^3 + (5*a*b*c - 4*a^2*d)*x)*sqrt(-
a*b*c + a^2*d)*sqrt(d*x^2 + c) + (3*a^2*b*c^2 - 4*a^3*c*d + (3*b^
3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*log
((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2*b
*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^4*b*c - a^5*d + (a^2*b^3*c - a
^3*b^2*d)*x^4 + 2*(a^3*b^2*c - a^4*b*d)*x^2)*sqrt(-a*b*c + a^2*d)
), 1/16*(2*((3*b^2*c - 2*a*b*d)*x^3 + (5*a*b*c - 4*a^2*d)*x)*sqrt
(a*b*c - a^2*d)*sqrt(d*x^2 + c) + (3*a^2*b*c^2 - 4*a^3*c*d + (3*b
^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*ar
ctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^
2 + c)*x)))/((a^4*b*c - a^5*d + (a^2*b^3*c - a^3*b^2*d)*x^4 + 2*(
a^3*b^2*c - a^4*b*d)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 1.24272, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.104 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^4} \, dx$

Optimal. Leaf size=199

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{cx\sqrt{c+dx^2}(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*
x^2)) + ((5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)
*(a + b*x^2)^2) + (b*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b
*x^2)^3) + (c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] time = 0.298351, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}+\frac{cx\sqrt{c+dx^2}(5bc-6ad)}{16a^3\left(a+bx^2\right)(bc-ad)}+\frac{x\left(c+dx^2\right)^{3/2}(5bc-6ad)}{24a^2\left(a+bx^2\right)^2(bc-ad)}+\frac{bx\left(c+dx^2\right)^{5/2}}{6a\left(a+bx^2\right)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*
x^2)) + ((5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)
*(a + b*x^2)^2) + (b*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b
*x^2)^3) + (c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[
a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c - a*d)^(3/2))
```

Rubi in Sympy [A] time = 44.8537, size = 175, normalized size = 0.88

$$-\frac{bx\left(c+dx^2\right)^{\frac{5}{2}}}{6a\left(a+bx^2\right)^3(ad-bc)}+\frac{x\left(c+dx^2\right)^{\frac{3}{2}}(6ad-5bc)}{24a^2\left(a+bx^2\right)^2(ad-bc)}+\frac{cx\sqrt{c+dx^2}\,(6ad-5bc)}{16a^3\left(a+bx^2\right)(ad-bc)}+\frac{c^2(6ad-5bc)\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{\frac{7}{2}}(ad-bc)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

```
[Out] -b*x*(c + d*x**2)**(5/2)/(6*a*(a + b*x**2)**3*(a*d - b*c)) + x*(c
 + d*x**2)**(3/2)*(6*a*d - 5*b*c)/(24*a**2*(a + b*x**2)**2*(a*d -
 b*c)) + c*x*sqrt(c + d*x**2)*(6*a*d - 5*b*c)/(16*a**3*(a + b*x**
2)*(a*d - b*c)) + c**2*(6*a*d - 5*b*c)*atanh(x*sqrt(a*d - b*c)/(s
qrt(a)*sqrt(c + d*x**2)))/(16*a**(7/2)*(a*d - b*c)**(3/2))
```

Mathematica [A] time = 0.413399, size = 179, normalized size = 0.9

$$\frac{\frac{3c^2(5bc-6ad)\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}-\frac{\sqrt{a}x\sqrt{c+dx^2}\left(-6a^3d\left(5c+2dx^2\right)+a^2b\left(33c^2-22cdx^2-4d^2x^4\right)+8ab^2cx^2\left(5c-dx^2\right)+15b^3c^2x^4\right)}{\left(a+bx^2\right)^3(ad-bc)}}{48a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*
c - d*x^2) - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2
 - 4*d^2*x^4)))/((-(b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c -
 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b
*c - a*d)^(3/2))/(48*a^(7/2))
```

Maple [B] time = 0.074, size = 13964, normalized size = 70.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)
```

Fricas [A] time = 0.620245, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(4*((15*b^3*c^2 - 8*a*b^2*c*d - 4*a^2*b*d^2)*x^5 + 2*(20*a
*b^2*c^2 - 11*a^2*b*c*d - 6*a^3*d^2)*x^3 + 3*(11*a^2*b*c^2 - 10*a
^3*c*d)*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c) + 3*(5*a^3*b*c^3
- 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 + 3*(5*a*b^3*c^3
- 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^2)*l
og((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 - 3*a^2
*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x^2 + c
))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/((a^6*b*c - a^7*d + (a^3*b^4*c -
 a^4*b^3*d)*x^6 + 3*(a^4*b^3*c - a^5*b^2*d)*x^4 + 3*(a^5*b^2*c -
a^6*b*d)*x^2)*sqrt(-a*b*c + a^2*d)), 1/96*(2*((15*b^3*c^2 - 8*a*b
^2*c*d - 4*a^2*b*d^2)*x^5 + 2*(20*a*b^2*c^2 - 11*a^2*b*c*d - 6*a^
3*d^2)*x^3 + 3*(11*a^2*b*c^2 - 10*a^3*c*d)*x)*sqrt(a*b*c - a^2*d)
*sqrt(d*x^2 + c) + 3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*
a*b^3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a
^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^2)*arctan(1/2*((b*c - 2*a*d)*x^2 -
a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x)))/((a^6*b*c - a^7*d
+ (a^3*b^4*c - a^4*b^3*d)*x^6 + 3*(a^4*b^3*c - a^5*b^2*d)*x^4 + 3
*(a^5*b^2*c - a^6*b*d)*x^2)*sqrt(a*b*c - a^2*d))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 19.5275, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=20

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0142504, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]

[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 4.93944, size = 15, normalized size = 0.75

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)

[Out] d*x/(b*c*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.017386, size = 20, normalized size = 1.

$$\frac{dx}{bc\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Maple [A] time = 0.004, size = 19, normalized size = 1.

$$\frac{dx}{bc}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x)
```

```
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.210956, size = 36, normalized size = 1.8

$$\frac{\sqrt{dx^2+c}dx}{bcdx^2+bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{d\int\frac{1}{c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}}\,dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
```

```
[Out] d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

GIAC/XCAS [A] time = 0.240655, size = 24, normalized size = 1.2

$$\frac{dx}{\sqrt{dx^2 + c}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] d*x/(sqrt(d*x^2 + c)*b*c)
```

3.106 $\int \frac{1}{\sqrt{1-x^2}(1+x^2)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rubi [A] time = 0.0270475, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 6.09703, size = 22, normalized size = 0.88

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{\sqrt{-x^2+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)

[Out] sqrt(2)*atan(sqrt(2)*x/sqrt(-x**2 + 1))/2
```

Mathematica [A] time = 0.0397045, size = 25, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]

[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Maple [A] time = 0.006, size = 28, normalized size = 1.1

$$-\frac{\sqrt{2}}{2}\arctan\left(\frac{x\sqrt{2}}{x^2-1}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)/(-x^2+1)^(1/2),x)

[Out] -1/2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+1)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 0.212849, size = 65, normalized size = 2.6

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)+\sqrt{2}\sqrt{-x^2+1}}{2\left(\sqrt{-x^2+1}x-x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*(x^2 - 1) + sqrt(2)*sqrt(-x^2 + 1
))/(sqrt(-x^2 + 1)*x - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

GIAC/XCAS [A] time = 0.236704, size = 69, normalized size = 2.76

$$\frac{1}{4}\sqrt{2}\left(\pi\operatorname{sign}(x) + 2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(pi*sign(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1
) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Rubi [A] time = 0.0555219, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Rubi in Sympy [A] time = 11.2787, size = 42, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] atanh(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x**2)))/(sqrt(a)*sqrt
(a*d - b*c))
```

Mathematica [A] time = 0.042639, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[b*c - a*d])
```

Maple [B] time = 0.015, size = 306, normalized size = 6.2

$$-\frac{1}{2}\ln\left(1\left(-2\,\frac{ad-bc}{b}+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}d+2\,\frac{d\sqrt{-ab}}{b}\left(x-\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$
$$+\frac{1}{2}\ln\left(1\left(-2\,\frac{ad-bc}{b}-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)+2\,\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}d-2\,\frac{d\sqrt{-ab}}{b}\left(x+\frac{\sqrt{-ab}}{b}\right)-\frac{ad-bc}{b}}\right)\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] -1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b+2*d*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*d-b*c)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*d+2*d*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*d-b*c
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/2/(-a*b)^(1/2)/(-(a*d-b*c)/b)
^(1/2)*ln((-2*(a*d-b*c)/b-2*d*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
+2*(-(a*d-b*c)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*d-2*d*(-a*b)^(1/2
)/b*(x+1/b*(-a*b)^(1/2))-(a*d-b*c)/b)^(1/2))/(x+1/b*(-a*b)^(1/2))
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259353, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{\left(\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2\right)\sqrt{-abc+a^2d}+4\left(\left(ab^2c^2-3\,a^2bcd+2\,a^3d^2\right)x^3-\left(a^2bc^2-a^3cd\right)x\right)\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\sqrt{-abc+a^2d}},\ \frac{\arctan\left(\frac{(bc-}{2\sqrt{abc}}\right.}{2\sqrt{abc}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] [1/4*log((((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3
*a*b*c^2 - 4*a^2*c*d)*x^2)*sqrt(-a*b*c + a^2*d) + 4*((a*b^2*c^2 -
 3*a^2*b*c*d + 2*a^3*d^2)*x^3 - (a^2*b*c^2 - a^3*c*d)*x)*sqrt(d*x
^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(-a*b*c + a^2*d), 1/2*a
rctan(1/2*((b*c - 2*a*d)*x^2 - a*c)/(sqrt(a*b*c - a^2*d)*sqrt(d*x
^2 + c)*x))/sqrt(a*b*c - a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [A] time = 0.238582, size = 95, normalized size = 1.94

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*
a*d)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.108 $\int \frac{-1+x^2}{(1+x^2)^{3/2}} \, dx$

Optimal. Leaf size=15

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi [A] time = 0.0155755, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + x^2)/(1 + x^2)^(3/2),x]

[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi in Sympy [A] time = 4.06519, size = 14, normalized size = 0.93

$$-\frac{2x}{\sqrt{x^2+1}} + \operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-1)/(x**2+1)**(3/2),x)

[Out] -2*x/sqrt(x**2 + 1) + asinh(x)
```

Mathematica [A] time = 0.0202053, size = 15, normalized size = 1.

$$\sinh^{-1}(x) - \frac{2x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Maple [A] time = 0.008, size = 14, normalized size = 0.9

$$Arcsinh\left(x\right)-2\,\frac{x}{\sqrt{x^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-1)/(x^2+1)^(3/2),x)
```

```
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

Maxima [A] time = 1.49141, size = 18, normalized size = 1.2

$$-\frac{2\,x}{\sqrt{x^{2}+1}}+\operatorname{arsinh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x/sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [A] time = 0.202848, size = 66, normalized size = 4.4

$$-\frac{\left(x^{2}-\sqrt{x^{2}+1}x+1\right)\log\left(-x+\sqrt{x^{2}+1}\right)+2}{x^{2}-\sqrt{x^{2}+1}x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x^2 - sqrt(x^2 + 1)*x + 1)*log(-x + sqrt(x^2 + 1)) + 2)/(x^2 -
 sqrt(x^2 + 1)*x + 1)
```

Sympy [A] time = 10.8157, size = 31, normalized size = 2.07

$$\frac{x^{2}\operatorname{asinh}\left(x\right)}{x^{2}+1}-\frac{2x}{\sqrt{x^{2}+1}}+\frac{\operatorname{asinh}\left(x\right)}{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 +
1)
```

GIAC/XCAS [A] time = 0.228812, size = 34, normalized size = 2.27

$$-\frac{2\,x}{\sqrt{x^2+1}} - \ln\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 - 1)/(x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] -2*x/sqrt(x^2 + 1) - ln(-x + sqrt(x^2 + 1))
```

3.109 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx$

Optimal. Leaf size=648

$$\frac{24192\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{36288\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}$$
$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(
5/3))/6175 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x
*(a - b*x^2)^(5/3)*(3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36288*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
 a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
+ (24192*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)])
```

Rubi [A] time = 1.20504, antiderivative size = 648, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{24192\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{36288\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}$$
$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(
5/3))/6175 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x
*(a - b*x^2)^(5/3)*(3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36288*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
 a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
+ (24192*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)])
```

Rubi in Sympy [A] time = 90.8448, size = 525, normalized size = 0.81

$$-\frac{36288\sqrt[4]{3}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{24192\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{72576a^4x}{1235\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x\left(a-bx^2\right)^{\frac{2}{3}}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{\frac{5}{3}}}{6175}$$

$$-\frac{378ax\left(a-bx^2\right)^{\frac{5}{3}}\left(3a+bx^2\right)}{475}-\frac{3x\left(a-bx^2\right)^{\frac{5}{3}}\left(3a+bx^2\right)^2}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
```

```
[Out] -36288*3**(1/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)*
*(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x
**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3
))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))
/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3
))/(1235*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**
(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 24192*sqrt(2)*
3**(3/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
+ (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(
1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1
/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)
) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(1235*b*x*sqrt(-a**(1/
3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a
 - b*x**2)**(1/3))**2)) + 72576*a**4*x/(1235*(a**(1/3)*(-1 + sqrt
(3)) + (a - b*x**2)**(1/3))) + 18144*a**3*x*(a - b*x**2)**(2/3)/1
235 - 23544*a**2*x*(a - b*x**2)**(5/3)/6175 - 378*a*x*(a - b*x**2
)**(5/3)*(3*a + b*x**2)/475 - 3*x*(a - b*x**2)**(5/3)*(3*a + b*x*
*2)**2/25
```

Mathematica [C] time = 0.0845421, size = 99, normalized size = 0.15

$$-\frac{3\left(-40320a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-15255a^4x+3390a^3bx^3+8992a^2b^2x^5+2626ab^3x^7+247b^4x^9\right)}{6175\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (-3*(-15255*a^4*x + 3390*a^3*b*x^3 + 8992*a^2*b^2*x^5 + 2626*a*b^
3*x^7 + 247*b^4*x^9 - 40320*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeom
etric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(6175*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{2}{3}}\left(bx^2+3a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^3\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3x^6+9\,ab^2x^4+27\,a^2bx^2+27\,a^3\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2
+ a)^(2/3), x)
```

Sympy [A] time = 11.4876, size = 136, normalized size = 0.21

$$27a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi
)/a) + 9*a**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_po
lar(2*I*pi)/a) + 9*a**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,),
b*x**2*exp_polar(2*I*pi)/a)/5 + a**(2/3)*b**3*x**7*hyper((-2/3, 7
/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

3.110 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx$

Optimal. Leaf size=617

$$\frac{10368\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}$$

$$-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))
/247 - (3*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(
1729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(
a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (10368*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.960829, antiderivative size = 617, normalized size of antiderivative = 1., number

of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{10368\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}$$

$$-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))
/247 - (3*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(
1729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(
a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (10368*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 59.5352, size = 496, normalized size = 0.8

$$-\frac{15552\sqrt[4]{3}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{10368\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{31104a^3x}{1729\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x\left(a-bx^2\right)^{\frac{2}{3}}}{1729}-\frac{252ax\left(a-bx^2\right)^{\frac{5}{3}}}{247}-\frac{3x\left(a-bx^2\right)^{\frac{5}{3}}\left(3a+bx^2\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

```
[Out] -15552*3**(1/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)*
*(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x
**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3
))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))
/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3
))/(1729*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**
(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 10368*sqrt(2)*
3**(3/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
+ (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(
1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1
/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)
) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(1729*b*x*sqrt(-a**(1/
3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a
 - b*x**2)**(1/3))**2)) + 31104*a**3*x/(1729*(a**(1/3)*(-1 + sqrt
(3)) + (a - b*x**2)**(1/3))) + 7776*a**2*x*(a - b*x**2)**(2/3)/17
29 - 252*a*x*(a - b*x**2)**(5/3)/247 - 3*x*(a - b*x**2)**(5/3)*(3
*a + b*x**2)/19
```

Mathematica [C] time = 0.0630779, size = 88, normalized size = 0.14

$$-\frac{3\left(-3456a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-1731a^3x+961a^2bx^3+679ab^2x^5+91b^3x^7\right)}{1729\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (-3*(-1731*a^3*x + 961*a^2*b*x^3 + 679*a*b^2*x^5 + 91*b^3*x^7 - 3
456*a^3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2,
(b*x^2)/a]))/(1729*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{2}{3}} \left(bx^2+3\,a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4+6\,abx^2+9\,a^2\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 7.90244, size = 99, normalized size = 0.16

$$9a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/
a) + 2*a**(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_pola
r(2*I*pi)/a) + a**(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x*
*2*exp_polar(2*I*pi)/a)/5
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

3.111 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\, dx$

Optimal. Leaf size=588

$$\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*
a^2*x)/(13*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))^2)])
```

Rubi [A] time = 0.806444, antiderivative size = 588, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*
a^2*x)/(13*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 39.1913, size = 469, normalized size = 0.8

$$-\frac{36\sqrt[4]{3}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{24\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{72a^2x}{13\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{18ax\left(a-bx^2\right)^{\frac{2}{3}}}{13}-\frac{3x\left(a-bx^2\right)^{\frac{5}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
```

```
[Out] -36*3**(1/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/
3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)
**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*e
lliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a
**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(
13*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*
(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 24*sqrt(2)*3**(3/4)*
a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x
**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*
(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + s
qrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b
*x**2)**(1/3))), -7 + 4*sqrt(3))/(13*b*x*sqrt(-a**(1/3)*(a**(1/3)
 - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)) + 72*a**2*x/(13*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**
2)**(1/3))) + 18*a*x*(a - b*x**2)**(2/3)/13 - 3*x*(a - b*x**2)**(
5/3)/13
```

Mathematica [C] time = 0.0495926, size = 76, normalized size = 0.13

$$-\frac{3\left(-8a^2x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-5a^2x+4abx^3+b^2x^5\right)}{13\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (-3*(-5*a^2*x + 4*a*b*x^3 + b^2*x^5 - 8*a^2*x*(1 - (b*x^2)/a)^(1/
3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(13*(a - b*x^2)^
(1/3))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{2}{3}} \left(bx^2+3a\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right) \left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+3a\right)\left(-bx^2+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 5.37092, size = 63, normalized size = 0.11

$$3a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(5/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/
a) + a**(2/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(
2*I*pi)/a)/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + 3\,a\right)\left(-bx^2 + a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

3.112 $\int \frac{\left(a-bx^2\right)^{2/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=740

$$-\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$+\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1
/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2
^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*
x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
nh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/
Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^
2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[
-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2])
```

Rubi [A] time = 0.778856, antiderivative size = 740, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$+\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1
/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2
^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*
x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
nh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/
Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*
x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^
2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[
-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 112.322, size = 784, normalized size = 1.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)
```

```
[Out] 3*3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
 + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*ell
iptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**
(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(2*
b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1
 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) - sqrt(2)*3**(3/4)*a**(1/
3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**
(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a**(1
/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)
) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)
**(1/3))), -7 + 4*sqrt(3))/(b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b
*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2
)) + 2**(1/3)*sqrt(a)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)*(1 - sqr
t(b)*x/sqrt(a))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))/(2*sqrt(
b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(a)*(1 - b*x**2/a)**(1/3)*
log((1 - sqrt(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt(b)*x/sqrt
(a))**(1/3))/(2*sqrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*s
qrt(a)*(1 - b*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1
 + sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a))**(1/3)))/
(3*sqrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*sqrt(a)*(1 - b
*x**2/a)**(1/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(a))**(2
/3)/(3*(1 + sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)/(3*sqrt(b)*(a
 - b*x**2)**(1/3)) - 3*x/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)*
*(1/3))
```

Mathematica [C] time = 0.235956, size = 162, normalized size = 0.22

$$\frac{9ax\left(a-bx^2\right)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(9aF_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-2bx^2\left(F_1\left(\frac{3}{2};-\frac{2}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (9*a*x*(a - b*x^2)^(2/3)*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -
(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, -2/3, 1, 3/2, (
b*x^2)/a, -(b*x^2)/(3*a)] - 2*b*x^2*(AppellF1[3/2, -2/3, 2, 5/2,
(b*x^2)/a, -(b*x^2)/(3*a)] + 2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)
/a, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{b x^2 + 3 a} \left(-b x^2 + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)

[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^2 + a\right)^{\frac{2}{3}}}{b x^2 + 3 a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a),x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - b x^2\right)^{\frac{2}{3}}}{3 a + b x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)

[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a),x, algorithm="giac")

[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

3.113 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx$

Optimal. Leaf size=584

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}-\frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
 b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ell
ipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*3^(3/4
)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^
(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.803382, antiderivative size = 584, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
 b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ell
ipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*3^(3/4
)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^
(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 46.8108, size = 452, normalized size = 0.77

$$\frac{x\left(a-bx^2\right)^{\frac{2}{3}}}{6a\left(3a+bx^2\right)}+\frac{x}{6a\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{12a^{\frac{2}{3}}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{18a^{\frac{2}{3}}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
```

```
[Out] x*(a - b*x**2)**(2/3)/(6*a*(3*a + b*x**2)) + x/(6*a*(a**(1/3)*(-1
 + sqrt(3)) + (a - b*x**2)**(1/3))) - 3**(1/4)*sqrt((a**(2/3) + a
**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1
+ sqrt(3)) + (a - b*x**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3)
 - (a - b*x**2)**(1/3))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) -
 (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(
1/3))), -7 + 4*sqrt(3))/(12*a**(2/3)*b*x*sqrt(-a**(1/3)*(a**(1/3)
 - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)) + sqrt(2)*3**(3/4)*sqrt((a**(2/3) + a**(1/3)*(a - b*x
**2)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a
- b*x**2)**(1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f
(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(
-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(18*a**(2/
3)*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*
(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2))
```

Mathematica [C] time = 0.0913135, size = 86, normalized size = 0.15

$$\frac{x\sqrt[3]{\frac{a-bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)}{18a\sqrt[3]{a-bx^2}}+\frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) + (x*((a - b*x^2)/a)^(1
/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(18*a*(a - b*x^2
)^(1/3))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{b^2x^4+6\,abx^2+9\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*x^2 + a)^(2/3)/(b^2*x^4 + 6*a*b*x^2 + 9*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^{2}\right)^{\frac{2}{3}}}{\left(3a+bx^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{2}{3}}}{\left(bx^{2}+3\,a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

3.114 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx$

Optimal. Leaf size=818

$$\frac{(a-bx^2)^{2/3}x}{36a^2(bx^2+3a)}-\frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{12a(bx^2+3a)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\,2^{2/3}a^{11/6}\sqrt{b}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{24\,3^{3/4}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{18\sqrt{2}\sqrt[4]{3}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/
3))/(36*a^2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^
(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*
a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(216*2^(2/3)*a^(
11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*
(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sq
rt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a -
 b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
 + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(18*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi [A] time = 1.30377, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{(a-bx^2)^{2/3}x}{36a^2(bx^2+3a)}-\frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{12a(bx^2+3a)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{24\,3^{3/4}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{18\sqrt{2}\sqrt[4]{3}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/
3))/(36*a^2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^
(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*
a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(216*2^(2/3)*a^(
11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*
(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sq
rt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a -
 b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
 + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(18*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.735232, size = 350, normalized size = 0.43

$$\frac{x\left(\frac{3(a-bx^2)(6a+bx^2)}{a^2}+\frac{5bx^2(3a+bx^2)F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a\left(2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}+\frac{54(3a+bx^2)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\right.\right.}\right.}{108\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*((3*(a - b*x^2)*(6*a + b*x^2))/a^2 + (54*(3*a + b*x^2)*AppellF
1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2
, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/
2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1
, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) + (5*b*x^2*(3*a + b*x^2)*Appe
llF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(a*(15*a*Appel
lF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-Appe
llF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[5/2,
 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)])))))/(108*(a - b*x^2)^(1
/3)*(3*a + b*x^2)^2)
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^3}\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3\,a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{2}{3}}}{\left(3a+bx^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{\left(bx^2+3\,a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3,x, algorithm="giac")

[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

3.115 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx$

Optimal. Leaf size=849

$$\frac{(a-bx^2)^{2/3}x}{144a^3(bx^2+3a)}-\frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{54a^2(bx^2+3a)^2}+\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)^3}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\,2^{2/3}a^{17/6}\sqrt{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{96\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/
3))/(54*a^2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*
a + b*x^2)) - x/(144*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))) + (7*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sq
rt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2
^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^
(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(2/3)*
a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^
(1/3)*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sq
rt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*
```

```
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)])
```

Rubi [A] time = 1.47146, antiderivative size = 849, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{(a-bx^2)^{2/3}x}{144a^3(bx^2+3a)}-\frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{54a^2(bx^2+3a)^2}+\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)^3}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{96\ 3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/
3))/(54*a^2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*
a + b*x^2)) - x/(144*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))) + (7*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sq
rt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2
^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^
(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(2/3)*
a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^
(1/3)*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sq
rt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
```

```
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.36096, size = 364, normalized size = 0.43

$$\frac{x\left(\frac{69a^2\left(3a+bx^2\right)^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\left(a-bx^2\right)\left(75a^2+26abx^2+3b^2x^4\right)+\frac{\cdots}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\cdots\right.\right.}\right.}{432a^3\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^3}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*((a - b*x^2)*(75*a^2 + 26*a*b*x^2 + 3*b^2*x^4) + (69*a^2*(3*a
+ b*x^2)^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])
/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b
*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + Ap
pellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) + (5*a*b*(3
*a*x + b*x^3)^2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3
*a)])/(15*a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]
 + 2*b*x^2*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)
] + AppellF1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(43
2*a^3*(a - b*x^2)^(1/3)*(3*a + b*x^2)^3)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+3a\right)^{4}}\left(-bx^{2}+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

```
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^{2}+a\right)^{\frac{2}{3}}}{\left(bx^{2}+3a\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2 + a\right)^{\frac{2}{3}}}{\left(bx^2 + 3a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

3.116 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^3 \, dx$

Optimal. Leaf size=668

$$\frac{3746304\sqrt{2}3^{3/4}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{5619456\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995}+\frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425}$$
$$-\frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725}-\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*
x^2)^(5/3))/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432
*a*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3
)*(3*a + b*x^2)^2)/31 - (11238912*a^5*x)/(267995*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) - (5619456*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (37463
04*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rubi [A] time = 1.29296, antiderivative size = 668, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3746304\sqrt{2}3^{3/4}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{5619456\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x\left(a-bx^2\right)^{2/3}}{267995}+\frac{1404864a^3x\left(a-bx^2\right)^{5/3}}{191425}$$
$$-\frac{33264a^2x\left(a-bx^2\right)^{8/3}}{14725}-\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*
x^2)^(5/3))/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432
*a*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3
)*(3*a + b*x^2)^2)/31 - (11238912*a^5*x)/(267995*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) - (5619456*3^(1/4)*Sqrt[2 + Sqrt[3]]
*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(
a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (37463
04*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rubi in Sympy [A] time = 102.624, size = 544, normalized size = 0.81

$$-\frac{5619456\sqrt[4]{3}a^{\frac{16}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3746304\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{16}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{11238912a^5x}{267995\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x\left(a-bx^2\right)^{\frac{2}{3}}}{267995}+\frac{1404864a^3x\left(a-bx^2\right)^{\frac{5}{3}}}{191425}$$
$$-\frac{33264a^2x\left(a-bx^2\right)^{\frac{8}{3}}}{14725}-\frac{432ax\left(a-bx^2\right)^{\frac{8}{3}}\left(3a+bx^2\right)}{775}-\frac{3x\left(a-bx^2\right)^{\frac{8}{3}}\left(3a+bx^2\right)^2}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
```

```
[Out] -5619456*3**(1/4)*a**(16/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2
)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b
*x**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1
/3))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3
))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt
(3))/(267995*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/
(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 3746304*sq
rt(2)*3**(3/4)*a**(16/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**
(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x*
*2)**(1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin(
(a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + s
qrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(267995*b*x*sqrt
(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(
3)) + (a - b*x**2)**(1/3))**2)) + 11238912*a**5*x/(267995*(a**(1/
3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))) + 2809728*a**4*x*(a - b
*x**2)**(2/3)/267995 + 1404864*a**3*x*(a - b*x**2)**(5/3)/191425
- 33264*a**2*x*(a - b*x**2)**(8/3)/14725 - 432*a*x*(a - b*x**2)**
(8/3)*(3*a + b*x**2)/775 - 3*x*(a - b*x**2)**(8/3)*(3*a + b*x**2)
**2/31
```

Mathematica [C] time = 0.0849968, size = 110, normalized size = 0.16

$$\frac{3\left(6243840a^5x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+5815935a^5x-5312355a^4bx^3-1675114a^3b^2x^5+749658a^2b^3x^7+378651ab^4x\right.}{1339975\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (3*(5815935*a^5*x - 5312355*a^4*b*x^3 - 1675114*a^3*b^2*x^5 + 749
658*a^2*b^3*x^7 + 378651*a*b^4*x^9 + 43225*b^5*x^11 + 6243840*a^5
*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)
/a]))/(1339975*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{5}{3}}\left(bx^2+3a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^4x^8+8\,ab^3x^6+18\,a^2b^2x^4-27\,a^4\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 8*a*b^3*x^6 + 18*a^2*b^2*x^4 - 27*a^4)*(-b*x
^2 + a)^(2/3), x)
```

Sympy [A] time = 21.0275, size = 139, normalized size = 0.21

$$27a^{\frac{14}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{18a^{\frac{8}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}-\frac{8a^{\frac{5}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}-\frac{a^{\frac{2}{3}}b^4x^9\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(14/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi
)/a) - 18*a**(8/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*ex
p_polar(2*I*pi)/a)/5 - 8*a**(5/3)*b**3*x**7*hyper((-2/3, 7/2), (9
/2,), b*x**2*exp_polar(2*I*pi)/a)/7 - a**(2/3)*b**4*x**9*hyper((-
2/3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)/9
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

3.117 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx$

Optimal. Leaf size=637

$$\frac{38016\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645}$$

$$+\frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(
5/3))/6175 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(
8/3)*(3*a + b*x^2))/25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))) - (57024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1
3/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b
*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2
]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.06391, antiderivative size = 637, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{38016\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645}$$
$$+\frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(
5/3))/6175 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(
8/3)*(3*a + b*x^2))/25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))) - (57024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1
3/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b
*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2
]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 69.5784, size = 515, normalized size = 0.81

$$-\frac{57024\sqrt[4]{3}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{38016\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{114048a^4x}{8645\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x\left(a-bx^2\right)^{\frac{2}{3}}}{8645}$$
$$+\frac{14256a^2x\left(a-bx^2\right)^{\frac{5}{3}}}{6175}-\frac{306ax\left(a-bx^2\right)^{\frac{8}{3}}}{475}-\frac{3x\left(a-bx^2\right)^{\frac{8}{3}}\left(3a+bx^2\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

```
[Out] -57024*3**(1/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)*
*(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x
**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3
))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))
/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3
))/(8645*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**
(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 38016*sqrt(2)*
3**(3/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
+ (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(
1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1
/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)
) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(8645*b*x*sqrt(-a**(1/
3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a
 - b*x**2)**(1/3))**2)) + 114048*a**4*x/(8645*(a**(1/3)*(-1 + sqr
t(3)) + (a - b*x**2)**(1/3))) + 28512*a**3*x*(a - b*x**2)**(2/3)/
8645 + 14256*a**2*x*(a - b*x**2)**(5/3)/6175 - 306*a*x*(a - b*x**
2)**(8/3)/475 - 3*x*(a - b*x**2)**(8/3)*(3*a + b*x**2)/25
```

Mathematica [C] time = 0.0715376, size = 99, normalized size = 0.16

$$\frac{3\left(63360a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+66315a^4x-72370a^3bx^3-4956a^2b^2x^5+9282ab^3x^7+1729b^4x^9\right)}{43225\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (3*(66315*a^4*x - 72370*a^3*b*x^3 - 4956*a^2*b^2*x^5 + 9282*a*b^3
*x^7 + 1729*b^4*x^9 + 63360*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeom
etric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{5}{3}} \left(bx^2+3a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^2 \left(-bx^2+a\right)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^3x^6+5\,ab^2x^4+3\,a^2bx^2-9\,a^3\right)\left(-bx^2+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 5*a*b^2*x^4 + 3*a^2*b*x^2 - 9*a^3)*(-b*x^2 +
 a)^(2/3), x)
```

Sympy [A] time = 15.4531, size = 131, normalized size = 0.21

$$9a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)
/a) - a**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar
(2*I*pi)/a) - a**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**
2*exp_polar(2*I*pi)/a) - a**(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9
/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

3.118 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right) \, dx$

Optimal. Leaf size=608

$$\frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))
/247 - (3*x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400
*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(
2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi [A] time = 0.928471, antiderivative size = 608, normalized size of antiderivative = 1., number

of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))
/247 - (3*x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[
3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400
*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(
2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi in Sympy [A] time = 48.1472, size = 488, normalized size = 0.8

$$-\frac{3600\sqrt[4]{3}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{2400\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{7200a^3x}{1729\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x\left(a-bx^2\right)^{\frac{2}{3}}}{1729}+\frac{180ax\left(a-bx^2\right)^{\frac{5}{3}}}{247}-\frac{3x\left(a-bx^2\right)^{\frac{8}{3}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

```
[Out] -3600*3**(1/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**
(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x*
*2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3)
)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/
(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3)
)/(1729*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(
1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 2400*sqrt(2)*3*
*(3/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) +
(a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/
3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3
)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3))
- (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(1729*b*x*sqrt(-a**(1/3)
*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a -
 b*x**2)**(1/3))**2)) + 7200*a**3*x/(1729*(a**(1/3)*(-1 + sqrt(3)
) + (a - b*x**2)**(1/3))) + 1800*a**2*x*(a - b*x**2)**(2/3)/1729
+ 180*a*x*(a - b*x**2)**(5/3)/247 - 3*x*(a - b*x**2)**(8/3)/19
```

Mathematica [C] time = 0.0607574, size = 88, normalized size = 0.14

$$\frac{3\left(800a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+929a^3x-1167a^2bx^3+147ab^2x^5+91b^3x^7\right)}{1729\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (3*(929*a^3*x - 1167*a^2*b*x^3 + 147*a*b^2*x^5 + 91*b^3*x^7 + 800
*a^3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*
x^2)/a]))/(1729*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \left(-bx^2+a\right)^{\frac{5}{3}} \left(bx^2+3\,a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right) \left(-bx^2+a\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(b^2x^4+2\,abx^2-3\,a^2\right)\left(-bx^2+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] time = 10.4756, size = 100, normalized size = 0.16

$$3a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3}-\frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/
a) - 2*a**(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_pola
r(2*I*pi)/a)/3 - a**(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*
x**2*exp_polar(2*I*pi)/a)/5
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

3.119 $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx$

Optimal. Leaf size=765

$$\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}-\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a]
)/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sq
rt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/
6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^(1/4)
*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)])
```

Rubi [A] time = 1.07211, antiderivative size = 765, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}$$
$$-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}-\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a]
)/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sq
rt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/
6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^(1/4)
*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)])
```

Rubi in Sympy [A] time = 163.58, size = 811, normalized size = 1.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
```

```
[Out] 48*3**(1/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3
) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)*
*(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*el
liptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a*
*(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(7
*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-
1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) - 32*sqrt(2)*3**(3/4)*a*
*(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**
2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a
**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqr
t(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x
**2)**(1/3))), -7 + 4*sqrt(3))/(7*b*x*sqrt(-a**(1/3)*(a**(1/3) -
(a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/
3))**2)) + 2*2**(1/3)*a**(3/2)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)
*(1 - sqrt(b)*x/sqrt(a))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))
/(sqrt(b)*(a - b*x**2)**(1/3)) - 2*2**(1/3)*a**(3/2)*(1 - b*x**2/
a)**(1/3)*log((1 - sqrt(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt
(b)*x/sqrt(a))**(1/3))/(sqrt(b)*(a - b*x**2)**(1/3)) - 4*2**(1/3)
*sqrt(3)*a**(3/2)*(1 - b*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)
*sqrt(3)*(1 + sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a)
)**(1/3)))/(3*sqrt(b)*(a - b*x**2)**(1/3)) - 4*2**(1/3)*sqrt(3)*a
**(3/2)*(1 - b*x**2/a)**(1/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*
x/sqrt(a))**(2/3)/(3*(1 + sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)
/(3*sqrt(b)*(a - b*x**2)**(1/3)) - 96*a*x/(7*(a**(1/3)*(-1 + sqrt
(3)) + (a - b*x**2)**(1/3))) - 3*x*(a - b*x**2)**(2/3)/7
```

Mathematica [C] time = 0.272178, size = 333, normalized size = 0.44

$$\frac{x\left(\frac{144a^3F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}-\frac{160a^2bx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2}\ldots\right.}{(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2}\ldots\right.\right.\right.}\ldots\right.}{7\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (x*(-3*a + 3*b*x^2 + (144*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/
a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2
, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2
)/a, -(b*x^2)/(3*a)]))) - (160*a^2*b*x^2*AppellF1[3/2, 1/3, 1, 5/
```

```
2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(15*a*AppellF1[3/2,
 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[5/2
, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[5/2, 4/3, 1,
 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(7*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{b x^2 + 3 a} \left(-b x^2 + a\right)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^2 + a\right)^{\frac{5}{3}}}{b x^2 + 3 a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - b x^2\right)^{\frac{5}{3}}}{3 a + b x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{bx^2+3\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

3.120 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$

Optimal. Leaf size=775

$$\frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)}-\frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt
[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*A
rcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sq
rt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(
a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (11*Sq
rt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))^2)])
```

Rubi [A] time = 1.05508, antiderivative size = 775, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\ 3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{2x\left(a-bx^2\right)^{2/3}}{3\left(3a+bx^2\right)}-\frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt
[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*A
rcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sq
rt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(
a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (11*Sq
rt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 158.993, size = 814, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
```

```
[Out] -11*3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/
3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)
**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*e
lliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a
**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(
6*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(
-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 11*sqrt(2)*3**(3/4)*a
**(1/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x*
*2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(
a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sq
rt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*
x**2)**(1/3))), -7 + 4*sqrt(3))/(9*b*x*sqrt(-a**(1/3)*(a**(1/3) -
 (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1
/3))**2)) - 2**(1/3)*sqrt(a)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)*(
1 - sqrt(b)*x/sqrt(a))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))/(
2*sqrt(b)*(a - b*x**2)**(1/3)) + 2**(1/3)*sqrt(a)*(1 - b*x**2/a)*
*(1/3)*log((1 - sqrt(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt(b)
*x/sqrt(a))**(1/3))/(2*sqrt(b)*(a - b*x**2)**(1/3)) + 2**(1/3)*sq
rt(3)*sqrt(a)*(1 - b*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)*sqr
t(3)*(1 + sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a))**(
1/3)))/(3*sqrt(b)*(a - b*x**2)**(1/3)) + 2**(1/3)*sqrt(3)*sqrt(a)
*(1 - b*x**2/a)**(1/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(
a))**(2/3)/(3*(1 + sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)/(3*sqr
t(b)*(a - b*x**2)**(1/3)) + 2*x*(a - b*x**2)**(2/3)/(3*(3*a + b*x
**2)) + 11*x/(3*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3)))
```

Mathematica [C] time = 0.227754, size = 320, normalized size = 0.41

$$\frac{x\left(-\frac{27a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\frac{55abx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15a\cdots}\cdots\right)}{9\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(6*a - 6*b*x^2 - (27*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*
x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b
```

```
*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*
a)])) + (55*a*b*x^2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2
)/(3*a)])/(15*a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3
*a)] + 2*b*x^2*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(
3*a)] + AppellF1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))
/(9*(a - b*x^2)^(1/3)*(3*a + b*x^2))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+3 a\right)^{2}}\left(-b x^{2}+a\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-b x^{2}+a\right)^{\frac{5}{3}}}{\left(b x^{2}+3 a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a-bx^2\right)^{\frac{5}{3}}}{\left(3a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$

Optimal. Leaf size=815

$$-\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{3(bx^2+3a)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\,2^{2/3}a^{5/6}\sqrt{b}}$$
$$+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\,3^{3/4}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{9\sqrt{2}\sqrt[4]{3}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))
/(18*a*(3*a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*S
qrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3)*a^(5/6)*Sqrt[
b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)
^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1
/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3
) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/3)
 - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*S
qrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.38559, antiderivative size = 815, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$-\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)}+\frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{(a-bx^2)^{2/3}x}{3(bx^2+3a)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\,2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\,2^{2/3}a^{5/6}\sqrt{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\,3^{3/4}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{9\sqrt{2}\sqrt[4]{3}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))
/(18*a*(3*a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*S
qrt[3]*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3)*a^(5/6)*Sqrt[
b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)
^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1
/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3
) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/3)
 - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*S
qrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 50.4056, size = 78, normalized size = 0.1

$$\frac{x\left(a-bx^2\right)^{\frac{2}{3}}}{3\left(3a+bx^2\right)^2}+\frac{4bx^3\left(a-bx^2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{243a^3\left(1-\frac{bx^2}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)
```

```
[Out] x*(a - b*x**2)**(2/3)/(3*(3*a + b*x**2)**2) + 4*b*x**3*(a - b*x**
2)**(2/3)*appellf1(3/2, 1/3, 2, 5/2, b*x**2/a, -b*x**2/(3*a))/(24
3*a**3*(1 - b*x**2/a)**(2/3))
```

Mathematica [C] time = 0.435836, size = 346, normalized size = 0.42

$$\frac{x\left(-\frac{5bx^2(3a+bx^2)F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\frac{27a(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF}\right.}{54\sqrt[3]{a-bx^2}\left(3a+bx^2\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(9*a - 12*b*x^2 + (3*b^2*x^4)/a + (27*a*(3*a + b*x^2)*AppellF1
[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2,
 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2
, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1,
 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) - (5*b*x^2*(3*a + b*x^2)*Appel
lF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(15*a*AppellF1[
3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1
[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[5/2, 4/3
, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(54*(a - b*x^2)^(1/3)*(3
*a + b*x^2)^2)
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^3}\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^2+a\right)^{\frac{5}{3}}}{\left(bx^2+3\,a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3,x, algorithm="giac")

[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=659

$$\frac{1264896\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{1897344\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}$$
$$-\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^
2)^(2/3)*(3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2)^2)/19 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (37946
88*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1
897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a
^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2)]) + (1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.16484, antiderivative size = 659, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{1264896\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{1897344\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}$$

$$-\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^
2)^(2/3)*(3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2)^2)/19 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (37946
88*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1
897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a
^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))^2)]) + (1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 113.285, size = 537, normalized size = 0.81

$$-\frac{1897344\sqrt[4]{3}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{1264896\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{13}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3794688a^4x}{8645\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{2006208a^3x\left(a-bx^2\right)^{\frac{2}{3}}}{43225}$$
$$-\frac{27a^2x\left(a-bx^2\right)^{\frac{2}{3}}\left(1632a+1344bx^2\right)}{6175}-\frac{18ax\left(a-bx^2\right)^{\frac{2}{3}}\left(3a+bx^2\right)^2}{19}-\frac{3x\left(a-bx^2\right)^{\frac{2}{3}}\left(3a+bx^2\right)^3}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)
```

```
[Out] -1897344*3**(1/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2
)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b
*x**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1
/3))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3
))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt
(3))/(8645*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a
**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 1264896*sqrt
(2)*3**(3/4)*a**(13/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1
/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2
)**(1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a
**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqr
t(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(8645*b*x*sqrt(-a*
*(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3))
+ (a - b*x**2)**(1/3))**2)) + 3794688*a**4*x/(8645*(a**(1/3)*(-1
+ sqrt(3)) + (a - b*x**2)**(1/3))) - 2006208*a**3*x*(a - b*x**2)*
*(2/3)/43225 - 27*a**2*x*(a - b*x**2)**(2/3)*(1632*a + 1344*b*x**
2)/6175 - 18*a*x*(a - b*x**2)**(2/3)*(3*a + b*x**2)**2/19 - 3*x*(
a - b*x**2)**(2/3)*(3*a + b*x**2)**3/25
```

Mathematica [C] time = 0.102544, size = 98, normalized size = 0.15

$$\frac{3x\left(2108160a^4\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-941085a^4+727830a^3bx^2+184044a^2b^2x^4+27482ab^3x^6+1729b^4x^8\right)}{43225\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-941085*a^4 + 727830*a^3*b*x^2 + 184044*a^2*b^2*x^4 + 27482
*a*b^3*x^6 + 1729*b^4*x^8 + 2108160*a^4*(1 - (b*x^2)/a)^(1/3)*Hyp
ergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/
3))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^4\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^4}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^4x^8+12\,ab^3x^6+54\,a^2b^2x^4+108\,a^3bx^2+81\,a^4}{\left(-bx^2+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2
 + 81*a^4)/(-b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 12.5092, size = 165, normalized size = 0.25

$$81a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+36a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{54a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{12a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}+\frac{b^4x^9\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\\frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)
```

```
[Out] 81*a**(11/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)
/a) + 36*a**(8/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_pol
ar(2*I*pi)/a) + 54*a**(5/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b
*x**2*exp_polar(2*I*pi)/a)/5 + 12*a**(2/3)*b**3*x**7*hyper((1/3,
7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7 + b**4*x**9*hyper((1/
3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)/(9*a**(1/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^4}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

3.123 $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=628

$$\frac{71712\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729}$$
$$-\frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3
)*(3*a + b*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19
 - (215136*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))) - (107568*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a -
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(
1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.00488, antiderivative size = 628, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$\frac{71712\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729}$$
$$-\frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3
)*(3*a + b*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19
 - (215136*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))) - (107568*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a -
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipt
icE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(
1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 81.7608, size = 507, normalized size = 0.81

$$-\frac{107568\sqrt[4]{3}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{71712\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{10}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{215136a^3x}{1729\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x\left(a-bx^2\right)^{\frac{2}{3}}}{1729}$$
$$-\frac{324ax\left(a-bx^2\right)^{\frac{2}{3}}\left(3a+bx^2\right)}{247}-\frac{3x\left(a-bx^2\right)^{\frac{2}{3}}\left(3a+bx^2\right)^2}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
```

```
[Out] -107568*3**(1/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)
**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*
x**2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/
3))*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3)
)/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(
3))/(1729*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a*
*(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 71712*sqrt(2)
*3**(3/4)*a**(10/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
 + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(
1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3
)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(1729*b*x*sqrt(-a**(1
/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (
a - b*x**2)**(1/3))**2)) + 215136*a**3*x/(1729*(a**(1/3)*(-1 + sq
rt(3)) + (a - b*x**2)**(1/3))) - 15768*a**2*x*(a - b*x**2)**(2/3)
/1729 - 324*a*x*(a - b*x**2)**(2/3)*(3*a + b*x**2)/247 - 3*x*(a -
 b*x**2)**(2/3)*(3*a + b*x**2)**2/19
```

Mathematica [C] time = 0.0757102, size = 88, normalized size = 0.14

$$\frac{3\left(23904a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-8343a^3x+7041a^2bx^3+1211ab^2x^5+91b^3x^7\right)}{1729\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*(-8343*a^3*x + 7041*a^2*b*x^3 + 1211*a*b^2*x^5 + 91*b^3*x^7 +
23904*a^3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2
, (b*x^2)/a]))/(1729*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^3 \frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3a\right)^3}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3x^6+9\,ab^2x^4+27\,a^2bx^2+27\,a^3}{\left(-bx^2+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)/(-b*x^2
+ a)^(1/3), x)
```

Sympy [A] time = 8.90166, size = 129, normalized size = 0.21

$$27a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
```

```
[Out] 27*a**(8/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/
a) + 9*a**(5/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar
(2*I*pi)/a) + 9*a**(2/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x*
*2*exp_polar(2*I*pi)/a)/5 + b**3*x**7*hyper((1/3, 7/2), (9/2,), b
*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^3}{\left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

3.124 $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=597

$$\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

```
[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/13 - (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (1620*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(
1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.827428, antiderivative size = 597, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/13 - (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (1620*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(
1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 51.6719, size = 478, normalized size = 0.8

$$-\frac{1620\sqrt[4]{3}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{1080\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3240a^2x}{91\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{198ax\left(a-bx^2\right)^{\frac{2}{3}}}{91}-\frac{3x\left(a-bx^2\right)^{\frac{2}{3}}\left(3a+bx^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
```

```
[Out] -1620*3**(1/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(
1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**
2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))
*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(
-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))
/(91*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3
)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 1080*sqrt(2)*3**(3
/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a -
 b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))*
*2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1
 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a
 - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(91*b*x*sqrt(-a**(1/3)*(a**(
1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**
2)**(1/3))**2)) + 3240*a**2*x/(91*(a**(1/3)*(-1 + sqrt(3)) + (a -
 b*x**2)**(1/3))) - 198*a*x*(a - b*x**2)**(2/3)/91 - 3*x*(a - b*x
**2)**(2/3)*(3*a + b*x**2)/13
```

Mathematica [C] time = 0.0647463, size = 77, normalized size = 0.13

$$\frac{3\left(360a^2x\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-87a^2x+80abx^3+7b^2x^5\right)}{91\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*(-87*a^2*x + 80*a*b*x^3 + 7*b^2*x^5 + 360*a^2*x*(1 - (b*x^2)/a
)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*
x^2)^(1/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \left(bx^2+3\,a\right)^2 \frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4+6\,abx^2+9\,a^2}{\left(-bx^2+a\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)/(-b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 6.53675, size = 94, normalized size = 0.16

$$9a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+2a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
```

```
[Out] 9*a**(5/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a
) + 2*a**(2/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(
2*I*pi)/a) + b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar
(2*I*pi)/a)/(5*a**(1/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal. Leaf size=568

$$\frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(
1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.774975, antiderivative size = 568, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(
1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 33.0945, size = 450, normalized size = 0.79

$$-\frac{36\sqrt[4]{3}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{24\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{72ax}{7\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{3x\left(a-bx^2\right)^{\frac{2}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
```

```
[Out] -36*3**(1/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/
3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)
**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*e
lliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a
**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(
7*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(
-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 24*sqrt(2)*3**(3/4)*a
**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x*
*2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(
a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sq
rt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*
x**2)**(1/3))), -7 + 4*sqrt(3))/(7*b*x*sqrt(-a**(1/3)*(a**(1/3) -
 (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1
/3))**2)) + 72*a*x/(7*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1
/3))) - 3*x*(a - b*x**2)**(2/3)/7
```

Mathematica [C] time = 0.0531181, size = 62, normalized size = 0.11

$$\frac{3x\left(8a\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-a+bx^2\right)}{7\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-a + b*x^2 + 8*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/
3, 1/2, 3/2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (bx^2 + 3a)\frac{1}{\sqrt[3]{-bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 3a}{(-bx^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + 3a}{(-bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Sympy [A] time = 4.31079, size = 60, normalized size = 0.11

$$3a^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
```

```
[Out] 3*a**(2/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a
) + b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/
(3*a**(1/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)} \, dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTa
nh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2
/3)*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.127058, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```


```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTa
nh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2
/3)*a^(5/6)*Sqrt[b])
```

Rubi in Sympy [A] time = 73.7032, size = 355, normalized size = 1.74

$$\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\sqrt[3]{2}\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}+\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}}-\frac{\sqrt{3}}{3}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] 2**(1/3)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)*(1 - sqrt(b)*x/sqrt(a
))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))/(8*sqrt(a)*sqrt(b)*(a
 - b*x**2)**(1/3)) - 2**(1/3)*(1 - b*x**2/a)**(1/3)*log((1 - sqrt
(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt(b)*x/sqrt(a))**(1/3))/
(8*sqrt(a)*sqrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b
*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(b)*x/
sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a))**(1/3)))/(12*sqrt(a)*s
qrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1
/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 +
sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)/(12*sqrt(a)*sqrt(b)*(a -
b*x**2)**(1/3))
```

Mathematica [C] time = 0.063994, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((a
 - b*x^2)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*
x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b
*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a,
-(b*x^2)/(3*a)])))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{b x^2 + 3 a} \frac{1}{\sqrt[3]{-b x^2 + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)

[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^2 + 3 a)(-b x^2 + a)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a - b x^2} \, (3a + b x^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx$

Optimal. Leaf size=787

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(
Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*
2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/
(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(
1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b])
- (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.21714, antiderivative size = 787, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(
Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*
2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/
(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(
1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b])
- (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 151.13, size = 813, normalized size = 1.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
```

```
[Out] x*(a - b*x**2)**(2/3)/(24*a**2*(3*a + b*x**2)) + x/(24*a**2*(a**(
1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))) + 2**(1/3)*(1 - b*x**
2/a)**(1/3)*log(2**(1/3)*(1 - sqrt(b)*x/sqrt(a))**(1/3) + (1 + sq
rt(b)*x/sqrt(a))**(2/3))/(32*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/3)
) - 2**(1/3)*(1 - b*x**2/a)**(1/3)*log((1 - sqrt(b)*x/sqrt(a))**(
2/3) + 2**(1/3)*(1 + sqrt(b)*x/sqrt(a))**(1/3))/(32*a**(3/2)*sqrt
(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1/3)
*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(b)*x/sqrt(a))**(2/3)
/(3*(1 - sqrt(b)*x/sqrt(a))**(1/3)))/(48*a**(3/2)*sqrt(b)*(a - b*
x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1/3)*atan(2**(2
/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 + sqrt(b)*x/sqrt
(a))**(1/3)) - sqrt(3)/3)/(48*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/3
)) - 3**(1/4)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a
- b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))
**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_
e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*
(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(48*a**(5
/3)*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)
*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + sqrt(2)*3**(3/4)*sq
rt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**(2/3)
)/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a**(1/3) -
 (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (
a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/
3))), -7 + 4*sqrt(3))/(72*a**(5/3)*b*x*sqrt(-a**(1/3)*(a**(1/3) -
 (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1
/3))**2))
```

Mathematica [C] time = 0.282102, size = 322, normalized size = 0.41

$$\frac{x\left(\frac{\frac{5abx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+3a-3bx^2}{a^2}+\frac{189F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9a\right.}{72\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*((189*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(
9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x
```

```
^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + Appe
llF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) + (3*a - 3*b*
x^2 + (5*a*b*x^2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(
3*a)])/(15*a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)
] + 2*b*x^2*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a
)] + AppellF1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)])))/a^2
))/(72*(a - b*x^2)^(1/3)*(3*a + b*x^2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)^3}\,dx$

Optimal. Leaf size=818

$$\frac{5\left(a-bx^2\right)^{2/3}x}{288a^3\left(bx^2+3a\right)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3}x}{48a^2\left(bx^2+3a\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\,2^{2/3}a^{17/6}\sqrt{b}}$$
$$-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{192\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$+\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{144\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)
^(2/3))/(288*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[
b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[
3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(
144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh[(Sqrt[b]*x)/Sq
rt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]*x)/(
a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(
17/6)*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(192*3^(3/4)*a^(8/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^
(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.4196, antiderivative size = 818, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{5\left(a-bx^2\right)^{2/3}x}{288a^3\left(bx^2+3a\right)}-\frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\left(a-bx^2\right)^{2/3}x}{48a^2\left(bx^2+3a\right)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\,2^{2/3}a^{17/6}\sqrt{b}}$$
$$-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{192\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$+\frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{144\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)
^(2/3))/(288*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[
b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[
3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(
144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh[(Sqrt[b]*x)/Sq
rt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]*x)/(
a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(
17/6)*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(192*3^(3/4)*a^(8/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^
(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.386385, size = 352, normalized size = 0.43

$$\frac{x\left(\frac{675a^2(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\frac{25abx^2(3a+bx^2)F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1(\cdots)}\cdots\right)}{864a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(3*(a - b*x^2)*(21*a + 5*b*x^2) + (675*a^2*(3*a + b*x^2)*Appel
lF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*AppellF1[1
/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[
3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3,
 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) + (25*a*b*x^2*(3*a + b*x^2)
*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(15*a*App
ellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-Ap
pellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[5/
2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(864*a^3*(a - b*x^
2)^(1/3)*(3*a + b*x^2)^2)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+3a)^3}\frac{1}{\sqrt[3]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3a)^3(-bx^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3a)^3(-bx^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

3.129 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=623

$$-\frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2538}{91}ax\left(a-bx^2\right)^{2/3}+\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a +
b*x^2))/13 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2
*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) - (6696*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.07112, antiderivative size = 623, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2538}{91}ax\left(a-bx^2\right)^{2/3}+\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a +
b*x^2))/13 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2
*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))^2)]) - (6696*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1
/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 82.7068, size = 503, normalized size = 0.81

$$\frac{10044\sqrt[4]{3}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{6696\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{20088a^2x}{91\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{4050ax\left(a-bx^2\right)^{\frac{2}{3}}}{91}+\frac{9x\left(a-bx^2\right)^{\frac{2}{3}}\left(6a+18bx^2\right)}{26}+\frac{6x\left(3a+bx^2\right)^2}{\sqrt[3]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
```

```
[Out] 10044*3**(1/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(
1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**
2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))
*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(
-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))
/(91*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3
)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) - 6696*sqrt(2)*3**(3
/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a -
 b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))*
*2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1
 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a
 - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(91*b*x*sqrt(-a**(1/3)*(a**(
1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**
2)**(1/3))**2)) - 20088*a**2*x/(91*(a**(1/3)*(-1 + sqrt(3)) + (a
- b*x**2)**(1/3))) + 4050*a*x*(a - b*x**2)**(2/3)/91 + 9*x*(a - b
*x**2)**(2/3)*(6*a + 18*b*x**2)/26 + 6*x*(3*a + b*x**2)**2/(a - b
*x**2)**(1/3)
```

Mathematica [C] time = 0.0970422, size = 76, normalized size = 0.12

$$-\frac{3x\left(2232a^2\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-3051a^2+132abx^2+7b^2x^4\right)}{91\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-3051*a^2 + 132*a*b*x^2 + 7*b^2*x^4 + 2232*a^2*(1 - (b*x^2
)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a -
 b*x^2)^(1/3))
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int \left(b x^{2}+3 a\right)^{3} \left(-b x^{2}+a\right)^{-\frac{4}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(b x^{2} + 3 \, a\right)}^{3}}{{\left(-b x^{2} + a\right)}^{\frac{4}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-\frac{b^{3} x^{6} + 9 \, a b^{2} x^{4} + 27 \, a^{2} b x^{2} + 27 \, a^{3}}{{\left(b x^{2} - a\right)} {\left(-b x^{2} + a\right)}^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)/((b*x^2
 - a)*(-b*x^2 + a)^(1/3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a + bx^2\right)^3}{\left(a - bx^2\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(4/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^3}{\left(-bx^2 + a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=592

$$-\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}+\frac{45}{7}x\left(a-bx^2\right)^{2/3}$$

```
[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3
) + (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (
162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2
)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.917698, antiderivative size = 592, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}+\frac{45}{7}x\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3
) + (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (
162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*
(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2
)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 53.3254, size = 473, normalized size = 0.8

$$\frac{162\sqrt[4]{3}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{108\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{324ax}{7\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}+\frac{45x\left(a-bx^2\right)^{\frac{2}{3}}}{7}+\frac{6x\left(3a+bx^2\right)}{\sqrt[3]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
```

```
[Out] 162*3**(1/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/
3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)
**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*e
lliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a
**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(
7*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(
-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) - 108*sqrt(2)*3**(3/4)*
a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x
**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*
(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + s
qrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b
*x**2)**(1/3))), -7 + 4*sqrt(3))/(7*b*x*sqrt(-a**(1/3)*(a**(1/3)
- (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(
1/3))**2)) - 324*a*x/(7*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))) + 45*x*(a - b*x**2)**(2/3)/7 + 6*x*(3*a + b*x**2)/(a - b*
x**2)**(1/3)
```

Mathematica [C] time = 0.065432, size = 62, normalized size = 0.1

$$-\frac{3x\left(36a\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-57a+bx^2\right)}{7\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-57*a + b*x^2 + 36*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2
F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(bx^2 + 3\,a\right)^2 \left(-bx^2 + a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^2}{\left(-bx^2 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{b^2x^4 + 6\,abx^2 + 9\,a^2}{(bx^2 - a)(-bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 6*a*b*x^2 + 9*a^2)/((b*x^2 - a)*(-b*x^2 + a)
^(1/3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(4/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

3.131 $\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx$

Optimal. Leaf size=561

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3)) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt
[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*
Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 0.773649, antiderivative size = 561, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3)) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b
*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellip
ticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt
[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*
Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 32.7853, size = 442, normalized size = 0.79

$$\frac{9\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[3]{a}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{9x}{\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}}+\frac{6x}{\sqrt[3]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)
```

```
[Out] 9*3**(1/4)*a**(1/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3)
 + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*ell
iptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**
(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(2*
b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1
 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) - 3*sqrt(2)*3**(3/4)*a**(
1/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)
**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a**
(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(
3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**
2)**(1/3))), -7 + 4*sqrt(3))/(b*x*sqrt(-a**(1/3)*(a**(1/3) - (a -
 b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))*
*2)) - 9*x/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3)) + 6*x/
(a - b*x**2)**(1/3)
```

Mathematica [C] time = 0.0507215, size = 51, normalized size = 0.09

$$-\frac{3x\left(\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-2\right)}{\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-2 + (1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2
, (b*x^2)/a]))/(a - b*x^2)^(1/3)
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int \left(b x^{2}+3 a\right) \left(-b x^{2}+a\right)^{-\frac{4}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b x^{2} + 3 \, a}{{\left(-b x^{2} + a\right)}^{\frac{4}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(-\frac{b x^{2} + 3 \, a}{{\left(b x^{2} - a\right)} {\left(-b x^{2} + a\right)}^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)/((b*x^2 - a)*(-b*x^2 + a)^(1/3)), x)
```

Sympy [A] time = 9.81075, size = 60, normalized size = 0.11

$$\frac{3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[3]{a}}+\frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)
```

```
[Out] 3*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/3
) + b*x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/
(3*a**(4/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

3.132 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$

Optimal. Leaf size=776

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}-\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)
]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*
(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sq
rt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3
)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*3^(1/
4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)])
```

Rubi [A] time = 1.21561, antiderivative size = 776, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}-\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)
]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*
(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sq
rt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3
)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*3^(1/
4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(
1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)])
```

Rubi in Sympy [A] time = 150.704, size = 809, normalized size = 1.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
```

```
[Out] -3*x/(8*a**2*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))) + 3
*x/(8*a**2*(a - b*x**2)**(1/3)) + 2**(1/3)*(1 - b*x**2/a)**(1/3)*
log(2**(1/3)*(1 - sqrt(b)*x/sqrt(a))**(1/3) + (1 + sqrt(b)*x/sqrt
(a))**(2/3))/(32*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)
*(1 - b*x**2/a)**(1/3)*log((1 - sqrt(b)*x/sqrt(a))**(2/3) + 2**(1
/3)*(1 + sqrt(b)*x/sqrt(a))**(1/3))/(32*a**(3/2)*sqrt(b)*(a - b*x
**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1/3)*atan(sqrt(3
)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 - sqr
t(b)*x/sqrt(a))**(1/3)))/(48*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/3)
) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1/3)*atan(2**(2/3)*sqrt(3)*
(1 - sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 + sqrt(b)*x/sqrt(a))**(1/3))
 - sqrt(3)/3)/(48*a**(3/2)*sqrt(b)*(a - b*x**2)**(1/3)) + 3*3**(1
/4)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)*
*(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*sqrt(
sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_e(asin((a*
*(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt
(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(16*a**(5/3)*b*x*sq
rt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqr
t(3)) + (a - b*x**2)**(1/3))**2)) - sqrt(2)*3**(3/4)*sqrt((a**(2/
3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3
)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a**(1/3) - (a - b*x*
*2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2
)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 +
 4*sqrt(3))/(8*a**(5/3)*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**
2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2))
```

Mathematica [C] time = 0.332492, size = 325, normalized size = 0.42

$$\frac{x\left(\frac{3-\frac{5abx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\left(3a+bx^2\right)\left(2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}}{a^2}-\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\cdots\right.}{(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a}\cdots\right.\right.\right.}\cdots\right)}{8\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*((-9*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((
3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/
```

```
(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)
/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))
) + (3 - (5*a*b*x^2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2
)/(3*a)])/((3*a + b*x^2)*(15*a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)
/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2
)/a, -(b*x^2)/(3*a)] + AppellF1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*
x^2)/(3*a)]))))/a^2))/(8*(a - b*x^2)^(1/3))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2 + 3\,a} \left(-bx^2 + a\right)^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a - bx^2)^{\frac{4}{3}} (3a + bx^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

3.133 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

Optimal. Leaf size=807

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(bx^2+3a)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{6\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a +
 b*x^2)) + x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)))
 + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(1
7/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b
*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b])
- ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6)*Sqrt[b]) + Ar
cTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))
]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*El
lipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*3^(3/
4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/3) - (a -
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3
^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.38853, antiderivative size = 807, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{x}{12a^3\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(bx^2+3a)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8\,3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{6\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a +
 b*x^2)) + x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)))
 + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(1
7/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b
*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b])
- ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6)*Sqrt[b]) + Ar
cTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))
]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*El
lipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*3^(3/
4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/3) - (a -
b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3
^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.33416, size = 323, normalized size = 0.4

$$\frac{x\left(\frac{27a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}-\frac{10abx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+15aF_1\left(\right.}\right.}{72a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(21*a + 6*b*x^2 + (27*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a
, -(b*x^2)/(3*a)])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b
*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(
b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3
*a)])) - (10*a*b*x^2*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^
2)/(3*a)])/(15*a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(
3*a)] + 2*b*x^2*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/
(3*a)] + AppellF1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)])))
)/(72*a^3*(a - b*x^2)^(1/3)*(3*a + b*x^2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3a)^2(-bx^2 + a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3a)^2(-bx^2 + a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

3.134 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx$

Optimal. Leaf size=849

$$-\frac{19\left(a-bx^2\right)^{2/3}x}{1152a^4\left(bx^2+3a\right)}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{768\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{576\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a
 - b*x^2)^(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a
^4*(3*a + b*x^2)) + (19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288
*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*ArcTan[(Sqrt[3]*a^(1/6)*(
a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(288*2^(2/3)*
Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(864
*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(
1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b
]) + (19*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(
a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sqrt[2]*3^(1/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
```

```
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.73607, antiderivative size = 849, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{19\left(a-bx^2\right)^{2/3}x}{1152a^4\left(bx^2+3a\right)}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\left(bx^2+3a\right)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{x}{48a^2\sqrt[3]{a-bx^2}\left(bx^2+3a\right)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}+\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\,2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{768\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{576\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a
 - b*x^2)^(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a
^4*(3*a + b*x^2)) + (19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288
*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*ArcTan[(Sqrt[3]*a^(1/6)*(
a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(288*2^(2/3)*
Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(864
*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(
1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b
]) + (19*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^
(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*
```

```
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(
a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sqrt[2]*3^(1/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.391881, size = 353, normalized size = 0.42

$$\frac{\frac{999a^2x(3a+bx^2)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+819a^2x-\frac{95abx^3(3a+bx^2)F_1\left(\frac{3}{2};\frac{1}{3},1;\frac{5}{2};\frac{bx^2}{a}\right.}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right.}}{3456a^4\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (819*a^2*x + 420*a*b*x^3 + 57*b^2*x^5 + (999*a^2*x*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*Appel
lF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-Appe
llF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2,
 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) - (95*a*b*x^3*(3*a + b
*x^2)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(15*
a*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2
*(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + Appell
F1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)])))/(3456*a^4*(a -
 b*x^2)^(1/3)*(3*a + b*x^2)^2)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+3\,a)^3}\left(-bx^2+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+3a)^3(-bx^2+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3a)^3(-bx^2 + a)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=653

$$\frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{18468\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{9x\left(3a+bx^2\right)^2}{2\sqrt[3]{a-bx^2}}$$

$$-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a +
 b*x^2))/13 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*
(3*a + b*x^2)^3)/(2*(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (18468*3^(1/4)*Sqrt[2 +
Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12
312*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi [A] time = 1.30025, antiderivative size = 653, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{18468\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{9x\left(3a+bx^2\right)^2}{2\sqrt[3]{a-bx^2}}$$
$$-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a +
 b*x^2))/13 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*
(3*a + b*x^2)^3)/(2*(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (18468*3^(1/4)*Sqrt[2 +
Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^
2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12
312*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rubi in Sympy [A] time = 131.245, size = 527, normalized size = 0.81

$$-\frac{18468\sqrt[4]{3}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{12312\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{7}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{36936a^2x}{91\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{4374ax\left(a-bx^2\right)^{\frac{2}{3}}}{91}$$
$$-\frac{81x\left(a-bx^2\right)^{\frac{2}{3}}\left(a+bx^2\right)}{13}-\frac{9x\left(3a+bx^2\right)^2}{2\sqrt[3]{a-bx^2}}+\frac{3x\left(3a+bx^2\right)^3}{2\left(a-bx^2\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)
```

```
[Out] -18468*3**(1/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**
(1/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x*
*2)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3)
)*elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/
(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3)
)/(91*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/
3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 12312*sqrt(2)*3**
(3/4)*a**(7/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a
 - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3)
)**2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*
(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) -
(a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(91*b*x*sqrt(-a**(1/3)*(a*
*(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x
**2)**(1/3))**2)) + 36936*a**2*x/(91*(a**(1/3)*(-1 + sqrt(3)) + (
a - b*x**2)**(1/3))) - 4374*a*x*(a - b*x**2)**(2/3)/91 - 81*x*(a
- b*x**2)**(2/3)*(a + b*x**2)/13 - 9*x*(3*a + b*x**2)**2/(2*(a -
b*x**2)**(1/3)) + 3*x*(3*a + b*x**2)**3/(2*(a - b*x**2)**(4/3))
```

Mathematica [C] time = 0.122663, size = 96, normalized size = 0.15

$$-\frac{3\left(1647a^3x-4104a^2x\left(a-bx^2\right)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)-4743a^2bx^3+177ab^2x^5+7b^3x^7\right)}{91\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3*(1647*a^3*x - 4743*a^2*b*x^3 + 177*a*b^2*x^5 + 7*b^3*x^7 - 41
04*a^2*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3,
 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*x^2)^(4/3))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \left(bx^2+3a\right)^4\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3a\right)^4}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^4x^8+12\,ab^3x^6+54\,a^2b^2x^4+108\,a^3bx^2+81\,a^4}{(b^2x^4-2\,abx^2+a^2)(-bx^2+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2
 + 81*a^4)/((b^2*x^4 - 2*a*b*x^2 + a^2)*(-b*x^2 + a)^(1/3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a + bx^2\right)^4}{\left(a - bx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**4/(a - b*x**2)**(7/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3\,a\right)^4}{\left(-bx^2 + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=596

$$\frac{108\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{27}{14}x\left(a-bx^2\right)^{2/3}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^
2)^(4/3)) - (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))) - (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2
)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) -
(a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*
Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.02787, antiderivative size = 596, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{27}{14}x\left(a-bx^2\right)^{2/3}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^
2)^(4/3)) - (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))) - (162*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x
^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2
)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((
a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) -
(a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*
Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [A] time = 66.3547, size = 476, normalized size = 0.8

$$-\frac{162\sqrt[4]{3}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}+\frac{108\sqrt{2}\cdot 3^{\frac{3}{4}}a^{\frac{4}{3}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)^2}}}+\frac{324ax}{7\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^2}\right)}-\frac{27x\left(a-bx^2\right)^{\frac{2}{3}}}{14}+\frac{3x\left(3a+bx^2\right)^2}{2\left(a-bx^2\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
```

```
[Out] -162*3**(1/4)*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1
/3) + (a - b*x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2
)**(1/3))**2)*sqrt(sqrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*
elliptic_e(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-
a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/
(7*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*
(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)) + 108*sqrt(2)*3**(3/4)
*a**(4/3)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*
x**2)**(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)
*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 +
sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a -
b*x**2)**(1/3))), -7 + 4*sqrt(3))/(7*b*x*sqrt(-a**(1/3)*(a**(1/3)
 - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**
(1/3))**2)) + 324*a*x/(7*(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)*
*(1/3))) - 27*x*(a - b*x**2)**(2/3)/14 + 3*x*(3*a + b*x**2)**2/(2
*(a - b*x**2)**(4/3))
```

Mathematica [C] time = 0.107591, size = 83, normalized size = 0.14

$$\frac{81a^2x+108ax\left(a-bx^2\right)\sqrt[3]{1-\frac{bx^2}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^2}{a}\right)+90abx^3-3b^2x^5}{7\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (81*a^2*x + 90*a*b*x^3 - 3*b^2*x^5 + 108*a*x*(a - b*x^2)*(1 - (b*
x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(7*(a
- b*x^2)^(4/3))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \left(bx^2 + 3a\right)^3 \left(-bx^2 + a\right)^{-\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + 3a\right)^3}{\left(-bx^2 + a\right)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3x^6 + 9\,ab^2x^4 + 27\,a^2bx^2 + 27\,a^3}{(b^2x^4 - 2\,abx^2 + a^2)(-bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)/((b^2*x^
4 - 2*a*b*x^2 + a^2)*(-b*x^2 + a)^(1/3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3a + bx^2)^3}{(a - bx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(7/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + 3\,a)^3}{(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=44

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

```
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^
(4/3))
```

Rubi [A] time = 0.0511061, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}+\frac{9x}{2\sqrt[3]{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^
(4/3))
```

Rubi in Sympy [A] time = 21.476, size = 37, normalized size = 0.84

$$\frac{9x}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)}{2(a-bx^2)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
```

```
[Out] 9*x/(2*(a - b*x**2)**(1/3)) + 3*x*(3*a + b*x**2)/(2*(a - b*x**2)*
*(4/3))
```

Mathematica [A] time = 0.0381237, size = 24, normalized size = 0.55

$$\frac{9ax-3bx^3}{(a-bx^2)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
```

```
[Out] (9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

Maple [A] time = 0.008, size = 24, normalized size = 0.6

$$3\,\frac{x\left(-bx^2+3\,a\right)}{\left(-bx^2+a\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x)
```

```
[Out] 3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

Maxima [A] time = 1.48278, size = 45, normalized size = 1.02

$$\frac{3\left(bx^3-3\,ax\right)}{\left(bx^2-a\right)\left(-bx^2+a\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

Fricas [A] time = 0.261818, size = 57, normalized size = 1.3

$$-\frac{3\left(bx^3-3\,ax\right)\left(-bx^2+a\right)^{\frac{2}{3}}}{b^2x^4-2\,abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+3\,a\right)^2}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

3.138 $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$

Optimal. Leaf size=590

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

```
[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*
x)/(4*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)
*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) - (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rubi [A] time = 0.790696, antiderivative size = 590, normalized size of antiderivative = 1., number

of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*
x)/(4*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)
*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) - (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1
/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rubi in Sympy [A] time = 42.61, size = 466, normalized size = 0.79

$$\frac{3x}{2\left(a-bx^{2}\right)^{\frac{4}{3}}}-\frac{9x}{4a\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^{2}}\right)}+\frac{9x}{4a\sqrt[3]{a-bx^{2}}}+\frac{9\sqrt[4]{3}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^{2}}+\left(a-bx^{2}\right)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^{2}}\right)^{2}}}\sqrt{\sqrt{3}+2}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^{2}}\right)E\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^{2}}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^{2}}}\right)\middle|-7+4\sqrt{3}\right)}{8a^{\frac{2}{3}}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^{2}}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^{2}}\right)^{2}}}}-\frac{3\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{a-bx^{2}}+\left(a-bx^{2}\right)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^{2}}\right)^{2}}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^{2}}\right)F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\left(1+\sqrt{3}\right)-\sqrt[3]{a-bx^{2}}}{-\sqrt[3]{a}\left(-1+\sqrt{3}\right)-\sqrt[3]{a-bx^{2}}}\right)\middle|-7+4\sqrt{3}\right)}{4a^{\frac{2}{3}}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^{2}}\right)}{\left(\sqrt[3]{a}\left(-1+\sqrt{3}\right)+\sqrt[3]{a-bx^{2}}\right)^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

```
[Out] 3*x/(2*(a - b*x**2)**(4/3)) - 9*x/(4*a*(a**(1/3)*(-1 + sqrt(3)) +
 (a - b*x**2)**(1/3))) + 9*x/(4*a*(a - b*x**2)**(1/3)) + 9*3**(1/
4)*sqrt((a**(2/3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**
(2/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*sqrt(s
qrt(3) + 2)*(a**(1/3) - (a - b*x**2)**(1/3))*elliptic_e(asin((a**
(1/3)*(1 + sqrt(3)) - (a - b*x**2)**(1/3))/(-a**(1/3)*(-1 + sqrt(
3)) - (a - b*x**2)**(1/3))), -7 + 4*sqrt(3))/(8*a**(2/3)*b*x*sqrt
(-a**(1/3)*(a**(1/3) - (a - b*x**2)**(1/3))/(a**(1/3)*(-1 + sqrt(
3)) + (a - b*x**2)**(1/3))**2)) - 3*sqrt(2)*3**(3/4)*sqrt((a**(2/
3) + a**(1/3)*(a - b*x**2)**(1/3) + (a - b*x**2)**(2/3))/(a**(1/3
)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2)*(a**(1/3) - (a - b*x*
*2)**(1/3))*elliptic_f(asin((a**(1/3)*(1 + sqrt(3)) - (a - b*x**2
)**(1/3))/(-a**(1/3)*(-1 + sqrt(3)) - (a - b*x**2)**(1/3))), -7 +
 4*sqrt(3))/(4*a**(2/3)*b*x*sqrt(-a**(1/3)*(a**(1/3) - (a - b*x**
2)**(1/3))/(a**(1/3)*(-1 + sqrt(3)) + (a - b*x**2)**(1/3))**2))
```

Mathematica [C] time = 0.0871279, size = 74, normalized size = 0.13

$$\frac{-3x\left(a-bx^{2}\right)\sqrt[3]{1-\frac{bx^{2}}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{bx^{2}}{a}\right)+15ax-9bx^{3}}{4a\left(a-bx^{2}\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (15*a*x - 9*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hyperge
ometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(4*a*(a - b*x^2)^(4/3))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int (bx^2 + 3a) \left(-bx^2 + a\right)^{-\frac{7}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 3a}{(-bx^2 + a)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + 3a}{(b^2x^4 - 2abx^2 + a^2)(-bx^2 + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)/((b^2*x^4 - 2*a*b*x^2 + a^2)*(-b*x^2 + a)^
(1/3)), x)
```

Sympy [A] time = 36.9502, size = 60, normalized size = 0.1

$$\frac{3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{4}{3}}}+\frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

```
[Out] 3*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(4/3
) + b*x**3*hyper((3/2, 7/3), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/
(3*a**(7/3))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + 3\,a}{(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

3.139 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$

Optimal. Leaf size=796

$$\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$+\frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{128a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$-\frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{32\sqrt{2}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/
3)) + (21*x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)))
 + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(1
7/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b
*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b])
- ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6)*Sqrt[b]) + Ar
cTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))
]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7*3^(3/4)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(32*Sqrt[2]*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.29136, antiderivative size = 796, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{32a^2(a-bx^2)^{4/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\,2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\,2^{2/3}a^{17/6}\sqrt{b}}+\frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{128a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}-\frac{7\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{32\sqrt{2}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/
3)) + (21*x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)))
 + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(1
7/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b
*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b])
- ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6)*Sqrt[b]) + Ar
cTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))
]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7*3^(3/4)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(32*Sqrt[2]*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.327704, size = 347, normalized size = 0.44

$$\frac{x\left(-\frac{153a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}-\frac{35abx^2F_1\left(\frac{3}{2};\frac{1}{3},1;\right.}{(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{5}{2};\frac{1}{3},2\right.\right.\right.}\right.}{64a^3\sqrt[3]{a-bx^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*((3*(9*a - 7*b*x^2))/(a - b*x^2) - (153*a^2*AppellF1[1/2, 1/3,
 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(9*a*AppellF1
[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF
1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/
3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)]))) - (35*a*b*x^2*AppellF1[3
/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((3*a + b*x^2)*(15*a
*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*
(-AppellF1[5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF
1[5/2, 4/3, 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)])))))/(64*a^3*(a -
b*x^2)^(1/3))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+3\,a}\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a - bx^2)^{\frac{7}{3}}\,(3a + bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)

[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

3.140 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx$

Optimal. Leaf size=827

$$\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(bx^2+3a\right)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{5x}{384a^3\left(a-bx^2\right)^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{512\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}-\frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{384\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(
1/3)) + x/(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*
a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTa
n[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[b])
+ (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) - ArcTanh[(
Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcTanh[(
Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128
*2^(2/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(512*3^(3/
4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqr
t[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi [A] time = 1.52737, antiderivative size = 827, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{79x}{768a^4\sqrt[3]{a-bx^2}}+\frac{x}{24a^2(a-bx^2)^{4/3}(bx^2+3a)}+\frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{5x}{384a^3(a-bx^2)^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}$$
$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\,2^{2/3}a^{23/6}\sqrt{b}}$$
$$+\frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{512\,3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$-\frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{384\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(
1/3)) + x/(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*
a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTa
n[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[b])
+ (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) - ArcTanh[(
Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcTanh[(
Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128
*2^(2/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a -
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(512*3^(3/
4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*El
```

```
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqr
t[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.427194, size = 346, normalized size = 0.42

$$\frac{x\left(-\frac{1161a^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}+\frac{897a^2-444abx^2-237b^2x^4}{a-bx^2}-\frac{395ab\ldots}{2bx^2\left(F_1\left(\frac{5}{2};\frac{4}{3},1;\frac{7}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F\ldots}\right.}{2304a^4\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*((897*a^2 - 444*a*b*x^2 - 237*b^2*x^4)/(a - b*x^2) - (1161*a^2
*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(9*a*Appe
llF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-App
ellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2
, 4/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])) - (395*a*b*x^2*Appell
F1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)])/(15*a*AppellF1[3
/2, 1/3, 1, 5/2, (b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[
5/2, 1/3, 2, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[5/2, 4/3,
 1, 7/2, (b*x^2)/a, -(b*x^2)/(3*a)]))))/(2304*a^4*(a - b*x^2)^(1/
3)*(3*a + b*x^2))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+3\,a\right)^2}\left(-bx^2+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)

[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)^2(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)

[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)^2(-bx^2 + a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

3.141 $\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx$

Optimal. Leaf size=252

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1
/3)*Sqrt[a]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1
/3)*(-a + b*x^2)^(1/3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[
3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*
2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - ArcTanh[((-a)^(1/3)*Sqrt[b]
*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1/3)))]/(2*2^(2/
3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.191675, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\,2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\,2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```


```
[Out] -ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1
/3)*Sqrt[a]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1
/3)*(-a + b*x^2)^(1/3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[
3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*
2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) - ArcTanh[((-a)^(1/3)*Sqrt[b]
*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^(1/3)))]/(2*2^(2/
3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 90.6388, size = 355, normalized size = 1.41

$$-\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\sqrt[3]{2}\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}+\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{-a+bx^2}}+\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{-a+bx^2}}+\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{-a+bx^2}}+\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}}-\frac{\sqrt{3}}{3}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{-a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

```
[Out] -2**(1/3)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)*(1 - sqrt(b)*x/sqrt(
a))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))/(8*sqrt(a)*sqrt(b)*(
-a + b*x**2)**(1/3)) + 2**(1/3)*(1 - b*x**2/a)**(1/3)*log((1 - sq
rt(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt(b)*x/sqrt(a))**(1/3)
)/(8*sqrt(a)*sqrt(b)*(-a + b*x**2)**(1/3)) + 2**(1/3)*sqrt(3)*(1
- b*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(b)
*x/sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a))**(1/3)))/(12*sqrt(a
)*sqrt(b)*(-a + b*x**2)**(1/3)) + 2**(1/3)*sqrt(3)*(1 - b*x**2/a)
**(1/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(a))**(2/3)/(3*(
1 + sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)/(12*sqrt(a)*sqrt(b)*(
-a + b*x**2)**(1/3))
```

Mathematica [C] time = 0.250457, size = 163, normalized size = 0.65

$$-\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{bx^2-a}\,(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] (-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((
-a + b*x^2)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (
b*x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2,
```

```
(b*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a
, -(b*x^2)/(3*a)])))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-3a}\frac{1}{\sqrt[3]{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2+3a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
```

```
[Out] -Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/
3)), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2+3\,a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

Optimal. Leaf size=202

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan
[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*
2^(2/3)*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]
/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(
a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqr
t[3]*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.103011, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan
[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*
2^(2/3)*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]
/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(
a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqr
t[3]*a^(5/6)*Sqrt[b])
```

Rubi in Sympy [A] time = 35.6769, size = 348, normalized size = 1.72

$$\frac{\sqrt[3]{2}\sqrt{3}\log\left(\sqrt{3}-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{\frac{5}{6}}\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(\sqrt{3}+\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{\frac{5}{6}}\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt{3}\log\left(\sqrt{3}b-\frac{b^{\frac{3}{2}}x}{\sqrt{a}}-\frac{\sqrt[3]{2}\sqrt{3}b\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{24a^{\frac{5}{6}}\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(\sqrt{3}b+\frac{b^{\frac{3}{2}}x}{\sqrt{a}}-\frac{\sqrt[3]{2}\sqrt{3}b\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}\right)}{24a^{\frac{5}{6}}\sqrt{b}}-\frac{\sqrt[3]{2}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}\sqrt{a}-\sqrt{b}x\right)}{3\sqrt[6]{a}\sqrt[3]{a+bx^2}}\right)}{12a^{\frac{5}{6}}\sqrt{b}}+\frac{\sqrt[3]{2}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\left(\sqrt{3}\sqrt{a}+\sqrt{b}x\right)}{3\sqrt[6]{a}\sqrt[3]{a+bx^2}}\right)}{12a^{\frac{5}{6}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

```
[Out] 2**(1/3)*sqrt(3)*log(sqrt(3) - sqrt(b)*x/sqrt(a))/(24*a**(5/6)*sq
rt(b)) - 2**(1/3)*sqrt(3)*log(sqrt(3) + sqrt(b)*x/sqrt(a))/(24*a*
*(5/6)*sqrt(b)) + 2**(1/3)*sqrt(3)*log(sqrt(3)*b - b**(3/2)*x/sqr
t(a) - 2**(1/3)*sqrt(3)*b*(a + b*x**2)**(1/3)/a**(1/3))/(24*a**(5
/6)*sqrt(b)) - 2**(1/3)*sqrt(3)*log(sqrt(3)*b + b**(3/2)*x/sqrt(a
) - 2**(1/3)*sqrt(3)*b*(a + b*x**2)**(1/3)/a**(1/3))/(24*a**(5/6)
*sqrt(b)) - 2**(1/3)*atan(sqrt(3)/3 + 2**(2/3)*(sqrt(3)*sqrt(a) -
 sqrt(b)*x)/(3*a**(1/6)*(a + b*x**2)**(1/3)))/(12*a**(5/6)*sqrt(b
)) + 2**(1/3)*atan(sqrt(3)/3 + 2**(2/3)*(sqrt(3)*sqrt(a) + sqrt(b
)*x)/(3*a**(1/6)*(a + b*x**2)**(1/3)))/(12*a**(5/6)*sqrt(b))
```

Mathematica [C] time = 0.233952, size = 166, normalized size = 0.82

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{(3a-bx^2)\sqrt[3]{a+bx^2}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/(
(3*a - b*x^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -
((b*x^2)/a), (b*x^2)/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2,
 -((b*x^2)/a), (b*x^2)/(3*a)] - AppellF1[3/2, 4/3, 1, 5/2, -((b*x
^2)/a), (b*x^2)/(3*a)])))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^{2}+3\,a}\frac{1}{\sqrt[3]{bx^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^{2}+a)^{\frac{1}{3}}(bx^{2}-3\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-3a\sqrt[3]{a+bx^{2}}+bx^{2}\sqrt[3]{a+bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
```

```
[Out] -Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2+a)^{\frac{1}{3}}(bx^2-3\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

```
[Out] -ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*S
qrt[d]) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) +
 2^(1/3)*(c + 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - Ar
cTanh[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[
(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2
*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi [A] time = 0.122218, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```


```
[Out] -ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*S
qrt[d]) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) +
 2^(1/3)*(c + 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - Ar
cTanh[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[
(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2
*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi in Sympy [A] time = 38.6386, size = 376, normalized size = 1.84

$$\frac{\sqrt[3]{2}\log\left(\sqrt{3}-\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{6}}\sqrt{d}}-\frac{\sqrt[3]{2}\log\left(\sqrt{3}+\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{8c^{\frac{5}{6}}\sqrt{d}}+\frac{\sqrt[3]{2}\log\left(3\sqrt{3}d-\frac{3\sqrt{3}d^{\frac{3}{2}}x}{\sqrt{c}}-\frac{3\sqrt[3]{2}\sqrt{3}d\sqrt[3]{c+3dx^2}}{\sqrt[3]{c}}\right)}{8c^{\frac{5}{6}}\sqrt{d}}-\frac{\sqrt[3]{2}\log\left(3\sqrt{3}d+\frac{3\sqrt{3}d^{\frac{3}{2}}x}{\sqrt{c}}-\frac{3\sqrt[3]{2}\sqrt{3}d\sqrt[3]{c+3dx^2}}{\sqrt[3]{c}}\right)}{8c^{\frac{5}{6}}\sqrt{d}}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\sqrt{3}\left(\sqrt{c}-\sqrt{d}x\right)}{3\sqrt[6]{c}\sqrt[3]{c+3dx^2}}\right)}{12c^{\frac{5}{6}}\sqrt{d}}+\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}+\frac{2^{\frac{2}{3}}\sqrt{3}\left(\sqrt{c}+\sqrt{d}x\right)}{3\sqrt[6]{c}\sqrt[3]{c+3dx^2}}\right)}{12c^{\frac{5}{6}}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
```

```
[Out] 2**(1/3)*log(sqrt(3) - sqrt(3)*sqrt(d)*x/sqrt(c))/(8*c**(5/6)*sqr
t(d)) - 2**(1/3)*log(sqrt(3) + sqrt(3)*sqrt(d)*x/sqrt(c))/(8*c**(
5/6)*sqrt(d)) + 2**(1/3)*log(3*sqrt(3)*d - 3*sqrt(3)*d**(3/2)*x/s
qrt(c) - 3*2**(1/3)*sqrt(3)*d*(c + 3*d*x**2)**(1/3)/c**(1/3))/(8*
c**(5/6)*sqrt(d)) - 2**(1/3)*log(3*sqrt(3)*d + 3*sqrt(3)*d**(3/2)
*x/sqrt(c) - 3*2**(1/3)*sqrt(3)*d*(c + 3*d*x**2)**(1/3)/c**(1/3))
/(8*c**(5/6)*sqrt(d)) - 2**(1/3)*sqrt(3)*atan(sqrt(3)/3 + 2**(2/3
)*sqrt(3)*(sqrt(c) - sqrt(d)*x)/(3*c**(1/6)*(c + 3*d*x**2)**(1/3)
))/(12*c**(5/6)*sqrt(d)) + 2**(1/3)*sqrt(3)*atan(sqrt(3)/3 + 2**(
2/3)*sqrt(3)*(sqrt(c) + sqrt(d)*x)/(3*c**(1/6)*(c + 3*d*x**2)**(1
/3)))/(12*c**(5/6)*sqrt(d))
```

Mathematica [C] time = 0.231827, size = 153, normalized size = 0.75

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{(c-dx^2)\sqrt[3]{c+3dx^2}\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3dx^2}{c},\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c -
 d*x^2)*(c + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d
*x^2)/c, (d*x^2)/c] + 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x
^2)/c, (d*x^2)/c] - AppellF1[3/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x
^2)/c])))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{-d x^2 + c} \frac{1}{\sqrt[3]{3\, d x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

```
[Out] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\, d x^2 + c)^{\frac{1}{3}} (d x^2 - c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-c\sqrt[3]{c + 3dx^2} + dx^2\sqrt[3]{c + 3dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
```

```
[Out] -Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1
/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3\,dx^2+c)^{\frac{1}{3}}(dx^2-c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTa
nh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2
/3)*a^(5/6)*Sqrt[b])
```

Rubi [A] time = 0.101798, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTa
nh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(2*2^(2
/3)*a^(5/6)*Sqrt[b])
```

Rubi in Sympy [A] time = 75.3001, size = 355, normalized size = 1.74

$$\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\sqrt[3]{2}\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}+\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt[3]{1-\frac{bx^2}{a}}\log\left(\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}\right)}{8\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(1+\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1-\frac{\sqrt{b}x}{\sqrt{a}}}}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}-\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1+\frac{\sqrt{b}x}{\sqrt{a}}}}-\frac{\sqrt{3}}{3}\right)}{12\sqrt{a}\sqrt{b}\sqrt[3]{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] 2**(1/3)*(1 - b*x**2/a)**(1/3)*log(2**(1/3)*(1 - sqrt(b)*x/sqrt(a
))**(1/3) + (1 + sqrt(b)*x/sqrt(a))**(2/3))/(8*sqrt(a)*sqrt(b)*(a
 - b*x**2)**(1/3)) - 2**(1/3)*(1 - b*x**2/a)**(1/3)*log((1 - sqrt
(b)*x/sqrt(a))**(2/3) + 2**(1/3)*(1 + sqrt(b)*x/sqrt(a))**(1/3))/
(8*sqrt(a)*sqrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b
*x**2/a)**(1/3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(b)*x/
sqrt(a))**(2/3)/(3*(1 - sqrt(b)*x/sqrt(a))**(1/3)))/(12*sqrt(a)*s
qrt(b)*(a - b*x**2)**(1/3)) - 2**(1/3)*sqrt(3)*(1 - b*x**2/a)**(1
/3)*atan(2**(2/3)*sqrt(3)*(1 - sqrt(b)*x/sqrt(a))**(2/3)/(3*(1 +
sqrt(b)*x/sqrt(a))**(1/3)) - sqrt(3)/3)/(12*sqrt(a)*sqrt(b)*(a -
b*x**2)**(1/3))
```

Mathematica [C] time = 0.0670864, size = 162, normalized size = 0.79

$$\frac{9axF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)+9aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},-\frac{bx^2}{3a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -(b*x^2)/(3*a)])/((a
 - b*x^2)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*
x^2)/a, -(b*x^2)/(3*a)] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b
*x^2)/a, -(b*x^2)/(3*a)] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a,
-(b*x^2)/(3*a)])))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b x^{2}+3 a}\frac{1}{\sqrt[3]{-b x^{2}+a}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^{2}+3 a)(-b x^{2}+a)^{\frac{1}{3}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a-b x^{2}}\,(3a+b x^{2})}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
```

```
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 3\,a)(-bx^2 + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}(c+dx^2)} \, dx$

Optimal. Leaf size=204

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[
(c^(1/6)*(c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2
*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/
(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*S
qrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c - 3*d*x^2)^(1/3)))])/(2*
2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi [A] time = 0.0975356, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[
(c^(1/6)*(c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2
*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/
(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*S
qrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c - 3*d*x^2)^(1/3)))])/(2*
2^(2/3)*c^(5/6)*Sqrt[d])
```

Rubi in Sympy [A] time = 85.0538, size = 410, normalized size = 2.01

$$\frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{3dx^2}{c}}\log\left(\sqrt[3]{2}\sqrt[3]{1-\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}}+\left(1+\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)^{\frac{2}{3}}\right)}{8\sqrt{c}\sqrt{d}\sqrt[3]{c-3dx^2}} - \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{1-\frac{3dx^2}{c}}\log\left(\left(1-\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{1+\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}}\right)}{8\sqrt{c}\sqrt{d}\sqrt[3]{c-3dx^2}} - \frac{\sqrt[3]{2}\sqrt[3]{1-\frac{3dx^2}{c}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}\left(1+\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1-\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}}}\right)}{4\sqrt{c}\sqrt{d}\sqrt[3]{c-3dx^2}} - \frac{\sqrt[3]{2}\sqrt[3]{1-\frac{3dx^2}{c}}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\left(1-\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)^{\frac{2}{3}}}{3\sqrt[3]{1+\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}}}-\frac{\sqrt{3}}{3}\right)}{4\sqrt{c}\sqrt{d}\sqrt[3]{c-3dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

```
[Out] 2**(1/3)*sqrt(3)*(1 - 3*d*x**2/c)**(1/3)*log(2**(1/3)*(1 - sqrt(3
)*sqrt(d)*x/sqrt(c))**(1/3) + (1 + sqrt(3)*sqrt(d)*x/sqrt(c))**(2
/3))/(8*sqrt(c)*sqrt(d)*(c - 3*d*x**2)**(1/3)) - 2**(1/3)*sqrt(3)
*(1 - 3*d*x**2/c)**(1/3)*log((1 - sqrt(3)*sqrt(d)*x/sqrt(c))**(2/
3) + 2**(1/3)*(1 + sqrt(3)*sqrt(d)*x/sqrt(c))**(1/3))/(8*sqrt(c)*
sqrt(d)*(c - 3*d*x**2)**(1/3)) - 2**(1/3)*(1 - 3*d*x**2/c)**(1/3)
*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(1 + sqrt(3)*sqrt(d)*x/sqrt(c)
)**(2/3)/(3*(1 - sqrt(3)*sqrt(d)*x/sqrt(c))**(1/3)))/(4*sqrt(c)*s
qrt(d)*(c - 3*d*x**2)**(1/3)) - 2**(1/3)*(1 - 3*d*x**2/c)**(1/3)*
atan(2**(2/3)*sqrt(3)*(1 - sqrt(3)*sqrt(d)*x/sqrt(c))**(2/3)/(3*(
1 + sqrt(3)*sqrt(d)*x/sqrt(c))**(1/3)) - sqrt(3)/3)/(4*sqrt(c)*sq
rt(d)*(c - 3*d*x**2)**(1/3))
```

Mathematica [C] time = 0.235873, size = 156, normalized size = 0.76

$$\frac{3cxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\,(c+dx^2)\left(2dx^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)+3cF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3dx^2}{c},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c
 - 3*d*x^2)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*
```

```
d*x^2)/c, -((d*x^2)/c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3
*d*x^2)/c, -((d*x^2)/c)] + AppellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c
, -((d*x^2)/c)])))
```

Maple [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\frac{1}{\sqrt[3]{-3\,dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

```
[Out] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+c)(-3\,dx^2+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
```

```
[Out] Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2 + c)(-3\,dx^2 + c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=113

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi [A] time = 0.0484057, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}(x)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1
/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2
/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rubi in Sympy [A] time = 13.9152, size = 144, normalized size = 1.27

$$\frac{\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{8}-\frac{\sqrt[3]{2}\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{8}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 2**(1/3)*log(2**(1/3)*(-x + 1)**(1/3) + (x + 1)**(2/3))/8 - 2**(1
/3)*log((-x + 1)**(2/3) + 2**(1/3)*(x + 1)**(1/3))/8 - 2**(1/3)*s
```

```
qrt(3)*atan(sqrt(3)/3 - 2**(2/3)*sqrt(3)*(x + 1)**(2/3)/(3*(-x +
1)**(1/3)))/12 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-x + 1)*
*(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/12
```

Mathematica [C] time = 0.16937, size = 118, normalized size = 1.04

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/((1 - x^2)^(1/3)*(
3 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, x^2, -x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 x^2, -x^2/3])))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/
(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(S
qrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi [A] time = 0.0456091, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\tan^{-1}(x)}{6\ 2^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -ArcTan[x]/(6*2^(2/3)) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/
(2*2^(2/3)) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(S
qrt[3]*(1 - 2^(1/3)*(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rubi in Sympy [A] time = 11.585, size = 192, normalized size = 1.76

$$\frac{\sqrt[3]{2}\sqrt{3}\log\left(-x+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(x+\sqrt{3}\right)}{24}+\frac{\sqrt[3]{2}\sqrt{3}\log\left(-x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\sqrt{3}\log\left(x-\sqrt[3]{2}\sqrt{3}\sqrt[3]{x^2+1}+\sqrt{3}\right)}{24}-\frac{\sqrt[3]{2}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\left(-x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt[3]{2}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\left(x+\sqrt{3}\right)}{3\sqrt[3]{x^2+1}}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] 2**(1/3)*sqrt(3)*log(-x + sqrt(3))/24 - 2**(1/3)*sqrt(3)*log(x +
sqrt(3))/24 + 2**(1/3)*sqrt(3)*log(-x - 2**(1/3)*sqrt(3)*(x**2 +
1)**(1/3) + sqrt(3))/24 - 2**(1/3)*sqrt(3)*log(x - 2**(1/3)*sqrt(
3)*(x**2 + 1)**(1/3) + sqrt(3))/24 - 2**(1/3)*atan(2**(2/3)*(-x +
 sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)/3)/12 + 2**(1/3)*atan(2
**(2/3)*(x + sqrt(3))/(3*(x**2 + 1)**(1/3)) + sqrt(3)/3)/12
```

Mathematica [C] time = 0.0558719, size = 124, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)}{(x^2-3)\sqrt[3]{x^2+1}\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,\frac{x^2}{3}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x
^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(App
ellF1[3/2, 1/3, 2, 5/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2,
 -x^2, x^2/3])))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+3}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
```

```
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=96

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1
 - x)^(1/3))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/
3)*(1 - x)^(1/3) + (1 + x)^(2/3)])/(2*2^(2/3))
```

Rubi [A] time = 0.0685237, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1
 - x)^(1/3))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/
3)*(1 - x)^(1/3) + (1 + x)^(2/3)])/(2*2^(2/3))
```

Rubi in Sympy [A] time = 6.05646, size = 87, normalized size = 0.91

$$-\frac{\sqrt[3]{2}\log\left(x^2+3\right)}{4}+\frac{3\sqrt[3]{2}\log\left(\sqrt[3]{2}\sqrt[3]{-x+1}+(x+1)^{\frac{2}{3}}\right)}{4}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2^{\frac{2}{3}}\sqrt{3}(x+1)^{\frac{2}{3}}}{3\sqrt[3]{-x+1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -2**(1/3)*log(x**2 + 3)/4 + 3*2**(1/3)*log(2**(1/3)*(-x + 1)**(1/
3) + (x + 1)**(2/3))/4 - 2**(1/3)*sqrt(3)*atan(sqrt(3)/3 - 2**(2/
3)*sqrt(3)*(x + 1)**(2/3)/(3*(-x + 1)**(1/3)))/2
```

Mathematica [C] time = 0.438495, size = 203, normalized size = 2.11

$$\frac{3x\left(\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+\frac{xF_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)\right)-6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*x*((9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*AppellF1[1/2
, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2,
x^2, -x^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])) + (x*App
ellF1[1, 1/3, 1, 2, x^2, -x^2/3])/(-6*AppellF1[1, 1/3, 1, 2, x^2,
 -x^2/3] + x^2*(AppellF1[2, 1/3, 2, 3, x^2, -x^2/3] - AppellF1[2,
 4/3, 1, 3, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{3-x}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] -integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{x^2\sqrt[3]{-x^2+1}+3\sqrt[3]{-x^2+1}}\,dx-\int\left(-\frac{3}{x^2\sqrt[3]{-x^2+1}+3\sqrt[3]{-x^2+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -Integral(x/(x**2*(-x**2 + 1)**(1/3) + 3*(-x**2 + 1)**(1/3)), x)
- Integral(-3/(x**2*(-x**2 + 1)**(1/3) + 3*(-x**2 + 1)**(1/3)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

Optimal. Leaf size=95

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 +
 x)^(1/3))])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^
(2/3) + 2^(1/3)*(1 + x)^(1/3)])/(2*2^(2/3))
```

Rubi [A] time = 0.0654004, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 +
 x)^(1/3))])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^
(2/3) + 2^(1/3)*(1 + x)^(1/3)])/(2*2^(2/3))
```

Rubi in Sympy [A] time = 5.63106, size = 87, normalized size = 0.92

$$\frac{\sqrt[3]{2}\log\left(x^2+3\right)}{4}-\frac{3\sqrt[3]{2}\log\left((-x+1)^{\frac{2}{3}}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{4}-\frac{\sqrt[3]{2}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}(-x+1)^{\frac{2}{3}}}{3\sqrt[3]{x+1}}-\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] 2**(1/3)*log(x**2 + 3)/4 - 3*2**(1/3)*log((-x + 1)**(2/3) + 2**(1
/3)*(x + 1)**(1/3))/4 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*(-
x + 1)**(2/3)/(3*(x + 1)**(1/3)) - sqrt(3)/3)/2
```

Mathematica [C] time = 0.237294, size = 203, normalized size = 2.14

$$\frac{3x\left(\frac{9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}{2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};x^2,-\frac{x^2}{3}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};x^2,-\frac{x^2}{3}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};x^2,-\frac{x^2}{3}\right)}+\frac{xF_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}{x^2\left(F_1\left(2;\frac{4}{3},1;3;x^2,-\frac{x^2}{3}\right)-F_1\left(2;\frac{1}{3},2;3;x^2,-\frac{x^2}{3}\right)\right)+6F_1\left(1;\frac{1}{3},1;2;x^2,-\frac{x^2}{3}\right)}\right)}{\sqrt[3]{1-x^2}\,(x^2+3)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (3*x*((9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -x^2/3])/(9*AppellF1[1/2
, 1/3, 1, 3/2, x^2, -x^2/3] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2,
x^2, -x^2/3] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -x^2/3])) + (x*App
ellF1[1, 1/3, 1, 2, x^2, -x^2/3])/(6*AppellF1[1, 1/3, 1, 2, x^2,
-x^2/3] + x^2*(-AppellF1[2, 1/3, 2, 3, x^2, -x^2/3] + AppellF1[2,
 4/3, 1, 3, x^2, -x^2/3]))))/((1 - x^2)^(1/3)*(3 + x^2))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{3+x}{x^2+3}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

```
[Out] int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[
b]*ArcTan[(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])
/(12*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a
 + b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi [A] time = 0.0877304, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[
b]*ArcTan[(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])
/(12*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a
 + b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi in Sympy [A] time = 38.0638, size = 54, normalized size = 0.36

$$\frac{bx\left(a+bx^2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{9a^2d\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*x*(a + b*x**2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, -b*x**2/a, -b*
x**2/(9*a))/(9*a**2*d*(1 + b*x**2/a)**(2/3))
```

Mathematica [C] time = 0.258171, size = 169, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\,(9a+bx^2)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)
])/(d*(a + b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1,
 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)] - 2*b*x^2*(AppellF1[3/2, 1/3,
 2, 5/2, -((b*x^2)/a), -(b*x^2)/(9*a)] + 3*AppellF1[3/2, 4/3, 1,
5/2, -((b*x^2)/a), -(b*x^2)/(9*a)])))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{bx^2+a}}\left(9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3
)), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a -
 b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rubi [A] time = 0.0867765, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a -
 b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rubi in Sympy [A] time = 38.0804, size = 53, normalized size = 0.35

$$-\frac{bx\left(a-bx^2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{9a^2d\left(1-\frac{bx^2}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] -b*x*(a - b*x**2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, b*x**2/a, b*x
**2/(9*a))/(9*a**2*d*(1 - b*x**2/a)**(2/3))
```

Mathematica [C] time = 0.273785, size = 167, normalized size = 1.09

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\left(9a-bx^2\right)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/
(d*(a - b*x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/
2, (b*x^2)/a, (b*x^2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2
, (b*x^2)/a, (b*x^2)/(9*a)] + 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2
)/a, (b*x^2)/(9*a)])))
```

Maple [F] time = 0.07, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{-bx^2+a}}\left(-9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/
3)), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2+a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a
+ b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rubi [A] time = 0.0889492, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```


```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a
+ b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rubi in Sympy [A] time = 39.3748, size = 51, normalized size = 0.34

$$\frac{bx\left(-a+bx^2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{9a^2d\left(1-\frac{bx^2}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*x*(-a + b*x**2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, b*x**2/a, b*x
**2/(9*a))/(9*a**2*d*(1 - b*x**2/a)**(2/3))
```

Mathematica [C] time = 0.249661, size = 168, normalized size = 1.11

$$-\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{bx^2-a}\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)+27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{9a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/
(d*(9*a - b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3
/2, (b*x^2)/a, (b*x^2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, (b*x^2)/(9*a)] + 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^
2)/a, (b*x^2)/(9*a)])))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{bx^2-a}}\left(-9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-a\right)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(
1/3)), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=153

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt
[b]*ArcTan[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)
])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) +
(-a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi [A] time = 0.0889332, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) - (Sqrt
[b]*ArcTan[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)
])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) +
(-a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rubi in Sympy [A] time = 39.9634, size = 58, normalized size = 0.38

$$-\frac{bx\left(-a-bx^2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{9a^2d\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] -b*x*(-a - b*x**2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, -b*x**2/a, -
b*x**2/(9*a))/(9*a**2*d*(1 + b*x**2/a)**(2/3))
```

Mathematica [C] time = 0.272171, size = 172, normalized size = 1.12

$$\frac{27abxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\,(9a+bx^2)\left(27aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)
])/(d*(-a - b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1
, 3/2, -((b*x^2)/a), -(b*x^2)/(9*a)] - 2*b*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, -((b*x^2)/a), -(b*x^2)/(9*a)] + 3*AppellF1[3/2, 4/3, 1,
 5/2, -((b*x^2)/a), -(b*x^2)/(9*a)])))
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{-bx^2-a}}\left(9\,\frac{ad}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1
/3)), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[
b]*ArcTan[(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])
/(12*2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2
 + b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0900336, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[
b]*ArcTan[(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])
/(12*2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2
 + b*x^2)^(1/3)))/(Sqrt[b]*x)])/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi in Sympy [A] time = 16.1674, size = 32, normalized size = 0.21

$$\frac{2^{\frac{2}{3}}bx\operatorname{appellf_1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{36d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
```

```
[Out] 2**(2/3)*b*x*appellf1(1/2, 1/3, 1, 3/2, -b*x**2/2, -b*x**2/18)/(3
6*d)
```

Mathematica [C] time = 0.232691, size = 148, normalized size = 0.98

$$-\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{bx^2+2}\,(bx^2+18)\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2)/2, -(b*x^2)/18])/(d*
(2 + b*x^2)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(
b*x^2)/2, -(b*x^2)/18] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^
2)/2, -(b*x^2)/18] + 3*AppellF1[3/2, 4/3, 1, 5/2, -(b*x^2)/2, -(b
*x^2)/18])))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{bx^2+2}}\left(18\,\frac{d}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
```

```
[Out] int(1/(b*x^2+2)^(1/3)/(18/b*d+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+2)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)
), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+2)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}$$

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[2])])/(12*2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2
+ b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d)
```

Rubi [A] time = 0.0797945, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\,2^{5/6}d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\,2^{5/6}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/
(Sqrt[b]*x)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x
)/(3*Sqrt[2])])/(12*2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2
+ b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d)
```

Rubi in Sympy [A] time = 25.286, size = 46, normalized size = 0.31

$$\frac{bx\left(bx^2-2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)}{36d\left(-\frac{bx^2}{2}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
```

```
[Out] b*x*(b*x**2 - 2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, b*x**2/2, b*x*
*2/18)/(36*d*(-b*x**2/2 + 1)**(2/3))
```

Mathematica [C] time = 0.273102, size = 148, normalized size = 1.01

$$\frac{27bxF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(bx^2-18\right)\sqrt[3]{bx^2-2}\left(bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)+27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{18}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-1
8 + b*x^2)*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x
^2)/2, (b*x^2)/18] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2,
 (b*x^2)/18] + 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18
])))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt[3]{bx^2-2}}\left(-18\,\frac{d}{b}+dx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
```

```
[Out] int(1/(b*x^2-2)^(1/3)/(-18/b*d+d*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2-2\right)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int\frac{1}{bx^2\sqrt[3]{bx^2-2}-18\sqrt[3]{bx^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
```

```
[Out] b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)
), x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2-2)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}(6d+dx^2)}\,dx$

Optimal. Leaf size=123

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 +
3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - Ar
cTanh[(2^(1/6)*(2^(1/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi [A] time = 0.0626472, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 +
3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - Ar
cTanh[(2^(1/6)*(2^(1/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi in Sympy [A] time = 11.6995, size = 29, normalized size = 0.24

$$\frac{2^{\frac{2}{3}}x\,\text{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] 2**(2/3)*x*appellf1(1/2, 1/3, 1, 3/2, -3*x**2/2, -x**2/6)/(12*d)
```

Mathematica [C] time = 0.185309, size = 136, normalized size = 1.11

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\left(x^2+6\right)\sqrt[3]{3x^2+2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6])/(d*(6 + x^2
)*(2 + 3*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x
^2/6] + x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -x^2/6] + 3*A
ppellF1[3/2, 4/3, 1, 5/2, (-3*x^2)/2, -x^2/6])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+6\,d}\frac{1}{\sqrt[3]{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x)
```

```
[Out] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)
/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}(-6d+dx^2)}\,dx$

Optimal. Leaf size=123

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] -ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
- ArcTanh[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (
2 - 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0584587, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] -ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
- ArcTanh[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (
2 - 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi in Sympy [A] time = 11.879, size = 27, normalized size = 0.22

$$-\frac{2^{\frac{2}{3}}x\,\text{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] -2**(2/3)*x*appellf1(1/2, 1/3, 1, 3/2, 3*x**2/2, x**2/6)/(12*d)
```

Mathematica [C] time = 0.188918, size = 136, normalized size = 1.11

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\,(x^2-6)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)
^(1/3)*(-6 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6]
 + x^2*(AppellF1[3/2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1
[3/2, 4/3, 1, 5/2, (3*x^2)/2, x^2/6])))
```

Maple [F] time = 0.077, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2-6\,d}\frac{1}{\sqrt[3]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2 \sqrt[3]{-3x^2+2}-6\sqrt[3]{-3x^2+2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] Integral(1/(x**2*(-3*x**2 + 2)**(1/3) - 6*(-3*x**2 + 2)**(1/3)),
x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx$

Optimal. Leaf size=119

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
+ ArcTanh[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (
-2 + 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi [A] time = 0.0560293, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
+ ArcTanh[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (
-2 + 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rubi in Sympy [A] time = 17.1748, size = 42, normalized size = 0.35

$$\frac{x\left(3x^2-2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{12d\left(-\frac{3x^2}{2}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] x*(3*x**2 - 2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, 3*x**2/2, x**2/6
)/(12*d*(-3*x**2/2 + 1)**(2/3))
```

Mathematica [C] time = 0.188333, size = 136, normalized size = 1.14

$$\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\left(x^2-6\right)\sqrt[3]{3x^2-2}\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)+9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{3x^2}{2},\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*
(-2 + 3*x^2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6
] + x^2*(AppellF1[3/2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF
1[3/2, 4/3, 1, 5/2, (3*x^2)/2, x^2/6])))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2-6\,d}\frac{1}{\sqrt[3]{3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x)
```

```
[Out] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2-6\,d\right)\left(3\,x^2-2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{3x^2-2}-6\sqrt[3]{3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
```

```
[Out] Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)
/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}(6d+dx^2)}\,dx$

Optimal. Leaf size=119

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

```
[Out] -ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2
- 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) +
ArcTanh[(2^(1/6)*(2^(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi [A] time = 0.0567701, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] -ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2
- 3*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) +
ArcTanh[(2^(1/6)*(2^(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rubi in Sympy [A] time = 17.1447, size = 49, normalized size = 0.41

$$-\frac{x\left(-3x^2-2\right)^{\frac{2}{3}}\operatorname{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{12d\left(\frac{3x^2}{2}+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] -x*(-3*x**2 - 2)**(2/3)*appellf1(1/2, 1/3, 1, 3/2, -3*x**2/2, -x*
*2/6)/(12*d*(3*x**2/2 + 1)**(2/3))
```

Mathematica [C] time = 0.203705, size = 136, normalized size = 1.14

$$-\frac{9xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-3x^2-2}\,(x^2+6)\left(x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)-9F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{x^2}{6}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -x^2/6])/(d*(-2 - 3*
x^2)^(1/3)*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -
x^2/6] + x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -x^2/6] + 3*
AppellF1[3/2, 4/3, 1, 5/2, (-3*x^2)/2, -x^2/6])))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+6\,d}\frac{1}{\sqrt[3]{-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x)
```

```
[Out] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
```

```
[Out] Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)),
x)/d
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

3.160 $\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx$

Optimal. Leaf size=70

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTa
nh[(Sqrt[3]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rubi [A] time = 0.0314233, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTa
nh[(Sqrt[3]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 5.12431, size = 19, normalized size = 0.27

$$\frac{x\,\mathrm{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

```
[Out] x*appellf1(1/2, 1/3, 1, 3/2, -x**2, -x**2/9)/9
```

Mathematica [C] time = 0.147685, size = 124, normalized size = 1.77

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{x^2+1}\,(x^2+9)\left(2x^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-x^2,-\frac{x^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-x^2,-\frac{x^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -x^2/9])/((1 + x^2)^(1/3)
*(9 + x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -x^2/9] + 2*x^2*
(AppellF1[3/2, 1/3, 2, 5/2, -x^2, -x^2/9] + 3*AppellF1[3/2, 4/3,
1, 5/2, -x^2, -x^2/9])))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{1}{x^2+9}\frac{1}{\sqrt[3]{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

```
[Out] int(1/(x^2+1)^(1/3)/(x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^2 + 1}\,(x^2 + 9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
```

```
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 9)(x^2 + 1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}(9+bx^2)}\,dx$

Optimal. Leaf size=104

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3
))^2/(3*Sqrt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x
^2)^(1/3)))/(Sqrt[b]*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rubi [A] time = 0.0533674, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}\,x}\right)}{12\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}\,x}\right)}{4\sqrt{3}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{3}\right)}{12\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3
))^2/(3*Sqrt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x
^2)^(1/3)))/(Sqrt[b]*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rubi in Sympy [A] time = 9.58172, size = 22, normalized size = 0.21

$$\frac{x\,\mathrm{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-bx^2,-\frac{bx^2}{9}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
```

```
[Out] x*appellf1(1/2, 1/3, 1, 3/2, -b*x**2, -b*x**2/9)/9
```

Mathematica [C] time = 0.16732, size = 137, normalized size = 1.32

$$-\frac{27xF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{bx^2+1}\,(bx^2+9)\left(2bx^2\left(F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)+3F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};-bx^2,-\frac{bx^2}{9}\right)\right)-27F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};-bx^2,-\frac{bx^2}{9}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -(b*x^2)/9])/((1 + b*
x^2)^(1/3)*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2),
-(b*x^2)/9] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -(b*x
^2)/9] + 3*AppellF1[3/2, 4/3, 1, 5/2, -(b*x^2), -(b*x^2)/9])))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+9}\frac{1}{\sqrt[3]{bx^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

```
[Out] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx^2 + 1}\,(bx^2 + 9)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
```

```
[Out] Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + 9)(bx^2 + 1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

3.162 $\int \frac{1}{\sqrt[3]{1-x^2}(9-x^2)}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x
/3]/12 - ArcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rubi [A] time = 0.0370406, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x
/3]/12 - ArcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rubi in Sympy [A] time = 8.49567, size = 15, normalized size = 0.2

$$\frac{x\,\text{appellf}_1\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,\frac{x^2}{9}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] x*appellf1(1/2, 1/3, 1, 3/2, x**2, x**2/9)/9
```

Mathematica [C] time = 0.0704206, size = 125, normalized size = 1.69

$$\frac{\sqrt[3]{\frac{x-1}{x-3}}\sqrt[3]{\frac{x+1}{x-3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};-\frac{4}{x-3},-\frac{2}{x-3}\right)-\sqrt[3]{\frac{x-1}{x+3}}\sqrt[3]{\frac{x+1}{x+3}}F_1\left(\frac{2}{3};\frac{1}{3},\frac{1}{3};\frac{5}{3};\frac{2}{x+3},\frac{4}{x+3}\right)}{4\sqrt[3]{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] (((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3,
 1/3, 1/3, 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1
/3)*((1 + x)/(3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x
), 4/(3 + x)])/(4*(1 - x^2)^(1/3))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+9}\frac{1}{\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/3)/(-x^2+9),x)
```

```
[Out] int(1/(-x^2+1)^(1/3)/(-x^2+9),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2\sqrt[3]{-x^2+1} - 9\sqrt[3]{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] -Integral(1/(x**2*(-x**2 + 1)**(1/3) - 9*(-x**2 + 1)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

3.163 $\int \left(a+bx^2\right)^{3/2} \sqrt{c+dx^2}\,dx$

Optimal. Leaf size=328

$$\frac{\sqrt{c}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{b}+7ac-\frac{2bc^2}{d}\right)}{15\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{15d}$$

```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[
c + d*x^2]) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(15*d) + (b*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c
]*(2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(3/2)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*c
 - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.73919, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{b}+7ac-\frac{2bc^2}{d}\right)}{15\sqrt{c+dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}{5d}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{15d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```



```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[
c + d*x^2]) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(15*d) + (b*x*Sqrt[a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c
]*(2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*d^(3/2)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*c
```

```
 - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 91.9174, size = 301, normalized size = 0.92

$$-\frac{\sqrt{a}\sqrt{c+dx^2}\left(3a^2d^2+7abcd-2b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15\sqrt{b}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}}{5d}+\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}\,(9ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{15d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(ad-\frac{bc}{3}\right)}{5d}+\frac{x\sqrt{c+dx^2}\left(3a^2d^2+7abcd-2b^2c^2\right)}{15d^2\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(c + d*x**2)*(3*a**2*d**2 + 7*a*b*c*d - 2*b**2*c**2)
*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(15*sqrt(b)*
d**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) + b*
x*sqrt(a + b*x**2)*(c + d*x**2)**(3/2)/(5*d) + c**(3/2)*sqrt(a +
b*x**2)*(9*a*d - b*c)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c
/(a*d))/(15*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c
 + d*x**2)) + 2*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d - b*c/3)
/(5*d) + x*sqrt(c + d*x**2)*(3*a**2*d**2 + 7*a*b*c*d - 2*b**2*c**
2)/(15*d**2*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.798564, size = 243, normalized size = 0.74

$$\frac{-2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2+7abcd-2b^2c^2\right)E}{15d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d + b*(c + 3*d*x^2))
- I*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- (2*I)*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*S
```

```
qrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
)/(15*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.068, size = 543, normalized size = 1.7

$$\frac{1}{(15\,bdx^4+15\,adx^2+15\,cx^2b+15\,ac)\,d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+9\sqrt{-\frac{b}{a}}x^5abd^3+4\sqrt{-\frac{b}{a}}x^5b^2cd^2+6\sqrt{-\frac{b}{a}}x^3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+
9*(-b/a)^(1/2)*x^5*a*b*d^3+4*(-b/a)^(1/2)*x^5*b^2*c*d^2+6*(-b/a)^
(1/2)*x^3*a^2*d^3+10*(-b/a)^(1/2)*x^3*a*b*c*d^2+(-b/a)^(1/2)*x^3*
b^2*c^2*d+6*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-8*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2
*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*b^2*c^3+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+6*(-b/a)^(1/2)*
x*a^2*c*d^2+(-b/a)^(1/2)*x*a*b*c^2*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)/d^2/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2\right)^{\frac{3}{2}} \sqrt{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

3.164 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=249

$$\frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{x\sqrt{a+bx^2}(ad+bc)}{3b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
(2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.421404, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{x\sqrt{a+bx^2}(ad+bc)}{3b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```


```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) +
(2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 57.1388, size = 214, normalized size = 0.86

$$\frac{2a^{\frac{3}{2}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3}-\frac{\sqrt{c}\sqrt{a+bx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,(ad+bc)}{3b\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] 2*a**(3/2)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -
a*d/(b*c) + 1)/(3*sqrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*s
qrt(a + b*x**2)) + x*sqrt(a + b*x**2)*sqrt(c + d*x**2)/3 - sqrt(c
)*sqrt(a + b*x**2)*(a*d + b*c)*elliptic_e(atan(sqrt(d)*x/sqrt(c))
, 1 - b*c/(a*d))/(3*b*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)
))*sqrt(c + d*x**2)) + x*sqrt(a + b*x**2)*(a*d + b*c)/(3*b*sqrt(c
 + d*x**2))
```

Mathematica [C] time = 0.44176, size = 198, normalized size = 0.8

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+bc)E\left(i\sin\right.}{3d\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt
[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.016, size = 328, normalized size = 1.3

$$\frac{1}{(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac)\,d}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}Ell\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a
)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c
^2+(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}\sqrt{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}\sqrt{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^2}\sqrt{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

3.165 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (
c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.275606, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (
c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 40.2712, size = 168, normalized size = 0.82

$$-\frac{\sqrt{a}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{c+dx^2}}{\sqrt{a+bx^2}}+\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*
d/(b*c) + 1)/(sqrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(
a + b*x**2)) + x*sqrt(c + d*x**2)/sqrt(a + b*x**2) + c**(3/2)*sqr
t(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/
(a*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)
)
```

Mathematica [A] time = 0.0819035, size = 86, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)
]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2
)/c])
```

Maple [A] time = 0.023, size = 101, normalized size = 0.5

$$\frac{c}{bdx^4+adx^2+cx^2b+ac}\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.166 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}} \, dx$

Optimal. Leaf size=84

$$\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c
*(a + b*x^2))])
```

Rubi [A] time = 0.0573714, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c
*(a + b*x^2))])
```

Rubi in Sympy [A] time = 8.98429, size = 71, normalized size = 0.85

$$\frac{\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) +
 1)/(sqrt(a)*sqrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a
 + b*x**2))
```

Mathematica [C] time = 0.529476, size = 133, normalized size = 1.58

$$\frac{x\left(c+dx^2\right)+\frac{ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}}}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(c + d*x^2) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/Sqrt[b/a])/(a*Sqrt[a + b*x^2]*Sqrt
[c + d*x^2])
```

Maple [A] time = 0.049, size = 181, normalized size = 2.2

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,a}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(x^3d\sqrt{-\frac{b}{a}}+EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-EllipticE\left(x\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*(x^3*d*(-b/a)^(1/2)+EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)+x*c*(-b/a)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)/a/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)

[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

3.167 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c
)])/(3*a^(3/2)*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c -
 a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.330143, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c
)])/(3*a^(3/2)*Sqrt[b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c -
 a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 48.2984, size = 197, normalized size = 0.83

$$\frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{\frac{3}{2}}}+\frac{d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{a}\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\left(ad-bc\right)}+\frac{\sqrt{c+dx^2}\left(ad-2bc\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3a^{\frac{3}{2}}\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] x*sqrt(c + d*x**2)/(3*a*(a + b*x**2)**(3/2)) + d*sqrt(c + d*x**2)
*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*sqrt(a)*s
qrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*
d - b*c)) + sqrt(c + d*x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(b
)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*a**(3/2)*sqrt(b)*sqrt(a*(c + d*x
**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c))
```

Mathematica [C] time = 0.769971, size = 243, normalized size = 1.03

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(2a^2d+ab\left(dx^2-3c\right)-2b^2cx^2\right)-2ic\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ic\left(\cdots\right.}{3a^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(2*a^2*d - 2*b^2*c*x^2 + a*b*(-3*c + d*x
^2)) + I*c*(-2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*
I)*c*(-(b*c) + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqr
t[b/a]*(-(b*c) + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.055, size = 617, normalized size = 2.6

$$\frac{1}{(3\,ad-3\,bc)\,a^2}\left(x^5abd^2\sqrt{-\frac{b}{a}}-2\,x^5b^2cd\sqrt{-\frac{b}{a}}+2\,EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^2abcd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-2\,EllipticF\left(x\sqrt{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(x^5*a*b*d^2*(-b/a)^(1/2)-2*x^5*b^2*c*d*(-b/a)^(1/2)+2*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)-2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^2*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)+2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2
*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*x^3*a^2*d^2*(-
b/a)^(1/2)-2*x^3*a*b*c*d*(-b/a)^(1/2)-2*x^3*b^2*c^2*(-b/a)^(1/2)+
2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*a*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)^(1/2)*((d*x^
```

```
2+c)/c)^(1/2)+2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*x*a^2*c*d*(-b/a)^(1/2)
-3*x*a*b*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)
/a^2/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2
+ a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

3.168 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

Optimal. Leaf size=309

$$-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*
Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2
*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c -
 a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
(2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^2*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.587501, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$-\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}(4bc-3ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)}+\frac{\sqrt{c+dx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```


```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*
Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2
*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(
Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c -
 a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
(2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^2*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 82.839, size = 274, normalized size = 0.89

$$\frac{x\sqrt{c+dx^2}}{5a\,(a+bx^2)^{\frac{5}{2}}}+\frac{x\sqrt{c+dx^2}\,(3ad-4bc)}{15a^2\,(a+bx^2)^{\frac{3}{2}}\,(ad-bc)}+\frac{2d\sqrt{c+dx^2}\,(3ad-2bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15a^{\frac{3}{2}}\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}+\frac{\sqrt{c+dx^2}\,(3a^2d^2-13abcd+8b^2c^2)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15a^{\frac{5}{2}}\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] x*sqrt(c + d*x**2)/(5*a*(a + b*x**2)**(5/2)) + x*sqrt(c + d*x**2)
*(3*a*d - 4*b*c)/(15*a**2*(a + b*x**2)**(3/2)*(a*d - b*c)) + 2*d*
sqrt(c + d*x**2)*(3*a*d - 2*b*c)*elliptic_f(atan(sqrt(b)*x/sqrt(a
)), -a*d/(b*c) + 1)/(15*a**(3/2)*sqrt(b)*sqrt(a*(c + d*x**2)/(c*(
a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)**2) + sqrt(c + d*x**2)
*(3*a**2*d**2 - 13*a*b*c*d + 8*b**2*c**2)*elliptic_e(atan(sqrt(b)
*x/sqrt(a)), -a*d/(b*c) + 1)/(15*a**(5/2)*sqrt(b)*sqrt(a*(c + d*x
**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)**2)
```

Mathematica [C] time = 0.896881, size = 285, normalized size = 0.92

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(3a^2d^2-13abcd+8b^2c^2\right)+3a^2(bc-ad)^2+a\left(a+bx^2\right)(ad-bc)(3ad-4bc)\right)+ic\sqrt{\frac{bx^2}{a}+1}\,(a \cdots}{15a^3\sqrt{\frac{b}{a}}\,(a+bx^2)\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*
(-4*b*c + 3*a*d)*(a + b*x^2) + (8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^
2)*(a + b*x^2)^2) + I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b^2*c^2 + 17*a*b*c*d - 9*a
^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^3*
Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.063, size = 1411, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x)
```

```
[Out] 1/15*(8*x^7*b^4*c^2*d*(-b/a)^(1/2)+9*x^5*a^3*b*d^3*(-b/a)^(1/2)+2
0*x^3*a*b^3*c^3*(-b/a)^(1/2)-13*x^7*a*b^3*c*d^2*(-b/a)^(1/2)+8*x^
5*b^4*c^3*(-b/a)^(1/2)+9*x^3*a^4*d^3*(-b/a)^(1/2)-3*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)+13*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+18*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)-17*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*x^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+
3*x^7*a^2*b^2*d^3*(-b/a)^(1/2)-30*x^5*a^2*b^2*c*d^2*(-b/a)^(1/2)+
7*x^5*a*b^3*c^2*d*(-b/a)^(1/2)-17*x^3*a^3*b*c*d^2*(-b/a)^(1/2)-18
*x^3*a^2*b^2*c^2*d*(-b/a)^(1/2)-26*x*a^3*b*c^2*d*(-b/a)^(1/2)+8*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+9*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)+8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b^2*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-3*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2
*b^2*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+9*x*a^4*c*d^2*(-
b/a)^(1/2)+15*x*a^2*b^2*c^3*(-b/a)^(1/2)-16*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)-17*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*
d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+13*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)+16*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c
^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-34*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)-6*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a
^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+26*EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)+9*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*x^4*a^2*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2))/(d*
x^2+c)^(1/2)/(-b/a)^(1/2)/(a*d-b*c)^2/a^3/(b*x^2+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 +
a^3)*sqrt(b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(7/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

3.169 $\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} dx$

Optimal. Leaf size=410

$$-\frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{35}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{2a^2d}{b}+9ac+\frac{bc^2}{d}\right)+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}+\frac{2x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{35b}$$

```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2]
)/(35*b^2*d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)
*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqr
t[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d
 + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*
d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(35*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi [A] time = 1.07085, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-18abcd+b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{1}{35}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{2a^2d}{b}+9ac+\frac{bc^2}{d}\right)+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}+\frac{2x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad)}{35b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2]
)/(35*b^2*d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)
*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqr
t[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d
 + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*
d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(35*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 141.37, size = 379, normalized size = 0.92

$$-\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}\left(a^2d^2-18abcd+b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{35b^{\frac{3}{2}}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{5}{2}}}{7d}$$
$$+\frac{2x\sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}(4ad-bc)}{35d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(a^2d^2+9abcd-2b^2c^2\right)}{35bd}$$
$$+\frac{2\sqrt{c}\sqrt{a+bx^2}\,(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{35b^2d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{2x\sqrt{a+bx^2}\,(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] -a**(3/2)*sqrt(c + d*x**2)*(a**2*d**2 - 18*a*b*c*d + b**2*c**2)*e
lliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(35*b**(3/2)*d
*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) + b*x*sq
rt(a + b*x**2)*(c + d*x**2)**(5/2)/(7*d) + 2*x*sqrt(a + b*x**2)*(
c + d*x**2)**(3/2)*(4*a*d - b*c)/(35*d) + x*sqrt(a + b*x**2)*sqrt
(c + d*x**2)*(a**2*d**2 + 9*a*b*c*d - 2*b**2*c**2)/(35*b*d) + 2*s
qrt(c)*sqrt(a + b*x**2)*(a*d + b*c)*(a**2*d**2 - 6*a*b*c*d + b**2
*c**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(35*b**
2*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2))))*sqrt(c + d*x**2)
) - 2*x*sqrt(a + b*x**2)*(a*d + b*c)*(a**2*d**2 - 6*a*b*c*d + b**
2*c**2)/(35*b**2*d*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.06385, size = 302, normalized size = 0.74

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(a^2d^2+abd\left(17c+8dx^2\right)+b^2\left(c^2+8cdx^2+5d^2x^4\right)\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^3d^3+8a^2bcd^2-\right.}{35bd^2\sqrt{\frac{b}{a}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8
*d*x^2) + b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 -
 5*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*d^3)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Maple [A] time = 0.028, size = 780, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/35*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(5*(-b/a)^(1/2)*x^9*b^3*d^4+
13*(-b/a)^(1/2)*x^7*a*b^2*d^4+13*(-b/a)^(1/2)*x^7*b^3*c*d^3+9*(-b
/a)^(1/2)*x^5*a^2*b*d^4+38*(-b/a)^(1/2)*x^5*a*b^2*c*d^3+9*(-b/a)^
(1/2)*x^5*b^3*c^2*d^2+(-b/a)^(1/2)*x^3*a^3*d^4+26*(-b/a)^(1/2)*x^
3*a^2*b*c*d^3+26*(-b/a)^(1/2)*x^3*a*b^2*c^2*d^2+(-b/a)^(1/2)*x^3*
b^3*c^3*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*a^3*c*d^3+8*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2
*d^2-11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^3*d+2*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4-2
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^3*c*d^3+10*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c^2*d^2+10*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b^2*c^3*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^4+(-b/a)^(1
/2)*x*a^3*c*d^3+17*(-b/a)^(1/2)*x*a^2*b*c^2*d^2+(-b/a)^(1/2)*x*a*
b^2*c^3*d)/b/d^2/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bdx^4+(bc+ad)x^2+ac\right)\sqrt{bx^2+a}\sqrt{dx^2+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

3.170 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx$

Optimal. Leaf size=336

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{15b^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{15b}$$

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*S
qrt[c + d*x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(15*b) + (d*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sq
rt[c]*(3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2
)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.729295, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{x\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)}{15b^2\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(-2a^2d^2+7abcd+3b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{15b}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*S
qrt[c + d*x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])/(15*b) + (d*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sq
rt[c]*(3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2
)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 98.0258, size = 299, normalized size = 0.89

$$-\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-9bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{\sqrt{a}\sqrt{c+dx^2}\left(2a^2d^2-7abcd-3b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15b^{\frac{3}{2}}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{dx\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{5b}-\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{ad}{3}-bc\right)}{5b}-\frac{x\sqrt{c+dx^2}\left(2a^2d^2-7abcd-3b^2c^2\right)}{15bd\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] -a**(3/2)*sqrt(c + d*x**2)*(a*d - 9*b*c)*elliptic_f(atan(sqrt(b)*
x/sqrt(a)), -a*d/(b*c) + 1)/(15*b**(3/2)*sqrt(a*(c + d*x**2)/(c*(
a + b*x**2)))*sqrt(a + b*x**2)) + sqrt(a)*sqrt(c + d*x**2)*(2*a**
2*d**2 - 7*a*b*c*d - 3*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(15*b**(3/2)*d*sqrt(a*(c + d*x**2)/(c*(a + b
*x**2)))*sqrt(a + b*x**2)) + d*x*(a + b*x**2)**(3/2)*sqrt(c + d*x
**2)/(5*b) - 2*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(a*d/3 - b*c)/
(5*b) - x*sqrt(c + d*x**2)*(2*a**2*d**2 - 7*a*b*c*d - 3*b**2*c**2
)/(15*b*d*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.716102, size = 246, normalized size = 0.73

$$\frac{-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(a^2d^2+2abcd-3b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)+ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(2a^2d^2-7abcd-3b^2c^2\right)E\left(\right.}{15bd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*b*c + a*d + 3*b*d*x^2)
+ I*c*(-3*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- I*c*(-3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(
15*b*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 545, normalized size = 1.6

$$\frac{1}{15\,d\,(bdx^4+adx^2+cx^2b+ac)\,b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^2d^3+4\sqrt{-\frac{b}{a}}x^5abd^3+9\sqrt{-\frac{b}{a}}x^5b^2cd^2+\sqrt{-\frac{b}{a}}x^3a^2d^3+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^2*d^3+
4*(-b/a)^(1/2)*x^5*a*b*d^3+9*(-b/a)^(1/2)*x^5*b^2*c*d^2+(-b/a)^(1
/2)*x^3*a^2*d^3+10*(-b/a)^(1/2)*x^3*a*b*c*d^2+6*(-b/a)^(1/2)*x^3*
b^2*c^2*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+2*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d
-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a*b*c^2*d+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+(-b/a)^(1/2)*x*a^
2*c*d^2+6*(-b/a)^(1/2)*x*a*b*c^2*d)/d/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)/b/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2 + a}(dx^2 + c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^2} \left(c + dx^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + a}(dx^2 + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

3.171 $\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3b^2\sqrt{c+dx^2}}-\frac{2\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (
d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(
2*b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(3*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*d)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.471558, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3b^2\sqrt{c+dx^2}}-\frac{2\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```


```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (
d*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(
2*b*c - a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(3*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*d)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b*Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 63.0559, size = 233, normalized size = 0.85

$$\frac{2\sqrt{a}\sqrt{c+dx^2}\,(ad-2bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{2x\sqrt{c+dx^2}\,(ad-2bc)}{3b\sqrt{a+bx^2}}-\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}\,(ad-3bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] 2*sqrt(a)*sqrt(c + d*x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(b)*
x/sqrt(a)), -a*d/(b*c) + 1)/(3*b**(3/2)*sqrt(a*(c + d*x**2)/(c*(a
 + b*x**2)))*sqrt(a + b*x**2)) + d*x*sqrt(a + b*x**2)*sqrt(c + d*
x**2)/(3*b) - 2*x*sqrt(c + d*x**2)*(a*d - 2*b*c)/(3*b*sqrt(a + b*
x**2)) - c**(3/2)*sqrt(a + b*x**2)*(a*d - 3*b*c)*elliptic_f(atan(
sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(3*a*b*sqrt(d)*sqrt(c*(a + b*x
**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.475835, size = 199, normalized size = 0.73

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-2bc)E\left(i\right.}{3b\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(-2*b*c + a*d)*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*b*Sqrt[b/a]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.022, size = 330, normalized size = 1.2

$$\frac{1}{(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac)\,b}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5bd^2+\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}Ell\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*x^5*b*d^2+(-b/a
)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d+a*c*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+4*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b*c^2+(-b/a)^(1/2)*x*a*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/b/(-b/
a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}}{\sqrt{bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

3.172 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{dx\sqrt{a+bx^2}(bc-2ad)}{ab^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) +
((b*c - a*d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*
Sqrt[d]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.460558, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$-\frac{dx\sqrt{a+bx^2}(bc-2ad)}{ab^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) +
((b*c - a*d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*
Sqrt[d]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 63.9753, size = 230, normalized size = 0.86

$$\frac{\sqrt{a}d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{x\sqrt{c+dx^2}\,(ad-bc)}{ab\sqrt{a+bx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,(2ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\,(2ad-bc)}{ab^2\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] sqrt(a)*d*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(b**(3/2)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) - x*sqrt(c + d*x**2)*(a*d - b*c)/(a*b*sqrt(a + b*x
**2)) - sqrt(c)*sqrt(d)*sqrt(a + b*x**2)*(2*a*d - b*c)*elliptic_e
(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(a*b**2*sqrt(c*(a + b*x*
*2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)) + d*x*sqrt(a + b*x**2)*(2
*a*d - b*c)/(a*b**2*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.481538, size = 191, normalized size = 0.72

$$\frac{(bc-ad)\left(x\sqrt{\frac{b}{a}}\left(c+dx^2\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)\right)-ic\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(2ad-bc)E\left(i\sinh^{-1}\left(\sqrt{\right.\right.}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(Sqr
t[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/
2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 332, normalized size = 1.2

$$\frac{1}{b\left(bdx^4+adx^2+cx^2b+ac\right)a}\left(-\sqrt{-\frac{b}{a}}x^3ad^2+\sqrt{-\frac{b}{a}}x^3bcd-ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+\sqrt{\frac{bx^2+}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x)
```

```
[Out] (-(-b/a)^(1/2)*x^3*a*d^2+(-b/a)^(1/2)*x^3*b*c*d-a*c*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b*c^2+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d-((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*b*c^2-(-b/a)^(1/2)*x*a*c*d+x*b*c^2*(-b/a)^(1/2))*(d*x^2+c)^
(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)

[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}}{\left(bx^2 + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2),x, algorithm="giac")

[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.173 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=229

$$\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b
*c + a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]],
1 - (a*d)/(b*c)])/(3*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c +
 d*x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*b*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.37155, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{2\sqrt{c+dx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```


```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b
*c + a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]],
1 - (a*d)/(b*c)])/(3*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c +
 d*x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*b*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 46.2652, size = 192, normalized size = 0.84

$$-\frac{x\sqrt{c+dx^2}\,(ad-bc)}{3ab\,(a+bx^2)^{\frac{3}{2}}}-\frac{d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{a}b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{2\sqrt{c+dx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3a^{\frac{3}{2}}b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] -x*sqrt(c + d*x**2)*(a*d - b*c)/(3*a*b*(a + b*x**2)**(3/2)) - d*s
qrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) +
1)/(3*sqrt(a)*b**(3/2)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt
(a + b*x**2)) + 2*sqrt(c + d*x**2)*(a*d + b*c)*elliptic_e(atan(sq
rt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*a**(3/2)*b**(3/2)*sqrt(a*(c
+ d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2))
```

Mathematica [C] time = 0.800394, size = 232, normalized size = 1.01

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(a^2d+ab\left(3c+2dx^2\right)+2b^2cx^2\right)-ic\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad+2bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)+2ic}{3a^3\left(\frac{b}{a}\right)^{3/2}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(a^2*d + 2*b^2*c*x^2 + a*b*(3*c + 2*d*x^
2)) + (2*I)*c*(b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c
*(2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^3*(b/a)^(3
/2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.036, size = 607, normalized size = 2.7

$$\frac{1}{3\,a^2b}\left(2\,x^5abd^2\sqrt{-\frac{b}{a}}+2\,x^5b^2cd\sqrt{-\frac{b}{a}}+EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^2abcd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}+2\,EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*x^5*a*b*d^2*(-b/a)^(1/2)+2*x^5*b^2*c*d*(-b/a)^(1/2)+Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^2*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b*c*d*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x
^2*b^2*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x^3*a^2*d^2*(-
b/a)^(1/2)+5*x^3*a*b*c*d*(-b/a)^(1/2)+2*x^3*b^2*c^2*(-b/a)^(1/2)+
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*a*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*EllipticE(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a^2*c*d*(-b/a)^(1/2)+3
*x*a*b*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/a^2/(-b/a)^(1/2)/(b*x^2+
a)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx^2+c)^{\frac{3}{2}}}{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)^(3/2)/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^
2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^{\frac{3}{2}}}{(bx^2 + a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.174 $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^{7/2}} \, dx$

Optimal. Leaf size=315

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2
*b*c + a*d)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8
*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*
c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
 (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.694953, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$-\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{\sqrt{c+dx^2}\left(-2a^2d^2-3abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\left(a+bx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```


```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2
*b*c + a*d)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8
*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*
c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) -
 (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 85.6364, size = 270, normalized size = 0.86

$$-\frac{x\sqrt{c+dx^2}\,(ad-bc)}{5ab\,(a+bx^2)^{\frac{5}{2}}}+\frac{2x\sqrt{c+dx^2}\,(ad+2bc)}{15a^2b\,(a+bx^2)^{\frac{3}{2}}}-\frac{d\sqrt{c+dx^2}\,(ad-4bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15a^{\frac{3}{2}}b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)}+\frac{\sqrt{c+dx^2}\,(2a^2d^2+3abcd-8b^2c^2)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15a^{\frac{5}{2}}b^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] -x*sqrt(c + d*x**2)*(a*d - b*c)/(5*a*b*(a + b*x**2)**(5/2)) + 2*x
*sqrt(c + d*x**2)*(a*d + 2*b*c)/(15*a**2*b*(a + b*x**2)**(3/2)) -
 d*sqrt(c + d*x**2)*(a*d - 4*b*c)*elliptic_f(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(15*a**(3/2)*b**(3/2)*sqrt(a*(c + d*x**2)/(c
*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)) + sqrt(c + d*x**2)*
(2*a**2*d**2 + 3*a*b*c*d - 8*b**2*c**2)*elliptic_e(atan(sqrt(b)*x
/sqrt(a)), -a*d/(b*c) + 1)/(15*a**(5/2)*b**(3/2)*sqrt(a*(c + d*x*
*2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c))
```

Mathematica [C] time = 0.995741, size = 285, normalized size = 0.9

$$\frac{x\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(-2a^2d^2-3abcd+8b^2c^2\right)+3a^2(bc-ad)^2+2a\left(a+bx^2\right)(ad+2bc)(bc-ad)\right)-ic\left(a+bx^2\right)^2 \cdots}{15a^4\left(\frac{b}{a}\right)^{3/2}\left(a+bx^2\right. \cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 2*a*(b*c - a*d)*(
2*b*c + a*d)*(a + b*x^2) + (8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*(a
 + b*x^2)^2) - I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*((-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*b^2*c^2 - 7*a*b*c*d - a^2*d^2)*E
llipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^4*(b/a)^(3/
2)*(b*c - a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.042, size = 1414, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x)
```

```
[Out] -1/15*(8*x^7*b^4*c^2*d*(-b/a)^(1/2)-6*x^5*a^3*b*d^3*(-b/a)^(1/2)+
20*x^3*a*b^3*c^3*(-b/a)^(1/2)-3*x^7*a*b^3*c*d^2*(-b/a)^(1/2)+8*x^
5*b^4*c^3*(-b/a)^(1/2)-x^3*a^4*d^3*(-b/a)^(1/2)+2*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)+3*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x
^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)-7*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/
2))*x^4*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*x^7
*a^2*b^2*d^3*(-b/a)^(1/2)-10*x^5*a^2*b^2*c*d^2*(-b/a)^(1/2)+17*x^
5*a*b^3*c^2*d*(-b/a)^(1/2)-17*x^3*a^3*b*c*d^2*(-b/a)^(1/2)+7*x^3*
a^2*b^2*c^2*d*(-b/a)^(1/2)-11*x*a^3*b*c^2*d*(-b/a)^(1/2)+8*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^4*c^3*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^4*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)+8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b
^2*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticE(x*(-b/
a)^(1/2),(a*d/b/c)^(1/2))*a^4*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)-8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^
3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-x*a^4*c*d^2*(-b/a)^(1/2
)+15*x*a^2*b^2*c^3*(-b/a)^(1/2)-16*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
-7*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c^2*d*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+3*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a^3*b*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+16
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^3*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-14*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)+4*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b*c*d^2*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+6*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^2*b^2
*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2))/(d*x^2+c)^(1/2)/a
^3/(a*d-b*c)/(-b/a)^(1/2)/(b*x^2+a)^(5/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)^(3/2)/((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2
+ a^3)*sqrt(b*x^2 + a)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

3.175 $\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$

Optimal. Leaf size=235

$$\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}+\frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}+\frac{2\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}(3b+2d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2
 + b*x^2]*Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*
Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[
2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*
b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]
)
```

Rubi [A] time = 0.377011, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3}+\frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}+\frac{2\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}(3b+2d)\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2
 + b*x^2]*Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*
Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[
2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*
b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]
)
```

Rubi in Sympy [A] time = 48.4445, size = 221, normalized size = 0.94

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3}+\frac{2x\left(\frac{3b}{2}+d\right)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}-\frac{2\sqrt{3}\left(\frac{3b}{2}+d\right)\sqrt{bx^2+2}E\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{3b\sqrt{d}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}+\frac{4\sqrt{2}\sqrt{dx^2+3}F\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{2}\right)\middle|1-\frac{2d}{3b}\right)}{3\sqrt{b}\sqrt{\frac{2dx^2+6}{3bx^2+6}}\sqrt{bx^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)/3 + 2*x*(3*b/2 + d)*sqrt(b*x*
*2 + 2)/(3*b*sqrt(d*x**2 + 3)) - 2*sqrt(3)*(3*b/2 + d)*sqrt(b*x**
2 + 2)*elliptic_e(atan(sqrt(3)*sqrt(d)*x/3), -3*b/(2*d) + 1)/(3*b
*sqrt(d)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sqrt(d*x**2 + 3)) +
4*sqrt(2)*sqrt(d*x**2 + 3)*elliptic_f(atan(sqrt(2)*sqrt(b)*x/2),
1 - 2*d/(3*b))/(3*sqrt(b)*sqrt((2*d*x**2 + 6)/(3*b*x**2 + 6))*sqr
t(b*x**2 + 2))
```

Mathematica [C] time = 0.172045, size = 127, normalized size = 0.54

$$\frac{\sqrt{b}dx\sqrt{bx^2+2}\sqrt{dx^2+3}+i\sqrt{3}(3b-2d)F\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-i\sqrt{3}(3b+2d)E\left(i\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{3\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] - I*Sqrt[3]*(3*b + 2
*d)*EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] + I*Sq
rt[3]*(3*b - 2*d)*EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)
/(3*b)])/(3*Sqrt[b]*d)
```

Maple [A] time = 0.038, size = 303, normalized size = 1.3

$$\frac{1}{(3\,bdx^4+9\,bx^2+6\,dx^2+18)\,b}\sqrt{bx^2+2}\sqrt{dx^2+3}\left(x^5b^2d\sqrt{-d}+3\,x^3b^2\sqrt{-d}+2\,x^3bd\sqrt{-d}+3\,EllipticF\left(1/3\,x\sqrt{3}\sqrt{-d},1/2\,\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)*(x^5*b^2*d*(-d)^(1/2)+3*x^3*b
^2*(-d)^(1/2)+2*x^3*b*d*(-d)^(1/2)+3*EllipticF(1/3*x*3^(1/2)*(-d)
^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2))*2^(1/2)*b*(b*x^2+2)^(1/2)
*(d*x^2+3)^(1/2)-2*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)
*2^(1/2)*(b/d)^(1/2))*2^(1/2)*d*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+3
*EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/
2))*2^(1/2)*b*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+2*EllipticE(1/3*x*3
^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2))*2^(1/2)*d*(b*x
^2+2)^(1/2)*(d*x^2+3)^(1/2)+6*x*b*(-d)^(1/2))/(b*d*x^4+3*b*x^2+2*
d*x^2+6)/(-d)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^2+2}\sqrt{dx^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)

[Out] Integral(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + 2}\sqrt{dx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

3.176 $\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x+\frac{2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}$$

```
[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1]
)/Sqrt[3]
```

Rubi [A] time = 0.0347745, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\sqrt{\frac{2}{3}}\sqrt{1-4x^4}x+\frac{2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1]
)/Sqrt[3]
```

Rubi in Sympy [A] time = 5.06177, size = 32, normalized size = 0.84

$$\frac{x\sqrt{-24x^4+6}}{3}+\frac{2\sqrt{3}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(-24*x**4 + 6)/3 + 2*sqrt(3)*elliptic_f(asin(sqrt(2)*x), -1
)/3
```

Mathematica [A] time = 0.0803951, size = 32, normalized size = 0.84

$$\frac{\sqrt{2-8x^4}x+2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] (x*Sqrt[2 - 8*x^4] + 2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]
```

Maple [B] time = 0.059, size = 75, normalized size = 2.

$$-\frac{\sqrt{2}}{36\,x^{4}-9}\sqrt{-6\,x^{2}+3}\sqrt{2\,x^{2}+1}\left(\sqrt{2}\sqrt{3}\sqrt{-6\,x^{2}+3}\sqrt{2\,x^{2}+1}\mathit{EllipticF}\left(x\sqrt{2},i\right)-12\,x^{5}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x)
```

```
[Out] -1/9*(-6*x^2+3)^(1/2)*2^(1/2)*(2*x^2+1)^(1/2)*(2^(1/2)*3^(1/2)*(-
6*x^2+3)^(1/2)*(2*x^2+1)^(1/2)*EllipticF(x*2^(1/2),I)-12*x^5+3*x)
/(4*x^4-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\,x^{2}+2}\sqrt{-6\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{4\,x^{2}+2}\sqrt{-6\,x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{6}\int\sqrt{-2x^2+1}\sqrt{2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(6)*Integral(sqrt(-2*x**2 + 1)*sqrt(2*x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

3.177 $\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx$

Optimal. Leaf size=20

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rubi [A] time = 0.0115642, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]

[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rubi in Sympy [A] time = 3.46163, size = 17, normalized size = 0.85

$$\frac{2\sqrt{6}x^3}{3}+\sqrt{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)

[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

Mathematica [A] time = 0.00358317, size = 15, normalized size = 0.75

$$\sqrt{6}\left(\frac{2x^3}{3}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*(x + (2*x^3)/3)
```

Maple [C] time = 0.007, size = 38, normalized size = 1.9

$$\frac{x\left(2x^{2}+3\right)}{6x^{2}+3}\sqrt{4x^{2}+2}\sqrt{6x^{2}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x)

[Out] 1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

Maxima [A] time = 1.52535, size = 23, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\sqrt{2}\left(2x^{3}+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

Fricas [A] time = 0.218863, size = 51, normalized size = 2.55

$$\frac{\left(2x^{3}+3x\right)\sqrt{6x^{2}+3}\sqrt{4x^{2}+2}}{3\left(2x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2),x, algorithm="fricas")

[Out] 1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

Sympy [A] time = 23.0732, size = 17, normalized size = 0.85

$$\frac{2\sqrt{6}x^{3}}{3}+\sqrt{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

GIAC/XCAS [A] time = 0.238672, size = 23, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\sqrt{2}\left(2\,x^{3}+3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

3.178 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*S
qrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/
(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.246113, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*S
qrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/
(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi in Sympy [A] time = 35.2931, size = 175, normalized size = 0.96

$$-\frac{\sqrt{2}\sqrt{b}\sqrt{dx^2+3}E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{2}\right)\middle|1-\frac{2d}{3b}\right)}{d\sqrt{\frac{2dx^2+6}{3bx^2+6}}\sqrt{bx^2+2}}+\frac{bx\sqrt{dx^2+3}}{d\sqrt{bx^2+2}}+\frac{\sqrt{3}\sqrt{bx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{\sqrt{d}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(b)*sqrt(d*x**2 + 3)*elliptic_e(atan(sqrt(2)*sqrt(b)
*x/2), 1 - 2*d/(3*b))/(d*sqrt((2*d*x**2 + 6)/(3*b*x**2 + 6))*sqrt
```

```
(b*x**2 + 2)) + b*x*sqrt(d*x**2 + 3)/(d*sqrt(b*x**2 + 2)) + sqrt(
3)*sqrt(b*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(d)*x/3), -3*b/(2
*d) + 1)/(sqrt(d)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sqrt(d*x**2
 + 3))
```

Mathematica [A] time = 0.0391646, size = 37, normalized size = 0.2

$$\frac{\sqrt{2}E\left(\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sq
rt[-d]
```

Maple [A] time = 0.023, size = 37, normalized size = 0.2

$$\sqrt{2}EllipticE\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2
))*2^(1/2)/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.179 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 +
(d*x^2)/c])) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/
2], (-4*d)/c])/(d*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.185635, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 +
(d*x^2)/c])) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/
2], (-4*d)/c])/(d*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 34.1494, size = 75, normalized size = 0.82

$$\frac{\sqrt{1+\frac{dx^2}{c}}\,(c+4d)\,F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}\,E\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(1 + d*x**2/c)*(c + 4*d)*elliptic_f(asin(x/2), -4*d/c)/(d*sqr
t(c + d*x**2)) - sqrt(c + d*x**2)*elliptic_e(asin(x/2), -4*d/c)/(
d*sqrt(1 + d*x**2/c))
```

Mathematica [A] time = 0.0568146, size = 60, normalized size = 0.66

$$\frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -c/(4*d)
])/(Sqrt[-(d/c)]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 78, normalized size = 0.9

$$\frac{1}{d}\left(cEllipticF\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+4\,EllipticF\left(x/2,2\sqrt{-\frac{d}{c}}\right)d-cEllipticE\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{dx^2+c}{c}}\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (c*EllipticF(1/2*x,2*(-d/c)^(1/2))+4*EllipticF(1/2*x,2*(-d/c)^(1/
2))*d-c*EllipticE(1/2*x,2*(-d/c)^(1/2)))*((d*x^2+c)/c)^(1/2)/(d*x
^2+c)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.180 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=150

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*Elliptic
E[ArcTan[x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c
*(4 + x^2))]) + (4*Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*
d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi [A] time = 0.187302, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*Elliptic
E[ArcTan[x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c
*(4 + x^2))]) + (4*Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*
d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi in Sympy [A] time = 27.3993, size = 129, normalized size = 0.86

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{2\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|1-\frac{4d}{c}\right)}{d\sqrt{\frac{4c+4dx^2}{c(x^2+4)}}\sqrt{x^2+4}}+\frac{8\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|1-\frac{4d}{c}\right)}{c\sqrt{\frac{4c+4dx^2}{c(x^2+4)}}\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] x*sqrt(c + d*x**2)/(d*sqrt(x**2 + 4)) - 2*sqrt(c + d*x**2)*ellipt
ic_e(atan(x/2), 1 - 4*d/c)/(d*sqrt((4*c + 4*d*x**2)/(c*(x**2 + 4)
))*sqrt(x**2 + 4)) + 8*sqrt(c + d*x**2)*elliptic_f(atan(x/2), 1 -
```

```
4*d/c)/(c*sqrt((4*c + 4*d*x**2)/(c*(x**2 + 4)))*sqrt(x**2 + 4))
```

Mathematica [A] time = 0.0464657, size = 60, normalized size = 0.4

$$\frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], c/(4*d)]
)/(Sqrt[-(d/c)]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.035, size = 53, normalized size = 0.4

$$2\frac{1}{\sqrt{dx^2+c}}EllipticE\left(x\sqrt{-\frac{d}{c}},1/2\sqrt{\frac{c}{d}}\right)\sqrt{\frac{dx^2+c}{c}}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 2*EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2))*((d*x^2+c)/c)^(1/2)/(
d*x^2+c)^(1/2)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(x**2 + 4)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

3.181 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rubi [A] time = 0.0268555, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]

[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 5.19888, size = 19, normalized size = 0.95

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 2/3)/3
```

Mathematica [A] time = 0.0297571, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Maple [A] time = 0.049, size = 29, normalized size = 1.5

$$\frac{\sqrt{2}}{6}\left(EllipticF\left(x, \frac{\sqrt{3}\sqrt{2}}{2}\right) + 2\,EllipticE\left(x, 1/2\,\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(EllipticF(x,1/2*3^(1/2)*2^(1/2))+2*EllipticE(x,1/2*3
^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + 1}}{\sqrt{-3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2 + 1}}{\sqrt{-3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="giac")

[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rubi [A] time = 0.0267, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]

[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rubi in Sympy [A] time = 5.18334, size = 20, normalized size = 0.95

$$\frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{1}{6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] 2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 1/6)/3
```

Mathematica [A] time = 0.0264415, size = 21, normalized size = 1.

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] time = 0.118, size = 24, normalized size = 1.1

$$\frac{2\sqrt{3}}{3}EllipticE\left(\frac{x\sqrt{3}\sqrt{2}}{2},\frac{\sqrt{3}\sqrt{2}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*3^(1/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),1/6*3^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(x-2\right)\left(x+2\right)}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.183 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Rubi [A] time = 0.0264847, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 5.26896, size = 19, normalized size = 0.95

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{8}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 8/3)/3
```

Mathematica [A] time = 0.0280647, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Maple [A] time = 0.111, size = 35, normalized size = 1.8

$$-\frac{\sqrt{2}}{12}\left(5\,EllipticF\left(2\,x,1/4\,\sqrt{3}\sqrt{2}\right)-8\,EllipticE\left(2\,x,1/4\,\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/12*2^(1/2)*(5*EllipticF(2*x,1/4*3^(1/2)*2^(1/2))-8*EllipticE(2
*x,1/4*3^(1/2)*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\left(2x-1\right)\left(2x+1\right)}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.184 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] EllipticE[ArcSin[x], -1]
```

Rubi [A] time = 0.0194454, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

```
[Out] EllipticE[ArcSin[x], -1]
```

Rubi in Sympy [A] time = 5.44906, size = 5, normalized size = 1.25

$$E(\operatorname{asin}(x)|-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] elliptic_e(asin(x), -1)
```

Mathematica [A] time = 0.0252287, size = 4, normalized size = 1.

$$E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
```

```
[Out] EllipticE[ArcSin[x], -1]
```

Maple [A] time = 0.02, size = 5, normalized size = 1.3

$$EllipticE(x, i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x)

[Out] EllipticE(x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1),x, algorithm="fricas")

[Out] integral(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)

[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

3.185 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\mid -\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0238563, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\mid -\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]

[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 5.1441, size = 20, normalized size = 1.

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle| -\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3
```

Mathematica [A] time = 0.0254786, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\mid -\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] time = 0.028, size = 25, normalized size = 1.3

$$\frac{\sqrt{3}}{3} EllipticE\left(\frac{x\sqrt{3}\sqrt{2}}{2}, \frac{i}{3}\sqrt{3}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),1/3*I*3^(1/2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-3x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.186 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rubi [A] time = 0.0239248, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]

[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rubi in Sympy [A] time = 4.86515, size = 22, normalized size = 1.05

$$\frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{1}{6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] 2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3
```

Mathematica [A] time = 0.0260063, size = 21, normalized size = 1.

$$\frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] time = 0.025, size = 25, normalized size = 1.2

$$\frac{2\sqrt{3}}{3} EllipticE\left(\frac{x\sqrt{3}\sqrt{2}}{2}, \frac{i}{6}\sqrt{3}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*3^(1/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),1/6*I*3^(1/2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

3.187 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rubi [A] time = 0.0255711, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]

[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 5.41111, size = 20, normalized size = 1.

$$\frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/3
```

Mathematica [A] time = 0.0252041, size = 20, normalized size = 1.

$$\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Maple [A] time = 0.036, size = 25, normalized size = 1.3

$$\frac{\sqrt{3}}{3} EllipticE\left(\frac{x\sqrt{3}\sqrt{2}}{2}, \frac{2\,i}{3}\sqrt{3}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*EllipticE(1/2*x*3^(1/2)*2^(1/2),2/3*I*3^(1/2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 1)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

3.188 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=13

$$2F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

```
[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rubi [A] time = 0.0511525, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$2F\left(\sin^{-1}(x)\middle|-1\right)-E\left(\sin^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]

[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rubi in Sympy [A] time = 13.1367, size = 14, normalized size = 1.08

$$-E\left(\operatorname{asin}(x)\middle|-1\right)+2F\left(\operatorname{asin}(x)\middle|-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)

[Out] -elliptic_e(asin(x), -1) + 2*elliptic_f(asin(x), -1)
```

Mathematica [C] time = 0.0223889, size = 12, normalized size = 0.92

$$-iE\left(i\sinh^{-1}(x)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]

[Out] (-I)*EllipticE[I*ArcSinh[x], -1]
```

Maple [A] time = 0.011, size = 14, normalized size = 1.1

$$-EllipticE(x, i) + 2\, EllipticF(x, i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x)

[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1),x, algorithm="fricas")

[Out] integral(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

3.189 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 + (5*EllipticF[ArcSin[x],
 -3/2])/(3*Sqrt[2])
```

Rubi [A] time = 0.0659232, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{5F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 + (5*EllipticF[ArcSin[x],
 -3/2])/(3*Sqrt[2])
```

Rubi in Sympy [A] time = 12.4154, size = 31, normalized size = 1.

$$-\frac{\sqrt{2}E\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{3}+\frac{5\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*elliptic_e(asin(x), -3/2)/3 + 5*sqrt(2)*elliptic_f(asin(
x), -3/2)/6
```

Mathematica [C] time = 0.0273486, size = 27, normalized size = 0.87

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3])/Sqrt[3]
```

Maple [A] time = 0.024, size = 33, normalized size = 1.1

$$\frac{\left(5\,\mathit{EllipticF}\left(x,i/2\sqrt{3}\sqrt{2}\right)-2\,\mathit{EllipticE}\left(x,i/2\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/6*(5*EllipticF(x,1/2*I*3^(1/2)*2^(1/2))-2*EllipticE(x,1/2*I*3^(
1/2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.190 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{7}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 + (7*Sqrt[2]*EllipticF[Ar
cSin[x/2], -6])/3
```

Rubi [A] time = 0.0670297, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{7}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -(Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 + (7*Sqrt[2]*EllipticF[Ar
cSin[x/2], -6])/3
```

Rubi in Sympy [A] time = 12.424, size = 31, normalized size = 0.89

$$-\frac{\sqrt{2}E\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-6\right)}{3}+\frac{7\sqrt{2}F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*elliptic_e(asin(x/2), -6)/3 + 7*sqrt(2)*elliptic_f(asin(
x/2), -6)/3
```

Mathematica [C] time = 0.0267695, size = 27, normalized size = 0.77

$$-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] time = 0.035, size = 37, normalized size = 1.1

$$\frac{\sqrt{2}}{3}\left(7\,EllipticF\left(x/2, i\sqrt{3}\sqrt{2}\right) - EllipticE\left(\frac{x}{2}, i\sqrt{3}\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*(7*EllipticF(1/2*x,I*3^(1/2)*2^(1/2))-EllipticE(1/2*x,I*3^(1/
2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 + 4}}{\sqrt{3\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2 + 4}}{\sqrt{3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.191 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{11F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)$$

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSi
n[2*x], -3/8])/(6*Sqrt[2])
```

Rubi [A] time = 0.0671248, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{11F\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)\middle|-\frac{3}{8}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSi
n[2*x], -3/8])/(6*Sqrt[2])
```

Rubi in Sympy [A] time = 12.4344, size = 36, normalized size = 1.03

$$-\frac{2\sqrt{2}E\left(\operatorname{asin}(2x)\middle|-\frac{3}{8}\right)}{3}+\frac{11\sqrt{2}F\left(\operatorname{asin}(2x)\middle|-\frac{3}{8}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] -2*sqrt(2)*elliptic_e(asin(2*x), -3/8)/3 + 11*sqrt(2)*elliptic_f(
asin(2*x), -3/8)/12
```

Mathematica [C] time = 0.0263359, size = 27, normalized size = 0.77

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -8/3])/Sqrt[3]
```

Maple [A] time = 0.029, size = 37, normalized size = 1.1

$$\frac{\left(11\,EllipticF\left(2\,x,i/4\sqrt{3}\sqrt{2}\right)-8\,EllipticE\left(2\,x,i/4\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/12*(11*EllipticF(2*x,1/4*I*3^(1/2)*2^(1/2))-8*EllipticE(2*x,1/4
*I*3^(1/2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(2x-1)(2x+1)}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.192 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+1}}+\frac{\sqrt{3x^2+2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1
+ x^2)]) + (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.131477, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+1}}+\frac{\sqrt{3x^2+2}\,F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1
+ x^2)]) + (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 19.3861, size = 119, normalized size = 0.91

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{x^2+1}}+\frac{\sqrt{2}\sqrt{3x^2+2}\,F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{2\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(3*x**2 + 2)/(3*sqrt(x**2 + 1)) - sqrt(2)*sqrt(3*x**2 + 2)*
elliptic_e(atan(x), -1/2)/(3*sqrt((3*x**2 + 2)/(x**2 + 1))*sqrt(x
**2 + 1)) + sqrt(2)*sqrt(3*x**2 + 2)*elliptic_f(atan(x), -1/2)/(2
*sqrt((3*x**2 + 2)/(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.0242048, size = 27, normalized size = 0.21

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[3]
```

Maple [A] time = 0.077, size = 36, normalized size = 0.3

$$-\frac{i}{6}\left(EllipticF\left(ix,\frac{\sqrt{3}\sqrt{2}}{2}\right)+2\,EllipticE\left(ix,1/2\,\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/6*I*(EllipticF(I*x,1/2*3^(1/2)*2^(1/2))+2*EllipticE(I*x,1/2*3^
(1/2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.193 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+4}}+\frac{2\sqrt{2}\sqrt{3x^2+2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4
+ x^2)]) + (2*Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])
/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```

Rubi [A] time = 0.144487, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+4}}+\frac{2\sqrt{2}\sqrt{3x^2+2}\,F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}-\frac{\sqrt{2}\sqrt{3x^2+2}\,E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*
EllipticE[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4
+ x^2)]) + (2*Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])
/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```

Rubi in Sympy [A] time = 19.0852, size = 114, normalized size = 0.84

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{2\sqrt{3x^2+2}\,E\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{\frac{12x^2+8}{2x^2+8}}\sqrt{x^2+4}}+\frac{4\sqrt{3x^2+2}\,F\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{\frac{12x^2+8}{2x^2+8}}\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] x*sqrt(3*x**2 + 2)/(3*sqrt(x**2 + 4)) - 2*sqrt(3*x**2 + 2)*ellipt
ic_e(atan(x/2), -5)/(3*sqrt((12*x**2 + 8)/(2*x**2 + 8))*sqrt(x**2
 + 4)) + 4*sqrt(3*x**2 + 2)*elliptic_f(atan(x/2), -5)/(sqrt((12*x
**2 + 8)/(2*x**2 + 8))*sqrt(x**2 + 4))
```

Mathematica [C] time = 0.0246886, size = 27, normalized size = 0.2

$$-\frac{2iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] time = 0.049, size = 34, normalized size = 0.3

$$-\frac{i}{3}\left(5\,EllipticF\left(i/2x,\sqrt{3}\sqrt{2}\right)+EllipticE\left(\frac{i}{2}x,\sqrt{3}\sqrt{2}\right)\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*(5*EllipticF(1/2*I*x,3^(1/2)*2^(1/2))+EllipticE(1/2*I*x,3^
(1/2)*2^(1/2)))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 4)/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

3.194 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=148

$$\frac{4\sqrt{3x^2+2}x}{3\sqrt{4x^2+1}}+\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}-\frac{2\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3
*x^2]*EllipticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2
)]*Sqrt[1 + 4*x^2]) + (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8
])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rubi [A] time = 0.16047, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{4\sqrt{3x^2+2}x}{3\sqrt{4x^2+1}}+\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}-\frac{2\sqrt{2}\sqrt{3x^2+2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3
*x^2]*EllipticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2
)]*Sqrt[1 + 4*x^2]) + (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8
])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rubi in Sympy [A] time = 20.2569, size = 121, normalized size = 0.82

$$\frac{4x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{2\sqrt{3x^2+2}E\left(\operatorname{atan}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{8x^2+2}}\sqrt{4x^2+1}}+\frac{\sqrt{3x^2+2}F\left(\operatorname{atan}(2x)|\frac{5}{8}\right)}{4\sqrt{\frac{3x^2+2}{8x^2+2}}\sqrt{4x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x*sqrt(3*x**2 + 2)/(3*sqrt(4*x**2 + 1)) - 2*sqrt(3*x**2 + 2)*el
liptic_e(atan(2*x), 5/8)/(3*sqrt((3*x**2 + 2)/(8*x**2 + 2))*sqrt(
4*x**2 + 1)) + sqrt(3*x**2 + 2)*elliptic_f(atan(2*x), 5/8)/(4*sqr
t((3*x**2 + 2)/(8*x**2 + 2))*sqrt(4*x**2 + 1))
```

Mathematica [C] time = 0.0252582, size = 27, normalized size = 0.18

$$-\frac{iE\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3])/Sqrt[3]
```

Maple [C] time = 0.113, size = 26, normalized size = 0.2

$$-\frac{i}{3}EllipticE\left(\frac{i}{2}\sqrt{3}\sqrt{2}x,\frac{2\sqrt{3}\sqrt{2}}{3}\right)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*EllipticE(1/2*I*3^(1/2)*2^(1/2)*x,2/3*3^(1/2)*2^(1/2))*3^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 1)/sqrt(3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

3.195 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt
[-1 + 2*x^2])
```

Rubi [A] time = 0.0510347, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt
[-1 + 2*x^2])
```

Rubi in Sympy [A] time = 10.4025, size = 37, normalized size = 0.92

$$\frac{\sqrt{2}\sqrt{-2x^2+1}E\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2\sqrt{2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-2*x**2 + 1)*elliptic_e(asin(sqrt(2)*x), 1/2)/(2*sqr
t(2*x**2 - 1))
```

Mathematica [A] time = 0.0345962, size = 35, normalized size = 0.88

$$\frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/Sqrt[-2 + 4*x
^2]
```

Maple [A] time = 0.048, size = 32, normalized size = 0.8

$$\frac{EllipticF\left(x,\sqrt{2}\right)+EllipticE\left(x,\sqrt{2}\right)}{2}\sqrt{-2\,x^{2}+1}\frac{1}{\sqrt{2\,x^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(x,2^(1/2))+EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/
(2*x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^{2}+1}}{\sqrt{2\,x^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^{2}+1}}{\sqrt{2\,x^{2}-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(2*x**2 - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

3.196 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(3bc-7ad)\left(15a^2d^2-11abcd+8b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{8\sqrt{c}\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}-\frac{8x\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^3\sqrt{c+dx^2}}-\frac{6bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{35d^2}+\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a
+ b*x^2])/(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d
 + 71*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*
b*(b*c - 2*a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (
b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*d) + (8*Sqrt[c]*(b*c -
2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c
]*(3*b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
105*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)
```

Rubi [A] time = 1.06507, antiderivative size = 423, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(3bc-7ad)\left(15a^2d^2-11abcd+8b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{8\sqrt{c}\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}-\frac{8x\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^3\sqrt{c+dx^2}}-\frac{6bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{35d^2}+\frac{bx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a
+ b*x^2])/(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d
 + 71*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*
b*(b*c - 2*a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (
b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*d) + (8*Sqrt[c]*(b*c -
2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[a + b*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c
]*(3*b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
105*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)
```

Rubi in Sympy [A] time = 130.858, size = 408, normalized size = 0.96

$$-\frac{8\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,(2ad-bc)\left(11a^2d^2-11abcd+6b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{105d^4\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}{7d}+\frac{6bx\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,(2ad-bc)}{35d^2}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}+\frac{8bx\sqrt{c+dx^2}\,(2ad-bc)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^4\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\,(7ad-3bc)\left(15a^2d^2-11abcd+8b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{105d^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -8*sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*(2*a*d - b*c)*(11*a**2*d**2 -
 11*a*b*c*d + 6*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(105*d**4*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) + b*x*(a + b*x**2)**(5/2)*sqrt(c + d*x**2)/(7*d) +
 6*b*x*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(2*a*d - b*c)/(35*d**
2) + b*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(71*a**2*d**2 - 71*a*b
*c*d + 24*b**2*c**2)/(105*d**3) + 8*b*x*sqrt(c + d*x**2)*(2*a*d -
 b*c)*(11*a**2*d**2 - 11*a*b*c*d + 6*b**2*c**2)/(105*d**4*sqrt(a
+ b*x**2)) + sqrt(c)*sqrt(a + b*x**2)*(7*a*d - 3*b*c)*(15*a**2*d*
*2 - 11*a*b*c*d + 8*b**2*c**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c))
, 1 - b*c/(a*d))/(105*d**(7/2)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2
)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 2.72893, size = 321, normalized size = 0.76

$$bdx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)\left(122a^2d^2+abd\left(66dx^2-89c\right)+3b^2\left(8c^2-6cdx^2+5d^2x^4\right)\right)-8ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(22a^3d^3\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(122*a^2*d^2 + a*b*d*(-8
9*c + 66*d*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - (8*I)*
b*c*(-6*b^3*c^3 + 23*a*b^2*c^2*d - 33*a^2*b*c*d^2 + 22*a^3*d^3)*S
```

```
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b
/a]*x], (a*d)/(b*c)] - I*(48*b^4*c^4 - 208*a*b^3*c^3*d + 353*a^2*
b^2*c^2*d^2 - 298*a^3*b*c*d^3 + 105*a^4*d^4)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 852, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/105*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(15*(-b/a)^(1/2)*x^9*b^4*d^
4+81*(-b/a)^(1/2)*x^7*a*b^3*d^4-3*(-b/a)^(1/2)*x^7*b^4*c*d^3+188*
(-b/a)^(1/2)*x^5*a^2*b^2*d^4-26*(-b/a)^(1/2)*x^5*a*b^3*c*d^3+6*(-
b/a)^(1/2)*x^5*b^4*c^2*d^2+122*(-b/a)^(1/2)*x^3*a^3*b*d^4+99*(-b/
a)^(1/2)*x^3*a^2*b^2*c*d^3-83*(-b/a)^(1/2)*x^3*a*b^3*c^2*d^2+24*(
-b/a)^(1/2)*x^3*b^4*c^3*d+105*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*d^4-298*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*a^3*b*c*d^3+353*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2-208*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4+176*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a^3*b*c*d^3-264*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^2+184*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4+122*(-b/
a)^(1/2)*x*a^3*b*c*d^3-89*(-b/a)^(1/2)*x*a^2*b^2*c^2*d^2+24*(-b/a
)^(1/2)*x*a*b^3*c^3*d)/d^4/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\sqrt{bx^{2}+a}}{\sqrt{dx^{2}+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 +
 a)/sqrt(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{\frac{7}{2}}}{\sqrt{dx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

3.197 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=344

$$\frac{x\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15d^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{15d^2}+\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2
*Sqrt[c + d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(15*d^2) + (b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d)
 - (Sqrt[c]*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(
5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sq
rt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.728836, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{x\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15d^2\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{15d^2}+\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2
*Sqrt[c + d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(15*d^2) + (b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d)
 - (Sqrt[c]*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(
5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sq
rt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 91.941, size = 328, normalized size = 0.95

$$\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15\sqrt{b}cd^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15d^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}}{5d}+\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(2ad-bc\right)}{15d^2}+\frac{bx\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)}{15d^3\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(c + d*x**2)*(15*a**2*d**2 - 11*a*b*c*d + 4*b**2*c**
2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(15*sqrt(b
)*c*d**2*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2))
- sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*(23*a**2*d**2 - 23*a*b*c*d + 8
*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(
15*d**3*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) +
 b*x*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(5*d) + 4*b*x*sqrt(a +
b*x**2)*sqrt(c + d*x**2)*(2*a*d - b*c)/(15*d**2) + b*x*sqrt(c + d
*x**2)*(23*a**2*d**2 - 23*a*b*c*d + 8*b**2*c**2)/(15*d**3*sqrt(a
+ b*x**2))
```

Mathematica [C] time = 0.845173, size = 260, normalized size = 0.76

$$\frac{-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(15a^3d^3-34a^2bcd^2+2\right.}{15d^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 11*a*d + 3*b*d
*x^2) - I*b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*(-8*b^3*c^3 + 27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a
^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSi
nh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])
```

Maple [A] time = 0.025, size = 615, normalized size = 1.8

$$\frac{1}{15\,d^3\left(bdx^4+adx^2+cx^2b+ac\right)}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}x^7b^3d^3+14\sqrt{-\frac{b}{a}}x^5ab^2d^3-\sqrt{-\frac{b}{a}}x^5b^3cd^2+11\sqrt{-\frac{b}{a}}x^3a^2bd^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^3*d^3+
14*(-b/a)^(1/2)*x^5*a*b^2*d^3-(-b/a)^(1/2)*x^5*b^3*c*d^2+11*(-b/a
)^(1/2)*x^3*a^2*b*d^3+10*(-b/a)^(1/2)*x^3*a*b^2*c*d^2-4*(-b/a)^(1
/2)*x^3*b^3*c^2*d+15*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*d^3-34*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a^2*b*c*d^2+27*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
^3*c^3+23*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d^2-23*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*
c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3+11*(-b/a)^(1/2)*x*a^2*b*c*d^2-4*
(-b/a)^(1/2)*x*a*b^2*c^2*d)/d^3/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b
/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\sqrt{bx^{2}+a}}{\sqrt{dx^{2}+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/sqrt(d*x^2 +
 c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^{2}\right)^{\frac{5}{2}}}{\sqrt{c+dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}}{\sqrt{dx^{2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

3.198 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2x\sqrt{a+bx^2}(bc-2ad)}{3d\sqrt{c+dx^2}}$$

```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.461426, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$-\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2\sqrt{c}\sqrt{a+bx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{2x\sqrt{a+bx^2}(bc-2ad)}{3d\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x
*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 61.7489, size = 236, normalized size = 0.91

$$-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx^2}\,(2ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}+\frac{2bx\sqrt{c+dx^2}\,(2ad-bc)}{3d^2\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\,(3ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -2*sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*(2*a*d - b*c)*elliptic_e(atan
(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*d**2*sqrt(a*(c + d*x**2)/
(c*(a + b*x**2)))*sqrt(a + b*x**2)) + b*x*sqrt(a + b*x**2)*sqrt(c
 + d*x**2)/(3*d) + 2*b*x*sqrt(c + d*x**2)*(2*a*d - b*c)/(3*d**2*s
qrt(a + b*x**2)) + sqrt(c)*sqrt(a + b*x**2)*(3*a*d - b*c)*ellipti
c_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(3*d**(3/2)*sqrt(c*(a
 + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.596973, size = 216, normalized size = 0.83

$$\frac{-i\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-5abcd+2b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)+bdx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(c+dx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}}{3d^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - (2*I)*b*c*(-(b*c) + 2*
a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqr
t[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.022, size = 399, normalized size = 1.5

$$\frac{1}{\left(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac\right)d^2}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5b^2d^2+\sqrt{-\frac{b}{a}}x^3abd^2+\sqrt{-\frac{b}{a}}x^3b^2cd+3\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b^2*d^2+(-b
/a)^(1/2)*x^3*a*b*d^2+(-b/a)^(1/2)*x^3*b^2*c*d+3*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a^2*d^2-5*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+4*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*
x*a*b*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^2/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

3.199 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])
```

Rubi [A] time = 0.27356, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])
```

Rubi in Sympy [A] time = 39.1481, size = 172, normalized size = 0.89

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) + b*x*sqrt(c + d*x**2)/(d*sqrt(a + b*x**2)) + sqrt
(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/
(a*d))/(sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2))))*sqrt(c + d*
x**2))
```

Mathematica [A] time = 0.0810411, size = 86, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*
x^2])
```

Maple [A] time = 0.018, size = 158, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,d}\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(aEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*Ellipti
cF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2)))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.200 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.0606109, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 10.5829, size = 73, normalized size = 0.84

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 -
b*c/(a*d))/(a*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(
c + d*x**2))
```

Mathematica [A] time = 0.0842963, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(
b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Maple [A] time = 0.023, size = 100, normalized size = 1.2

$$\frac{1}{bdx^4+adx^2+cx^2b+ac}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b
*x^2+a)/a)^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.201 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=273

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*S
qrt[c + d*x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.403152, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)}-\frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*S
qrt[c + d*x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 59.0922, size = 160, normalized size = 0.59

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)}-\frac{\sqrt{b}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{a}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(d)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c
)), 1 - b*c/(a*d))/(a*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(
c + d*x**2)*(a*d - b*c)) - sqrt(b)*sqrt(c + d*x**2)*elliptic_e(at
an(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(sqrt(a)*sqrt(a*(c + d*x**
2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.402386, size = 112, normalized size = 0.41

$$\frac{\frac{ad\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)\middle|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}}-bx\left(c+dx^2\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*x*(c + d*x^2)) + (a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/Sqrt[-(d/c)])/(
a*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.037, size = 248, normalized size = 0.9

$$\frac{1}{a\,(ad-bc)\,(bdx^4+adx^2+cx^2b+ac)}\left(-x^3bd\sqrt{-\frac{b}{a}}+EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)ad\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-EllipticF\left(x\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-x^3*b*d*(-b/a)^(1/2)+EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
a*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-EllipticF(x*(-b/a)^(1
```

```
/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)-x*b*c*(-b/a)^(1/2))*(d*x^2+c)^(1/2)*(b*x^2+a
)^(1/2)/a/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

<u>GIAC/XCAS</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

3.202 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=255

$$\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sq
rt[b]*(b*c - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/
Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x
^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*
c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.403865, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sq
rt[b]*(b*c - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/
Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x
^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*
c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 58.2299, size = 221, normalized size = 0.87

$$-\frac{bx\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d\sqrt{c+dx^2}\,(3ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3\sqrt{a}\sqrt{b}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}-\frac{2\sqrt{b}\sqrt{c+dx^2}\,(2ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3a^{\frac{3}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*x*sqrt(c + d*x**2)/(3*a*(a + b*x**2)**(3/2)*(a*d - b*c)) + d*s
qrt(c + d*x**2)*(3*a*d - b*c)*elliptic_f(atan(sqrt(b)*x/sqrt(a)),
 -a*d/(b*c) + 1)/(3*sqrt(a)*sqrt(b)*c*sqrt(a*(c + d*x**2)/(c*(a +
 b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)**2) - 2*sqrt(b)*sqrt(c +
d*x**2)*(2*a*d - b*c)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b
*c) + 1)/(3*a**(3/2)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a
 + b*x**2)*(a*d - b*c)**2)
```

Mathematica [C] time = 1.07125, size = 261, normalized size = 1.02

$$\frac{-i\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-5abcd+2b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+bx\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(-5a^2d+ab\left(3c-4d\right.\right.}{3a^2\sqrt{\frac{b}{a}}\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(-5*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 4
*d*x^2)) - (2*I)*b*c*(-(b*c) + 2*a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(3/2)
*Sqrt[c + d*x^2])
```

Maple [B] time = 0.041, size = 752, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(-4*x^5*a*b^2*d^2*(-b/a)^(1/2)+2*x^5*b^3*c*d*(-b/a)^(1/2)+3*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b*d^2*((d*x^2+c)
/c)^(1/2)*((b*x^2+a)/a)^(1/2)-5*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*x^2*a*b^2*c*d*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)+2*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^3*c^2*((d*x^2+c)/
c)^(1/2)*((b*x^2+a)/a)^(1/2)+4*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*x^2*a*b^2*c*d*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)-2*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^3*c^2*((d*x^2+c)/c
)^(1/2)*((b*x^2+a)/a)^(1/2)-5*x^3*a^2*b*d^2*(-b/a)^(1/2)-x^3*a*b^
2*c*d*(-b/a)^(1/2)+2*x^3*b^3*c^2*(-b/a)^(1/2)+3*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a^3*d^2*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a
)^(1/2)-5*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d*((d
*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a*b^2*c^2*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
+4*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d*((d*x^2+c)
/c)^(1/2)*((b*x^2+a)/a)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*b^2*c^2*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)-5*x*a^
2*b*c*d*(-b/a)^(1/2)+3*x*a*b^2*c^2*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/
(a*d-b*c)^2/(-b/a)^(1/2)/a^2/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)^{\frac{5}{2}} \sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}} \sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

3.203 $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=334

$$\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{a+bx^2}(bc-ad)^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*
(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2
)^(3/2)) + (Sqrt[b]*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])
/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(
c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 11*a*b*c*d + 15*
a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.703619, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{4bx\sqrt{c+dx^2}(bc-2ad)}{15a^2\left(a+bx^2\right)^{3/2}(bc-ad)^2}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{a+bx^2}(bc-ad)^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```


```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*
(b*c - 2*a*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2
)^(3/2)) + (Sqrt[b]*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])
/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(
c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 11*a*b*c*d + 15*
a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 120.703, size = 301, normalized size = 0.9

$$-\frac{bx\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{\frac{5}{2}}(ad-bc)}-\frac{4bx\sqrt{c+dx^2}\,(2ad-bc)}{15a^2\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)^2}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\left(15a^2d^2-11abcd+4b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{15a^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)^3}-\frac{\sqrt{b}\sqrt{c+dx^2}\left(23a^2d^2-23abcd+8b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15a^{\frac{5}{2}}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -b*x*sqrt(c + d*x**2)/(5*a*(a + b*x**2)**(5/2)*(a*d - b*c)) - 4*b
*x*sqrt(c + d*x**2)*(2*a*d - b*c)/(15*a**2*(a + b*x**2)**(3/2)*(a
*d - b*c)**2) + sqrt(c)*sqrt(d)*sqrt(a + b*x**2)*(15*a**2*d**2 -
11*a*b*c*d + 4*b**2*c**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 -
 b*c/(a*d))/(15*a**3*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c
 + d*x**2)*(a*d - b*c)**3) - sqrt(b)*sqrt(c + d*x**2)*(23*a**2*d*
*2 - 23*a*b*c*d + 8*b**2*c**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a))
, -a*d/(b*c) + 1)/(15*a**(5/2)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2
)))*sqrt(a + b*x**2)*(a*d - b*c)**3)
```

Mathematica [C] time = 1.05424, size = 301, normalized size = 0.9

$$\frac{bx\sqrt{\frac{b}{a}}\left(c+dx^2\right)\left(\left(a+bx^2\right)^2\left(23a^2d^2-23abcd+8b^2c^2\right)+3a^2(bc-ad)^2+4a\left(a+bx^2\right)(bc-2ad)(bc-ad)\right)+i\sqrt{\frac{bx^2}{a}+1}\,(}{15a^3\sqrt{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 4*a*(b*c - 2*a*
d)*(b*c - a*d)*(a + b*x^2) + (8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2
)*(a + b*x^2)^2) + I*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*(b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b^3*c^3 + 27*a*b^2*c^2*d
 - 34*a^2*b*c*d^2 + 15*a^3*d^3)*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^3*(a + b*x^2)^(5/2)
*Sqrt[c + d*x^2])
```

Maple [B] time = 0.049, size = 1607, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(23*x^7*a*b^4*c*d^2*(-b/a)^(1/2)+54*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*x^2*a^2*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)+46*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^3*b
^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-46*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^3*c^2*d*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)-34*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*x^4*a^2*b^3*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+27*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a*b^4*c^2*d*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+23*EllipticE(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*x^4*a^2*b^3*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)-23*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a*b^4*c^2*d*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-20*x^3*a*b^4*c^3*(-b/a)^(
1/2)-15*x*a^2*b^3*c^3*(-b/a)^(1/2)+15*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a^5*d^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-23
*x^7*a^2*b^3*d^3*(-b/a)^(1/2)-8*x^7*b^5*c^2*d*(-b/a)^(1/2)-54*x^5
*a^3*b^2*d^3*(-b/a)^(1/2)+16*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*x^2*a*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-34*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*b*c*d^2*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)+27*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a^3*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+23*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^4*b*c*d^2*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)-23*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a^3*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+15*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*a^3*b^2*d^3*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+30*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*x^2*a^4*b*d^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)-16*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^4*c^3*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-68*EllipticF(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*x^2*a^3*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)+35*x^5*a^2*b^3*c*d^2*(-b/a)^(1/2)+3*x^5*a*b^4*c^2*d*(-b/
a)^(1/2)-13*x^3*a^3*b^2*c*d^2*(-b/a)^(1/2)+43*x^3*a^2*b^3*c^2*d*(
-b/a)^(1/2)-34*x*a^4*b*c*d^2*(-b/a)^(1/2)-34*x^3*a^4*b*d^3*(-b/a)
^(1/2)+41*x*a^3*b^2*c^2*d*(-b/a)^(1/2)-8*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*x^4*b^5*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)+8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^4*b^5*c^3*((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-8*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*a^2*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)+8*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^3*c^3*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-8*x^5*b^5*c^3*(-b/a)^(1/2))/(d*x
^2+c)^(1/2)/(a*d-b*c)^3/(-b/a)^(1/2)/a^3/(b*x^2+a)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{2}} \sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{2}} \sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

3.204 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=445

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{15cd^3}+\frac{x\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)}{15cd^3\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5cd^2}-\frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*
Sqrt[a + b*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a +
 b*x^2)^(5/2))/(c*d*Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*
d + 15*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (
b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*c*d^2)
- ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*
d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rubi [A] time = 1.01467, antiderivative size = 445, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{b\sqrt{c}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{15cd^3} + \frac{x\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)}{15cd^3\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5cd^2} - \frac{x\left(a+bx^2\right)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*
Sqrt[a + b*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a +
 b*x^2)^(5/2))/(c*d*Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*
d + 15*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (
b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*c*d^2)
- ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*
d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rubi in Sympy [A] time = 143.453, size = 418, normalized size = 0.94

$$\frac{a^{\frac{3}{2}}\sqrt{b}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{15cd^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}} - \frac{bx\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}(5ad-6bc)}{5cd^2} - \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(15a^2d^2-43abcd+24b^2c^2\right)}{15cd^3} + \frac{x\left(a+bx^2\right)^{\frac{5}{2}}(ad-bc)}{cd\sqrt{c+dx^2}} - \frac{x\sqrt{a+bx^2}\left(15a^3d^3-103a^2bcd^2+128ab^2c^2d-48b^3c^3\right)}{15cd^3\sqrt{c+dx^2}} + \frac{\sqrt{a+bx^2}\left(15a^3d^3-103a^2bcd^2+128ab^2c^2d-48b^3c^3\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{\frac{7}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] a**(3/2)*sqrt(b)*sqrt(c + d*x**2)*(45*a**2*d**2 - 61*a*b*c*d + 24
*b**2*c**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(
15*c*d**3*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2))
 - b*x*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(5*a*d - 6*b*c)/(5*c*
d**2) - b*x*sqrt(a + b*x**2)*sqrt(c + d*x**2)*(15*a**2*d**2 - 43*
a*b*c*d + 24*b**2*c**2)/(15*c*d**3) + x*(a + b*x**2)**(5/2)*(a*d
- b*c)/(c*d*sqrt(c + d*x**2)) - x*sqrt(a + b*x**2)*(15*a**3*d**3
- 103*a**2*b*c*d**2 + 128*a*b**2*c**2*d - 48*b**3*c**3)/(15*c*d**
3*sqrt(c + d*x**2)) + sqrt(a + b*x**2)*(15*a**3*d**3 - 103*a**2*b
*c*d**2 + 128*a*b**2*c**2*d - 48*b**3*c**3)*elliptic_e(atan(sqrt(
d)*x/sqrt(c)), 1 - b*c/(a*d))/(15*sqrt(c)*d**(7/2)*sqrt(c*(a + b*
x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 2.09831, size = 318, normalized size = 0.71

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)\left(15a^3d^3-45a^2bcd^2+ab^2cd\left(61c+16dx^2\right)-3b^3c\left(8c^2+2cdx^2-d^2x^4\right)\right)+4ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(-15a\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-45*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*
c*d*(61*c + 16*d*x^2) - 3*b^3*c*(8*c^2 + 2*c*d*x^2 - d^2*x^4)) +
I*b*c*(-48*b^3*c^3 + 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d
^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + (4*I)*b*c*(12*b^3*c^3 - 38*a*b^2*c^2*
d + 41*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[
b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.061, size = 755, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/15*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(3*(-b/a)^(1/2)*x^7*b^4*c*d^
3+19*(-b/a)^(1/2)*x^5*a*b^3*c*d^3-6*(-b/a)^(1/2)*x^5*b^4*c^2*d^2+
```

```
15*(-b/a)^(1/2)*x^3*a^3*b*d^4-29*(-b/a)^(1/2)*x^3*a^2*b^2*c*d^3+5
5*(-b/a)^(1/2)*x^3*a*b^3*c^2*d^2-24*(-b/a)^(1/2)*x^3*b^4*c^3*d+60
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^3*b*c*d^3-164*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^
2+152*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d-48*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4-15
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^3*b*c*d^3+103*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d^
2-128*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*d+48*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^4*c^4+15
*x*a^4*d^4*(-b/a)^(1/2)-45*(-b/a)^(1/2)*x*a^3*b*c*d^3+61*(-b/a)^(
1/2)*x*a^2*b^2*c^2*d^2-24*(-b/a)^(1/2)*x*a*b^3*c^3*d)/d^4/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 +
 a)/(d*x^2 + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.205 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=346

$$-\frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{3cd^2\sqrt{c+dx^2}}+\frac{\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2b\sqrt{c}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3cd^2}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] -((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c*d^
2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[
c + d*x^2]) + (b*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(3*c*d^2) + ((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*S
qrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.716793, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$-\frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{3cd^2\sqrt{c+dx^2}}+\frac{\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{2b\sqrt{c}\sqrt{a+bx^2}(2bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3cd^2}-\frac{x\left(a+bx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(3*c*d^
2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[
c + d*x^2]) + (b*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])/(3*c*d^2) + ((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*S
qrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 97.8097, size = 318, normalized size = 0.92

$$\frac{2a^{\frac{3}{2}}\sqrt{b}\sqrt{c+dx^2}\,(3ad-2bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3cd^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}} - \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3ad-4bc)}{3cd^2} + \frac{x\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)}{cd\sqrt{c+dx^2}} - \frac{x\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)}{3cd^2\sqrt{c+dx^2}} + \frac{\sqrt{a+bx^2}\left(3a^2d^2-13abcd+8b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{\frac{5}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] 2*a**(3/2)*sqrt(b)*sqrt(c + d*x**2)*(3*a*d - 2*b*c)*elliptic_f(at
an(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*c*d**2*sqrt(a*(c + d*x*
*2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)) - b*x*sqrt(a + b*x**2)*sq
rt(c + d*x**2)*(3*a*d - 4*b*c)/(3*c*d**2) + x*(a + b*x**2)**(3/2)
*(a*d - b*c)/(c*d*sqrt(c + d*x**2)) - x*sqrt(a + b*x**2)*(3*a**2*
d**2 - 13*a*b*c*d + 8*b**2*c**2)/(3*c*d**2*sqrt(c + d*x**2)) + sq
rt(a + b*x**2)*(3*a**2*d**2 - 13*a*b*c*d + 8*b**2*c**2)*elliptic_
e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(3*sqrt(c)*d**(5/2)*sqr
t(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.822653, size = 256, normalized size = 0.74

$$\frac{-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(9a^2d^2-17abcd+8b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-13abcd+8b^{2}\right.}{3cd^3\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(4*c +
 d*x^2)) + I*b*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*b*c*(8*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.036, size = 539, normalized size = 1.6

$$\frac{1}{(3\,bdx^4+3\,adx^2+3\,cx^2b+3\,ac)\,d^3c}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^5b^3cd^2+3\sqrt{-\frac{b}{a}}x^3a^2bd^3-5\sqrt{-\frac{b}{a}}x^3ab^2cd^2+4\sqrt{-\frac{b}{a}}x^3b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] 1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^5*b^3*c*d^2+3
*(-b/a)^(1/2)*x^3*a^2*b*d^3-5*(-b/a)^(1/2)*x^3*a*b^2*c*d^2+4*(-b/
a)^(1/2)*x^3*b^3*c^2*d+9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d^2-17*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*a*b^2*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3-3*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*a^2*b*c*d^2+13*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^2*c^2*d-8*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b^3*c^3+3*x*a^3*d^3*(-b/a)^(1/2)-6*(-b/a)^(1/2)*x*a^2*b*c*d^2+4*(
-b/a)^(1/2)*x*a*b^2*c^2*d)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d^3/c/(-
b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(d*x^2 + c)^
(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^{\frac{5}{2}}}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.206 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}(2bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*
c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.438079, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}(2bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```


```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*
c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2
))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 61.0852, size = 223, normalized size = 0.86

$$\frac{a^{\frac{3}{2}}\sqrt{b}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{cd\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{x\sqrt{a+bx^2}\,(ad-2bc)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\,(ad-bc)}{cd\sqrt{c+dx^2}}+\frac{\sqrt{a+bx^2}\,(ad-2bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{\frac{3}{2}}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] a**(3/2)*sqrt(b)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(c*d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*s
qrt(a + b*x**2)) - x*sqrt(a + b*x**2)*(a*d - 2*b*c)/(c*d*sqrt(c +
 d*x**2)) + x*sqrt(a + b*x**2)*(a*d - b*c)/(c*d*sqrt(c + d*x**2))
 + sqrt(a + b*x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(d)*x/sqrt(
c)), 1 - b*c/(a*d))/(sqrt(c)*d**(3/2)*sqrt(c*(a + b*x**2)/(a*(c +
 d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.490192, size = 196, normalized size = 0.76

$$\frac{(ad-bc)\left(dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)-2ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-2bc)E\left(i\sinh\right.}{cd^2\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ell
ipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(b*c) + a*d)*(Sqr
t[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[
b/a]*c*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.033, size = 345, normalized size = 1.3

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,d^2c}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}x^3abd^2-\sqrt{-\frac{b}{a}}x^3b^2cd+2\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticF\left(x\sqrt{-\frac{b}{a}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*x^3*a*b*d^2-(-b/a)^
(1/2)*x^3*b^2*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)
^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+x*a^2*
d^2*(-b/a)^(1/2)-(-b/a)^(1/2)*x*a*b*c*d)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)/d^2/c/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)

[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.207 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])
```

Rubi [A] time = 0.0532417, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*S
qrt[c + d*x^2])
```

Rubi in Sympy [A] time = 8.28142, size = 71, normalized size = 0.85

$$\frac{\sqrt{a+bx^2}\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] sqrt(a + b*x**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d
))/(sqrt(c)*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c
+ d*x**2))
```

Mathematica [C] time = 0.520193, size = 136, normalized size = 1.62

$$\frac{\frac{x(a+bx^2)}{c}+\frac{ia\sqrt{\frac{b}{a}}\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{d}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2))/c + (I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/d)/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])
```

Maple [A] time = 0.026, size = 188, normalized size = 2.2

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,dc}\sqrt{bx^2+a}\sqrt{dx^2+c}\left(x^3bd\sqrt{-\frac{b}{a}}+EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}-EllipticE$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(x^3*b*d*(-b/a)^(1/2)+EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a*d*(-b/a)^(1/2))/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)/d/c/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

3.208 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b
*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.402502, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261

$$\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b
*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 58.6867, size = 228, normalized size = 1.18

$$\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)}-\frac{bx\sqrt{c+dx^2}}{c\sqrt{a+bx^2}\,(ad-bc)}+\frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}\,(ad-bc)}-\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a
)), -a*d/(b*c) + 1)/(c*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt
(a + b*x**2)*(a*d - b*c)) - b*x*sqrt(c + d*x**2)/(c*sqrt(a + b*x*
*2)*(a*d - b*c)) + d*x*sqrt(a + b*x**2)/(c*sqrt(c + d*x**2)*(a*d
- b*c)) - b*sqrt(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sq
rt(c)), 1 - b*c/(a*d))/(a*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x
**2)))*sqrt(c + d*x**2)*(a*d - b*c))
```

Mathematica [A] time = 0.412183, size = 112, normalized size = 0.58

$$\frac{\frac{bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}-dx\left(a+bx^2\right)}{c\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (-(d*x*(a + b*x^2)) + (b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(
c*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.034, size = 144, normalized size = 0.7

$$\frac{1}{c\left(ad-bc\right)\left(bdx^4+adx^2+cx^2b+ac\right)}\left(x^3bd\sqrt{-\frac{b}{a}}-EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}+xad\sqrt{-\frac{b}{a}}\right)\sqrt{dx^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (x^3*b*d*(-b/a)^(1/2)-EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+x*a*d*(-b/a)^(1/2))*(d
*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.209 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=242

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*
(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])
```

Rubi [A] time = 0.339549, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*
(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])
```

Rubi in Sympy [A] time = 51.8775, size = 207, normalized size = 0.86

$$-\frac{2\sqrt{a}\sqrt{b}d\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}+\frac{dx}{c\sqrt{a+bx^2}\sqrt{c+dx^2}\,(ad-bc)}+\frac{\sqrt{b}\sqrt{c+dx^2}\,(ad+bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{a}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] -2*sqrt(a)*sqrt(b)*d*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/s
qrt(a)), -a*d/(b*c) + 1)/(c*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))
*sqrt(a + b*x**2)*(a*d - b*c)**2) + d*x/(c*sqrt(a + b*x**2)*sqrt(
c + d*x**2)*(a*d - b*c)) + sqrt(b)*sqrt(c + d*x**2)*(a*d + b*c)*e
lliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(sqrt(a)*c*sqr
t(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)**
2)
```

Mathematica [C] time = 1.24342, size = 224, normalized size = 0.93

$$\frac{\sqrt{\frac{b}{a}}\left(x\sqrt{\frac{b}{a}}\left(a^2d^2+abd^2x^2+b^2c\left(c+dx^2\right)\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}}\right.}{bc\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2
)) + I*b*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*
d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

Maple [A] time = 0.038, size = 354, normalized size = 1.5

$$\frac{1}{ac\,(ad-bc)^2\,(bdx^4+adx^2+cx^2b+ac)}\left(\sqrt{-\frac{b}{a}}x^3abd^2+\sqrt{-\frac{b}{a}}x^3b^2cd-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] ((-b/a)^(1/2)*x^3*a*b*d^2+(-b/a)^(1/2)*x^3*b^2*c*d-((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*a*b*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-((b*
x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*b^2*c^2+x*a^2*d^2*(-b/a)^(1/2)+x*b^2*c^2*(-b/a)^(1/2
))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/(-b/a)^(1/2)/a/(a*d-b*c)^2/(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bdx^4+(bc+ad)x^2+ac)\sqrt{bx^2+a}\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

3.210 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2bx(bc-3ad)}{3a^2\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*
(b*c - 3*a*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2]) + (Sqrt[d]*(2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a
^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*
(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.674622, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217

$$\frac{\sqrt{d}\sqrt{a+bx^2}\left(-3a^2d^2-7abcd+2b^2c^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{2bx(bc-3ad)}{3a^2\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-9ad)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bx}{3a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```


```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*
(b*c - 3*a*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2]) + (Sqrt[d]*(2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a
^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*
(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^
2])
```

Rubi in Sympy [A] time = 96.8982, size = 284, normalized size = 0.88

$$\frac{dx}{c\left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)}+\frac{bx\sqrt{c+dx^2}\,(3ad+bc)}{3ac\left(a+bx^2\right)^{\frac{3}{2}}(ad-bc)^2}-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,(9ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)^3}+\frac{\sqrt{b}\sqrt{c+dx^2}\left(3a^2d^2+7abcd-2b^2c^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{3a^{\frac{3}{2}}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] d*x/(c*(a + b*x**2)**(3/2)*sqrt(c + d*x**2)*(a*d - b*c)) + b*x*sq
rt(c + d*x**2)*(3*a*d + b*c)/(3*a*c*(a + b*x**2)**(3/2)*(a*d - b*
c)**2) - b*sqrt(c)*sqrt(d)*sqrt(a + b*x**2)*(9*a*d - b*c)*ellipti
c_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(3*a**2*sqrt(c*(a + b
*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)*(a*d - b*c)**3) + sqrt(
b)*sqrt(c + d*x**2)*(3*a**2*d**2 + 7*a*b*c*d - 2*b**2*c**2)*ellip
tic_e(atan(sqrt(b)*x/sqrt(a)), -a*d/(b*c) + 1)/(3*a**(3/2)*c*sqrt
(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*(a*d - b*c)**3
)
```

Mathematica [C] time = 1.84406, size = 337, normalized size = 1.04

$$\frac{2ibc\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2d^2-4abcd+b^2c^2\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+ibc\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}\left(3a^2\right.}{3a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 2*b^4*c^2*x^2*(c + d*
x^2) + a^2*b^2*d*(8*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + a*b^3*c*(-3*c^
2 + 4*c*d*x^2 + 7*d^2*x^4)) + I*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a
^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(b^2*c^2 -
4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*
Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [B] time = 0.044, size = 964, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(-3*x^5*a^2*b^2*d^3*(-b/a)^(1/2)-7*x^5*a*b^3*c*d^2*(-b/a)^(1
/2)+2*x^5*b^4*c^2*d*(-b/a)^(1/2)+6*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*x^2*a^2*b^2*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)-8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^2*d*
((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*x^2*b^4*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)+3*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*a^2*b^2*c*d
^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+7*EllipticE(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*x^2*a*b^3*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2
+c)/c)^(1/2)-2*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^2*b^4*
c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-6*x^3*a^3*b*d^3*(-b/a
)^(1/2)-8*x^3*a^2*b^2*c*d^2*(-b/a)^(1/2)-4*x^3*a*b^3*c^2*d*(-b/a)
^(1/2)+2*x^3*b^4*c^3*(-b/a)^(1/2)+6*EllipticF(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a^3*b*c*d^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-
8*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b^2*c^2*d*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+2*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*a*b^3*c^3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+3*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^3*b*c*d^2*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)+7*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^
(1/2))*a^2*b^2*c^2*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-2*El
lipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b^3*c^3*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)-3*x*a^4*d^3*(-b/a)^(1/2)-8*x*a^2*b^2*c^2
*d*(-b/a)^(1/2)+3*x*a*b^3*c^3*(-b/a)^(1/2))/(d*x^2+c)^(1/2)/(a*d-
b*c)^3/a^2/(-b/a)^(1/2)/c/(b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2dx^6 + (b^2c + 2\,abd)x^4 + a^2c + (2\,abc + a^2d)x^2)\sqrt{bx^2 + a}\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c
 + a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)^{\frac{5}{2}}\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{2}}(dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

3.211 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.061582, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 10.4541, size = 73, normalized size = 0.84

$$\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 -
b*c/(a*d))/(a*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(
c + d*x**2))
```

Mathematica [A] time = 0.0848358, size = 86, normalized size = 0.99

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(
b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*
x^2])
```

Maple [A] time = 0., size = 100, normalized size = 1.2

$$\frac{1}{bdx^4+adx^2+cx^2b+ac}EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b
*x^2+a)/a)^(1/2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(-b/a)^(1/2)/(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.212 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\, dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.162074, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 37.3199, size = 75, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 + d*x**2/c)*elliptic_f(asin(sqr
t(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*sqrt(a - b*x**2)*sqrt(c + d
*x**2))
```

Mathematica [A] time = 0.0959843, size = 87, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/
a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A] time = 0.048, size = 106, normalized size = 1.2

$$-\frac{1}{bdx^4-adx^2+cx^2b-ac}\mathit{EllipticF}\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] -EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*(-
(b*x^2-a)/a)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/(
b*d*x^4-a*d*x^2+b*c*x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - bx^2}\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2 + a}\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.213 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*
Sqrt[c - d*x^2])
```

Rubi [A] time = 0.162079, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*
Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 37.4818, size = 75, normalized size = 0.86

$$\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*elliptic_f(asin(sqr
t(d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(d)*sqrt(a + b*x**2)*sqrt(c - d
*x**2))
```

Mathematica [A] time = 0.10233, size = 89, normalized size = 1.02

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(
b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c -
 d*x^2])
```

Maple [A] time = 0.044, size = 106, normalized size = 1.2

$$-\frac{1}{bdx^4+adx^2-cx^2b-ac}EllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\sqrt{bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] -EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*((b*x^2+a)/a)^(1/2)*(-
(d*x^2-c)/c)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(
b*d*x^4+a*d*x^2-b*c*x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.214 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqr
t[c - d*x^2])
```

Rubi [A] time = 0.171007, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08

$$\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqr
t[c - d*x^2])
```

Rubi in Sympy [A] time = 54.7996, size = 73, normalized size = 0.83

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 - d*x**2/c)*elliptic_f(asin(sqr
t(b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*sqrt(a - b*x**2)*sqrt(c - d*
x**2))
```

Mathematica [A] time = 0.0925442, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}F\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/
a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.041, size = 108, normalized size = 1.2

$$\frac{1}{bdx^4-adx^2-cx^2b+ac}EllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-bx^2+a}\sqrt{-dx^2+c}\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*(-(b*x^2-a)/a)^(1/2)*(-(
d*x^2-c)/c)^(1/2)*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(
b*d*x^4-a*d*x^2-b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a - bx^2}\sqrt{c - dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^2 + a}\sqrt{-dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

3.215 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rubi [A] time = 0.025319, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]

[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.22481, size = 14, normalized size = 1.17

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{5}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), -5/2)/2
```

Mathematica [A] time = 0.0326783, size = 12, normalized size = 1.

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Maple [A] time = 0.043, size = 17, normalized size = 1.4

$$\frac{\mathit{EllipticF}\left(x, \frac{i}{2}\sqrt{2}\sqrt{5}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2)*5^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.216 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rubi [A] time = 0.0273416, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]

[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.24884, size = 12, normalized size = 1.2

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), -2)/2
```

Mathematica [C] time = 0.0512027, size = 58, normalized size = 5.8

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2\sqrt{-2x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I/2)*Sqrt[1 - x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]
*x], -1/2])/Sqrt[1 + x^2 - 2*x^4]
```

Maple [A] time = 0.038, size = 14, normalized size = 1.4

$$\frac{EllipticF\left(x, i\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2+1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.217 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.0248665, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.2395, size = 14, normalized size = 1.17

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(x), -3/2)/2
```

Mathematica [A] time = 0.0311283, size = 12, normalized size = 1.

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Maple [A] time = 0.025, size = 17, normalized size = 1.4

$$\frac{\mathit{EllipticF}\left(x, \frac{i}{2}\sqrt{3}\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*I*3^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.218 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rubi [A] time = 0.0232372, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.95543, size = 12, normalized size = 1.2

$$\frac{\sqrt{2}F\left(\operatorname{asin}\left(x\right)\middle|-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(x), -1)/2
```

Mathematica [A] time = 0.031862, size = 10, normalized size = 1.

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] time = 0.03, size = 10, normalized size = 1.

$$\frac{\mathit{EllipticF}\left(x,i\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [A] time = 33.9879, size = 73, normalized size = 7.3

$$-\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}+\frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4,
 1, 5/4), (0,)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3/2)) + sqrt(2
)*meijerg(((-1/4, 0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4,
1/4, 1/4)), x**(-4))/(16*pi**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rubi [A] time = 0.0228961, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.94708, size = 14, normalized size = 1.17

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(x), -1/2)/2
```

Mathematica [C] time = 0.0326594, size = 18, normalized size = 1.5

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[2]], -2]
```

Maple [A] time = 0.037, size = 14, normalized size = 1.2

$$\frac{EllipticF\left(x, \frac{i}{2}\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x)

[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")

[Out] integral(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rubi [A] time = 0.026604, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]

[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 8.28715, size = 12, normalized size = 1.

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), 1/2)/2
```

Mathematica [A] time = 0.0300406, size = 12, normalized size = 1.

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Maple [A] time = 0.035, size = 13, normalized size = 1.1

$$\frac{\sqrt{2}}{2}EllipticF\left(x,\frac{\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(-x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.221 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rubi [A] time = 0.00771095, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\tanh^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]

[Out] ArcTanh[x]/Sqrt[2]
```

Rubi in Sympy [A] time = 2.77489, size = 8, normalized size = 1.

$$\frac{\sqrt{2}\,\operatorname{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)

[Out] sqrt(2)*atanh(x)/2
```

Mathematica [B] time = 0.00920111, size = 26, normalized size = 3.25

$$-\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] -((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

Maple [A] time = 0.052, size = 8, normalized size = 1.

$$\frac{Artanh(x)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

Maxima [A] time = 1.54241, size = 22, normalized size = 2.75

$$\frac{1}{4}\sqrt{2}(\log(x+1) - \log(x-1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(log(x + 1) - log(x - 1))
```

Fricas [A] time = 0.249576, size = 95, normalized size = 11.88

$$\frac{1}{8}\sqrt{2}\log\left(\frac{4\left(x^3+x\right)\sqrt{-x^2+1}\sqrt{-2x^2+2}-\sqrt{2}\left(x^6+5x^4-5x^2-1\right)}{x^6-3x^4+3x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((4*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2) - sq
rt(2)*(x^6 + 5*x^4 - 5*x^2 - 1))/(x^6 - 3*x^4 + 3*x^2 - 1))
```

Sympy [A] time = 8.93193, size = 22, normalized size = 2.75

$$-\sqrt{2}\left(\begin{cases} -\frac{\operatorname{acoth}(x)}{2} & \text{for}\ x^2 > 1 \\ -\frac{\operatorname{atanh}(x)}{2} & \text{for}\ x^2 < 1 \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(2)*Piecewise((-acoth(x)/2, x**2 > 1), (-atanh(x)/2, x**2 <
1))
```

GIAC/XCAS [A] time = 0.265517, size = 28, normalized size = 3.5

$$\frac{1}{4}\sqrt{2}\ln\left(|x+1|\right)-\frac{1}{4}\sqrt{2}\ln\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*ln(abs(x + 1)) - 1/4*sqrt(2)*ln(abs(x - 1))
```

3.222 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Rubi [A] time = 0.0257913, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]

[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.28155, size = 12, normalized size = 1.

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), 3/2)/2
```

Mathematica [A] time = 0.0350116, size = 20, normalized size = 1.67

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Maple [A] time = 0.028, size = 16, normalized size = 1.3

$$\frac{\sqrt{2}}{2} EllipticF\left(x, \frac{\sqrt{3}\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticF(x,1/2*3^(1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(-3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.223 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=10

$$\frac{F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rubi [A] time = 0.0277691, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.38738, size = 10, normalized size = 1.

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)|2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(x), 2)/2
```

Mathematica [A] time = 0.0452139, size = 16, normalized size = 1.6

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], 1/2]/2
```

Maple [A] time = 0.031, size = 11, normalized size = 1.1

$$\frac{EllipticF\left(x, \sqrt{2}\right) \sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + 1}\sqrt{-4\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-x^2 + 1}\sqrt{-4\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{-2x^2+1}\sqrt{-x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-2*x**2 + 1)*sqrt(-x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.224 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}} dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rubi [A] time = 0.0255724, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 5.40413, size = 12, normalized size = 1.

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|\frac{5}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(x), 5/2)/2
```

Mathematica [A] time = 0.0369177, size = 20, normalized size = 1.67

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)\middle|\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], 2/5]/Sqrt[5]
```

Maple [A] time = 0.062, size = 16, normalized size = 1.3

$$\frac{\sqrt{2}}{2} EllipticF\left(x, \frac{\sqrt{2}\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticF(x,1/2*2^(1/2)*5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(-5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.225 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{5x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^
2]*Sqrt[(2 + 5*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0310118, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{5x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^
2]*Sqrt[(2 + 5*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.53648, size = 48, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{5x^2+2}F\left(\operatorname{atan}(x)\middle|-\frac{3}{2}\right)}{2\sqrt{\frac{5x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(5*x**2 + 2)*elliptic_f(atan(x), -3/2)/(2*sqrt((5*x**
2 + 2)/(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.029018, size = 19, normalized size = 0.37

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 5/2])/Sqrt[2]
```

Maple [A] time = 0.102, size = 20, normalized size = 0.4

$$-\frac{i}{2}EllipticF\left(ix, \frac{\sqrt{2}\sqrt{5}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*2^(1/2)*5^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.226 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{2x^2+1}F\left(\tan^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]
*Sqrt[(1 + 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0333253, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{2x^2+1}F\left(\tan^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]
*Sqrt[(1 + 2*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.19043, size = 46, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{4x^2+2}F\left(\operatorname{atan}(x)\middle|-1\right)}{2\sqrt{\frac{4x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(4*x**2 + 2)*elliptic_f(atan(x), -1)/(2*sqrt((4*x**2
+ 2)/(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.0415428, size = 17, normalized size = 0.35

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|2\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 2])/Sqrt[2]
```

Maple [A] time = 0.057, size = 15, normalized size = 0.3

$$-\frac{i}{2}EllipticF\left(ix,\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{x^2+1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.227 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^
2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0317199, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^
2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.58249, size = 48, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{3x^2+2}F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{2\sqrt{\frac{3x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(3*x**2 + 2)*elliptic_f(atan(x), -1/2)/(2*sqrt((3*x**
2 + 2)/(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.0286916, size = 19, normalized size = 0.37

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 3/2])/Sqrt[2]
```

Maple [A] time = 0.022, size = 20, normalized size = 0.4

$$-\frac{i}{2} EllipticF\left(ix, \frac{\sqrt{3}\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,1/2*3^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3x^2+2}\sqrt{x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.228 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=8

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

```
[Out] ArcTan[x]/Sqrt[2]
```

Rubi [A] time = 0.00793174, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Rubi in Sympy [A] time = 2.52358, size = 8, normalized size = 1.

$$\frac{\sqrt{2}\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*atan(x)/2
```

Mathematica [A] time = 0.00708218, size = 8, normalized size = 1.

$$\frac{\tan^{-1}(x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Maple [A] time = 0.031, size = 8, normalized size = 1.

$$\frac{\arctan(x)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] 1/2*arctan(x)*2^(1/2)
```

Maxima [A] time = 1.52895, size = 9, normalized size = 1.12

$$\frac{1}{2}\sqrt{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(x)
```

Fricas [A] time = 0.244895, size = 46, normalized size = 5.75

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2x^2+2}\sqrt{x^2+1}x}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4
- 1))
```

Sympy [A] time = 10.5482, size = 8, normalized size = 1.

$$\frac{\sqrt{2}\,\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*atan(x)/2
```

GIAC/XCAS [A]   time = 0.225048, size = 9, normalized size = 1.12

$$\frac{1}{2}\sqrt{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(x)
```

3.229 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*
Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0285857, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*
Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 4.87794, size = 42, normalized size = 0.89

$$\frac{\sqrt{2}\sqrt{x^2+2}F\left(\operatorname{atan}(x)|\frac{1}{2}\right)}{2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(x**2 + 2)*elliptic_f(atan(x), 1/2)/(2*sqrt((x**2 + 2
)/(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.0288724, size = 19, normalized size = 0.4

$$-\frac{iF\left(i\sinh^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], 1/2])/Sqrt[2]
```

Maple [C] time = 0.034, size = 15, normalized size = 0.3

$$-i\mathrm{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] -I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.230 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi [A] time = 0.0228177, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rubi in Sympy [A] time = 4.98975, size = 12, normalized size = 1.2

$$F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)

[Out] elliptic_f(asin(sqrt(2)*x/2), -2)
```

Mathematica [C] time = 0.0400808, size = 19, normalized size = 1.9

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [A] time = 0.036, size = 14, normalized size = 1.4

$$EllipticF\left(\frac{x\sqrt{2}}{2}, i\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + 2)*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

3.231 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rubi [A] time = 0.0228852, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rubi in Sympy [A] time = 4.42667, size = 12, normalized size = 1.2

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), -1)/2
```

Mathematica [A] time = 0.0337112, size = 10, normalized size = 1.

$$\frac{F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] time = 0.009, size = 10, normalized size = 1.

$$\frac{EllipticF\left(x,i\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Sympy [A] time = 33.2787, size = 76, normalized size = 7.6

$$\frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4
, 1, 5/4), (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-
1/4, 0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), e
xp_polar(-2*I*pi)/x**4)/(16*pi**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

3.232 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0258697, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 5.03698, size = 20, normalized size = 1.

$$\frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3
```

Mathematica [A] time = 0.0331416, size = 20, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] time = 0.023, size = 25, normalized size = 1.3

$$\frac{\sqrt{3}}{3} EllipticF\left(\frac{x\sqrt{3}\sqrt{2}}{2}, \frac{i}{3}\sqrt{3}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*EllipticF(1/2*x*3^(1/2)*2^(1/2),1/3*I*3^(1/2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2+1}\sqrt{-3x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*x**2 + 2)*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.233 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rubi [A] time = 0.0260297, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rubi in Sympy [A] time = 4.93511, size = 14, normalized size = 0.88

$$\frac{F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)

[Out] elliptic_f(asin(sqrt(2)*x), -1/2)/2
```

Mathematica [A] time = 0.0389256, size = 16, normalized size = 1.

$$\frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Maple [A] time = 0.03, size = 15, normalized size = 0.9

$$\frac{EllipticF\left(x\sqrt{2}, \frac{i}{2}\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int \frac{1}{\sqrt{-2x^2+1}\sqrt{x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-2*x**2 + 1)*sqrt(x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

3.234 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rubi [A] time = 0.0304281, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]

[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rubi in Sympy [A] time = 4.95618, size = 20, normalized size = 1.

$$\frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|-\frac{2}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)

[Out] sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/5
```

Mathematica [A] time = 0.0343294, size = 20, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Maple [A] time = 0.052, size = 25, normalized size = 1.3

$$\frac{\sqrt{5}}{5} EllipticF\left(\frac{\sqrt{5}\sqrt{2}x}{2}, \frac{i}{5}\sqrt{5}\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] 1/5*5^(1/2)*EllipticF(1/2*5^(1/2)*2^(1/2)*x,1/5*I*5^(1/2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^2+2}\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-5*x**2 + 2)*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

3.235 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi [A] time = 0.0458628, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi in Sympy [A] time = 8.69443, size = 31, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|-\frac{5}{2}\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), -5/2)/(2*sqrt(x**2 -
1))
```

Mathematica [A] time = 0.0326258, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Maple [A] time = 0.046, size = 43, normalized size = 1.3

$$-\frac{i}{5}EllipticF\left(\frac{i}{2}\sqrt{5}\sqrt{2}x, \frac{i}{5}\sqrt{5}\sqrt{2}\right)\sqrt{5}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] -1/5*I*EllipticF(1/2*I*5^(1/2)*2^(1/2)*x,1/5*I*5^(1/2)*2^(1/2))*(
-x^2+1)^(1/2)*5^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{5x^2+2}\sqrt{x^2-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.236 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0505064, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi in Sympy [A] time = 8.66463, size = 29, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|-2\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), -2)/(2*sqrt(x**2 - 1)
)
```

Mathematica [A] time = 0.0433599, size = 30, normalized size = 1.

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] time = 0.043, size = 34, normalized size = 1.1

$$-\frac{i}{2}EllipticF\left(ix\sqrt{2},\frac{i}{2}\sqrt{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x*2^(1/2),1/2*I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{4x^2+2}\sqrt{x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.237 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi [A] time = 0.0465169, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi in Sympy [A] time = 8.6114, size = 31, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), -3/2)/(2*sqrt(x**2 -
1))
```

Mathematica [A] time = 0.030831, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Maple [A] time = 0.037, size = 43, normalized size = 1.3

$$-\frac{i}{3}EllipticF\left(\frac{i}{2}\sqrt{3}\sqrt{2}x,\frac{i}{3}\sqrt{3}\sqrt{2}\right)\sqrt{3}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*I*EllipticF(1/2*I*3^(1/2)*2^(1/2)*x,1/3*I*3^(1/2)*2^(1/2))*(
-x^2+1)^(1/2)*3^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.238 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)$$

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rubi [A] time = 0.0318773, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rubi in Sympy [A] time = 5.25223, size = 41, normalized size = 1.64

$$-\frac{\sqrt{x^2-1}\sqrt{2x^2+2}\sqrt{-x^4+1}F\left(\operatorname{asin}(x)|-1\right)}{-2x^4+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(x**2 - 1)*sqrt(2*x**2 + 2)*sqrt(-x**4 + 1)*elliptic_f(asin(
x), -1)/(-2*x**4 + 2)
```

Mathematica [A] time = 0.0379615, size = 30, normalized size = 1.2

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [C] time = 0.029, size = 30, normalized size = 1.2

$$-\frac{i}{2}EllipticF\left(ix,i\right)\sqrt{2}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*EllipticF(I*x,I)*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Sympy [A] time = 33.0774, size = 75, normalized size = 3.

$$\frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2}, 1, 1 & \frac{3}{4}, \frac{3}{4}, \frac{5}{4} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4} & 0 \end{matrix} \middle| \frac{e^{2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} & 1 \\ 0, \frac{1}{2}, 0 & -\frac{1}{4}, \frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4
, 1, 5/4), (0,)), exp_polar(2*I*pi)/x**4)/(16*pi**(3/2)) - sqrt(2
)*I*meijerg(((-1/4, 0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4
, 1/4, 1/4)), x**(-4))/(16*pi**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi [A] time = 0.0438645, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Rubi in Sympy [A] time = 8.31563, size = 31, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|-\frac{1}{2}\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), -1/2)/(2*sqrt(x**2 -
1))
```

Mathematica [A] time = 0.0295661, size = 32, normalized size = 1.

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2
])
```

Maple [A] time = 0.035, size = 34, normalized size = 1.1

$$-iEllipticF\left(\frac{i}{2}x\sqrt{2}, i\sqrt{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] -I*EllipticF(1/2*I*x*2^(1/2),I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

3.240 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=12

$$-F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

```
[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]
```

Rubi [A] time = 0.0236243, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]

[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]
```

Rubi in Sympy [A] time = 6.41561, size = 12, normalized size = 1.

$$-F\left(\operatorname{acos}\left(\frac{\sqrt{2}x}{2}\right)\middle|2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)

[Out] -elliptic_f(acos(sqrt(2)*x/2), 2)
```

Mathematica [B] time = 0.0379881, size = 47, normalized size = 3.92

$$\frac{\sqrt{1-x^2}\sqrt{1-\frac{x^2}{2}}F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{-x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[x], 1/2])/Sqrt[-2
 + 3*x^2 - x^4]
```

Maple [A] time = 0.032, size = 28, normalized size = 2.3

$$1 EllipticF\left(\frac{x\sqrt{2}}{2}, \sqrt{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] EllipticF(1/2*x*2^(1/2),2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)

[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(-x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

3.241 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

```
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rubi [A] time = 0.0108957, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$-\frac{\sqrt{x^2-1}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]

[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rubi in Sympy [A] time = 2.70064, size = 22, normalized size = 0.76

$$\frac{\sqrt{-2x^2+2}\,\operatorname{atanh}(x)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)

[Out] sqrt(-2*x**2 + 2)*atanh(x)/(2*sqrt(x**2 - 1))
```

Mathematica [A] time = 0.019222, size = 40, normalized size = 1.38

$$\frac{\left(x^2-1\right)\left(\log(1-x)-\log(x+1)\right)}{2\sqrt{2}\sqrt{-\left(x^2-1\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] ((-1 + x^2)*(Log[1 - x] - Log[1 + x]))/(2*Sqrt[2]*Sqrt[-(-1 + x^2
)^2])
```

Maple [A] time = 0.013, size = 26, normalized size = 0.9

$$\frac{\sqrt{2}Artanh\left(x\right)}{2}\sqrt{-2\,x^{2}+2}\frac{1}{\sqrt{2\,x^{2}-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*(-2*x^2+2)^(1/2)*2^(1/2)/(2*x^2-2)^(1/2)*arctanh(x)
```

Maxima [A] time = 1.54485, size = 22, normalized size = 0.76

$$\frac{1}{4}\,i\,\sqrt{2}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] 1/4*I*sqrt(2)*(log(x + 1) - log(x - 1))
```

Fricas [A] time = 0.23926, size = 46, normalized size = 1.59

$$\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^{2}-1}\sqrt{-2\,x^{2}+2}x}{x^{4}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4
- 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2+1}\sqrt{x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 + 1)*sqrt(x**2 - 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.242 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2]
)
```

Rubi [A] time = 0.0467806, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2]
)
```

Rubi in Sympy [A] time = 8.65308, size = 29, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|\frac{3}{2}\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), 3/2)/(2*sqrt(x**2 - 1
))
```

Mathematica [A] time = 0.036968, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{2}{3}\right)}{\sqrt{3}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/(Sqrt[3]*Sqrt
[-1 + x^2])
```

Maple [A] time = 0.031, size = 32, normalized size = 1.

$$\frac{\sqrt{2}}{2}\mathit{EllipticF}\left(x, \frac{\sqrt{3}\sqrt{2}}{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*3^(1/2)*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)

[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(-3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.243 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.050285, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rubi in Sympy [A] time = 8.66625, size = 27, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)|2\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), 2)/(2*sqrt(x**2 - 1))
```

Mathematica [A] time = 0.0437039, size = 36, normalized size = 1.2

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{2\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2]*x], 1/2])/(2*Sqrt[-1 + x^
2])
```

Maple [A] time = 0.032, size = 27, normalized size = 0.9

$$\frac{\mathit{EllipticF}\left(x,\sqrt{2}\right)\sqrt{2}}{2}\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-2x^2+1}\sqrt{x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-2*x**2 + 1)*sqrt(x**2 - 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.244 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2]
)
```

Rubi [A] time = 0.047416, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2]
)
```

Rubi in Sympy [A] time = 8.72697, size = 29, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|\frac{5}{2}\right)}{2\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(-x**2 + 1)*elliptic_f(asin(x), 5/2)/(2*sqrt(x**2 - 1
))
```

Mathematica [A] time = 0.034042, size = 40, normalized size = 1.25

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], 2/5])/(Sqrt[5]*Sqrt
[-1 + x^2])
```

Maple [A] time = 0.064, size = 32, normalized size = 1.

$$\frac{\sqrt{2}}{2}EllipticF\left(x,\frac{\sqrt{2}\sqrt{5}}{2}\right)\sqrt{-x^2+1}\frac{1}{\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2)*5^(1/2))*2^(1/2)*(-x^2+1)^(1/2)/(x^2-
1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(-5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.245 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{5x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x
^2]*Sqrt[(2 + 5*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0342631, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{5x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x
^2]*Sqrt[(2 + 5*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.69693, size = 53, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{5x^2+2}F\left(\operatorname{atan}(x)\middle|-\frac{3}{2}\right)}{2\sqrt{-\frac{-5x^2-2}{x^2+1}}\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(5*x**2 + 2)*elliptic_f(atan(x), -3/2)/(2*sqrt(-(-5*x
**2 - 2)/(x**2 + 1))*sqrt(-x**2 - 1))
```

Mathematica [C] time = 0.0375772, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{x^2+1}F\left(i\sinh^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 5/2])/(Sqrt[2]*Sqrt[-
1 - x^2])
```

Maple [A] time = 0.09, size = 42, normalized size = 0.8

$$\frac{i}{5}\sqrt{5}\mathit{EllipticF}\left(\frac{i}{2}\sqrt{5}\sqrt{2}x,\frac{\sqrt{2}\sqrt{5}}{5}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x)
```

```
[Out] 1/5*I*EllipticF(1/2*I*5^(1/2)*2^(1/2)*x,1/5*2^(1/2)*5^(1/2))/(x^2
+1)^(1/2)*5^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{5x^2+2}\sqrt{-x^2-1}}{5x^4+7x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)/(5*x^4 + 7*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(5*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.246 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{2x^2+1}F\left(\tan^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2
]*Sqrt[(1 + 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0372169, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{2x^2+1}F\left(\tan^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2
]*Sqrt[(1 + 2*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.72887, size = 51, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{4x^2+2}F\left(\operatorname{atan}(x)\middle|-1\right)}{2\sqrt{-\frac{-4x^2-2}{x^2+1}}\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(4*x**2 + 2)*elliptic_f(atan(x), -1)/(2*sqrt(-(-4*x**
2 - 2)/(x**2 + 1))*sqrt(-x**2 - 1))
```

Mathematica [C] time = 0.0432214, size = 37, normalized size = 0.73

$$-\frac{i\sqrt{x^2+1}F\left(i\sinh^{-1}(x)\middle|2\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 2])/(Sqrt[2]*Sqrt[-1
- x^2])
```

Maple [A] time = 0.06, size = 33, normalized size = 0.7

$$\frac{i}{2}EllipticF\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x*2^(1/2),1/2*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}{2\,(2\,x^4+3\,x^2+1)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(-1/2*sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)/(2*x^4 + 3*x^2 + 1),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.247 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x
^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0351287, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{3x^2+2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x
^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.75099, size = 53, normalized size = 1.

$$\frac{\sqrt{2}\sqrt{3x^2+2}F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{2\sqrt{-\frac{-3x^2-2}{x^2+1}}\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(3*x**2 + 2)*elliptic_f(atan(x), -1/2)/(2*sqrt(-(-3*x
**2 - 2)/(x**2 + 1))*sqrt(-x**2 - 1))
```

Mathematica [C] time = 0.0337163, size = 39, normalized size = 0.74

$$-\frac{i\sqrt{x^2+1}F\left(i\sinh^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 3/2])/(Sqrt[2]*Sqrt[-
1 - x^2])
```

Maple [A] time = 0.092, size = 42, normalized size = 0.8

$$\frac{i}{3}\sqrt{3}EllipticF\left(\frac{i}{2}\sqrt{3}\sqrt{2}x, \frac{\sqrt{3}\sqrt{2}}{3}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*I*EllipticF(1/2*I*3^(1/2)*2^(1/2)*x,1/3*3^(1/2)*2^(1/2))/(x^2
+1)^(1/2)*3^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}{3\,x^4+5\,x^2+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 - 1}\sqrt{3x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(3*x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^2 + 2}\sqrt{-x^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.248 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rubi [A] time = 0.0113364, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt{x^2+1}\tan^{-1}(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rubi in Sympy [A] time = 3.12783, size = 24, normalized size = 0.86

$$\frac{\sqrt{2x^2+2}\,\mathrm{atan}\,(x)}{2\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2*x**2 + 2)*atan(x)/(2*sqrt(-x**2 - 1))
```

Mathematica [A] time = 0.0168138, size = 26, normalized size = 0.93

$$\frac{\left(x^2+1\right)\tan^{-1}(x)}{\sqrt{2}\sqrt{-\left(x^2+1\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

Maple [A] time = 0.01, size = 24, normalized size = 0.9

$$-\frac{\arctan(x)\sqrt{2}}{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x)
```

```
[Out] -1/2*(-x^2-1)^(1/2)*2^(1/2)/(x^2+1)^(1/2)*arctan(x)
```

Maxima [A] time = 1.52956, size = 9, normalized size = 0.32

$$-\frac{1}{2}i\sqrt{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(2)*arctan(x)
```

Fricas [A] time = 0.233575, size = 136, normalized size = 4.86

$$\frac{1}{8}\sqrt{2}\left(\log\left(\frac{2\sqrt{2}\left(x^4+\sqrt{2}\sqrt{2x^2+2}\sqrt{-x^2-1}x-1\right)}{x^4+2x^2+1}\right)-\log\left(-\frac{2\sqrt{2}\left(x^4-\sqrt{2}\sqrt{2x^2+2}\sqrt{-x^2-1}x-1\right)}{x^4+2x^2+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(log(2*sqrt(2)*(x^4 + sqrt(2)*sqrt(2*x^2 + 2)*sqrt(-x
^2 - 1)*x - 1)/(x^4 + 2*x^2 + 1)) - log(-2*sqrt(2)*(x^4 - sqrt(2)
*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x - 1)/(x^4 + 2*x^2 + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-x^2-1}\sqrt{x^2+1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]
*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0313948, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]
*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 5.4532, size = 48, normalized size = 0.98

$$\frac{\sqrt{2}\sqrt{x^2+2}F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2\sqrt{-\frac{-x^2-2}{x^2+1}}\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(x**2 + 2)*elliptic_f(atan(x), 1/2)/(2*sqrt(-(-x**2 -
 2)/(x**2 + 1))*sqrt(-x**2 - 1))
```

Mathematica [C] time = 0.0537498, size = 53, normalized size = 1.08

$$-\frac{i\sqrt{x^2+1}\sqrt{x^2+2}F\left(i\sinh^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-\left(x^2+1\right)\left(x^2+2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x], 1/2])/(
Sqrt[2]*Sqrt[-((1 + x^2)*(2 + x^2))])
```

Maple [C] time = 0.028, size = 33, normalized size = 0.7

$$\frac{i}{2}\sqrt{2}EllipticF\left(ix,\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^2+2}\sqrt{-x^2-1}}{x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^2 + 2)*sqrt(-x^2 - 1)/(x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rubi [A] time = 0.0511797, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]

[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rubi in Sympy [A] time = 12.0299, size = 31, normalized size = 1.

$$\frac{\sqrt{x^2+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-2\right)}{\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)

[Out] sqrt(x**2 + 1)*elliptic_f(asin(sqrt(2)*x/2), -2)/sqrt(-x**2 - 1)
```

Mathematica [C] time = 0.0342343, size = 39, normalized size = 1.26

$$-\frac{i\sqrt{x^2+1}F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], -1/2])/(Sqrt[2]*Sqrt[
-1 - x^2])
```

Maple [A] time = 0.03, size = 34, normalized size = 1.1

$$\frac{i}{2} EllipticF\left(ix, \frac{i}{2}\sqrt{2}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*2^(1/2))/(x^2+1)^(1/2)*2^(1/2)*(-x^2-1)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+2}\sqrt{-x^2-1}}{x^4-x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)/(x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 - 1}\sqrt{-x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(-x**2 + 2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + 2}\sqrt{-x^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

3.251 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-\frac{1}{x^4}}x^2F\left(\csc^{-1}(x)\middle|-1\right)}{\sqrt{2-2x^2}\sqrt{-x^2-1}}$$

```
[Out] -((Sqrt[1 - x^(-4)]*x^2*EllipticF[ArcCsc[x], -1])/(Sqrt[2 - 2*x^2
]*Sqrt[-1 - x^2]))
```

Rubi [A] time = 0.0399208, antiderivative size = 65, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)\middle|\frac{1}{2}\right)}{2\sqrt{-x^2-1}\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-
1 + x^2]], 1/2])/(2*Sqrt[-1 - x^2]*Sqrt[1 - x^2])
```

Rubi in Sympy [A] time = 5.64475, size = 42, normalized size = 1.

$$-\frac{\sqrt{-2x^2+2}\sqrt{-x^2-1}\sqrt{-x^4+1}F\left(\operatorname{asin}(x)\middle|-1\right)}{-2x^4+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] -sqrt(-2*x**2 + 2)*sqrt(-x**2 - 1)*sqrt(-x**4 + 1)*elliptic_f(asi
n(x), -1)/(-2*x**4 + 2)
```

Mathematica [A] time = 0.0477581, size = 30, normalized size = 0.71

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{2}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]

[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] time = 0.012, size = 30, normalized size = 0.7

$$\frac{i}{2} EllipticF\left(ix, i\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)

[Out] 1/2*I*EllipticF(I*x,I)*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}{2\,(x^4-1)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="fricas")

[Out] integral(1/2*sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)/(x^4 - 1), x)
```

Sympy [A] time = 35.2287, size = 73, normalized size = 1.74

$$\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}-\frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4,
1, 5/4), (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*meijerg(((-1/4,
 0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_p
olar(-2*I*pi)/x**4)/(16*pi**(3/2))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

3.252 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqr
t[-1 - x^2])
```

Rubi [A] time = 0.0508459, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqr
t[-1 - x^2])
```

Rubi in Sympy [A] time = 8.8835, size = 39, normalized size = 0.98

$$\frac{\sqrt{3}\sqrt{x^2+1}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(x**2 + 1)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/(3*sqr
t(-x**2 - 1))
```

Mathematica [A] time = 0.034818, size = 40, normalized size = 1.

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqr
t[-1 - x^2])
```

Maple [A] time = 0.034, size = 37, normalized size = 0.9

$$\frac{i}{2} EllipticF\left(ix, \frac{i}{2}\sqrt{3}\sqrt{2}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*3^(1/2)*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)
^(1/2)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}{3\,x^4+x^2-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)/(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*x**2 + 2)*sqrt(-x**2 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

3.253 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x
^2])
```

Rubi [A] time = 0.0525185, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x
^2])
```

Rubi in Sympy [A] time = 8.84701, size = 32, normalized size = 0.89

$$\frac{\sqrt{x^2+1}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 + 1)*elliptic_f(asin(sqrt(2)*x), -1/2)/(2*sqrt(-x**2 -
1))
```

Mathematica [A] time = 0.0449215, size = 36, normalized size = 1.

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x
^2])
```

Maple [A] time = 0.026, size = 34, normalized size = 0.9

$$\frac{i}{2}EllipticF\left(ix, i\sqrt{2}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}{2\,(2\,x^4+x^2-1)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)/(2*x^4 + x^2 - 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\int\frac{1}{\sqrt{-2x^2+1}\sqrt{-x^2-1}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(2)*Integral(1/(sqrt(-2*x**2 + 1)*sqrt(-x**2 - 1)), x)/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

3.254 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqr
t[-1 - x^2])
```

Rubi [A] time = 0.0522014, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqr
t[-1 - x^2])
```

Rubi in Sympy [A] time = 8.90966, size = 39, normalized size = 0.98

$$\frac{\sqrt{5}\sqrt{x^2+1}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|-\frac{2}{5}\right)}{5\sqrt{-x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] sqrt(5)*sqrt(x**2 + 1)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/(5*sq
rt(-x**2 - 1))
```

Mathematica [A] time = 0.0377513, size = 40, normalized size = 1.

$$\frac{\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqr
t[-1 - x^2])
```

Maple [A] time = 0.035, size = 37, normalized size = 0.9

$$\frac{i}{2}EllipticF\left(ix,\frac{i}{2}\sqrt{2}\sqrt{5}\right)\sqrt{2}\sqrt{-x^2-1}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x)
```

```
[Out] 1/2*I*EllipticF(I*x,1/2*I*2^(1/2)*5^(1/2))*2^(1/2)/(x^2+1)^(1/2)*
(-x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}{5\,x^4+3\,x^2-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)/(5*x^4 + 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^2+2}\sqrt{-x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-5*x**2 + 2)*sqrt(-x**2 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

3.255 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*
Sqrt[c - d*x^2])
```

Rubi [A] time = 0.15265, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*
Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 29.8198, size = 75, normalized size = 0.86

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{a+bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(a + b*x**2)*elliptic_e(asin(sqrt(
d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(d)*sqrt(1 + b*x**2/a)*sqrt(c - d
*x**2))
```

Mathematica [A] time = 0.0940049, size = 87, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x
], -((b*c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2
])
```

Maple [A] time = 0.017, size = 106, normalized size = 1.2

$$\frac{a}{-bdx^4-adx^2+cx^2b+ac}\sqrt{bx^2+a}\sqrt{-dx^2+c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}EllipticE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a
)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4-a*
d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.256 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]
*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.162785, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]
*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 37.9228, size = 76, normalized size = 0.84

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(-a - b*x**2)*elliptic_e(asin(sqrt
(d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(d)*sqrt(1 + b*x**2/a)*sqrt(c -
d*x**2))
```

Mathematica [A] time = 0.0818334, size = 90, normalized size = 1.

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*
x], -((b*c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^
2])
```

Maple [B] time = 0.024, size = 171, normalized size = 1.9

$$\frac{1}{(bdx^4+adx^2-cx^2b-ac)\,d}\left(-aEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcEllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{a}{b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*
(-b/a)^(1/2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(-a*d
/b/c)^(1/2)))*(-b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/
2)*(-(d*x^2-c)/c)^(1/2)/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.257 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*
Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.154724, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*
Sqrt[-c + d*x^2])
```

Rubi in Sympy [A] time = 30.1953, size = 75, normalized size = 0.85

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{a+bx^2}\,E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(a + b*x**2)*elliptic_e(asin(sqrt(
d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(d)*sqrt(1 + b*x**2/a)*sqrt(-c +
d*x**2))
```

Mathematica [A] time = 0.0773277, size = 88, normalized size = 1.

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x
], -((b*c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^
2])
```

Maple [B] time = 0.02, size = 168, normalized size = 1.9

$$\frac{1}{(bdx^4+adx^2-cx^2b-ac)\,d}\sqrt{bx^2+a}\sqrt{dx^2-c}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}\left(aEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcEllipticF\left(x\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c
)^(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*Ellip
ticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2
),(-a*d/b/c)^(1/2)))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/sqrt(-c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.258 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]
*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.161963, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]
*Sqrt[-c + d*x^2])
```

Rubi in Sympy [A] time = 37.1882, size = 76, normalized size = 0.84

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(-a - b*x**2)*elliptic_e(asin(sqrt
(d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(d)*sqrt(1 + b*x**2/a)*sqrt(-c +
 d*x**2))
```

Mathematica [A] time = 0.0720752, size = 91, normalized size = 1.

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*
x], -((b*c)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x
^2])
```

Maple [A] time = 0.019, size = 110, normalized size = 1.2

$$\frac{a}{bdx^4+adx^2-cx^2b-ac}\sqrt{-bx^2-a}\sqrt{dx^2-c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}EllipticE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (-b*x^2-a)^(1/2)*(d*x^2-c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a
)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4+a*d
*x^2-b*c*x^2-a*c)/(d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="fricas")

[Out] integral(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)

[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.259 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqr
t[c - d*x^2])
```

Rubi [A] time = 0.160418, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqr
t[c - d*x^2])
```

Rubi in Sympy [A] time = 42.1631, size = 73, normalized size = 0.83

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(a - b*x**2)*elliptic_e(asin(sqrt(
d)*x/sqrt(c)), b*c/(a*d))/(sqrt(d)*sqrt(1 - b*x**2/a)*sqrt(c - d*
x**2))
```

Mathematica [A] time = 0.0861279, size = 88, normalized size = 1.

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x
], (b*c)/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.018, size = 109, normalized size = 1.2

$$\frac{a}{bdx^4-adx^2-cx^2b+ac}\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}EllipticE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2
-a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4-a*
d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="fricas")

[Out] integral(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(a - b*x**2)/sqrt(c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

3.260 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sq
rt[c - d*x^2])
```

Rubi [A] time = 0.159725, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sq
rt[c - d*x^2])
```

Rubi in Sympy [A] time = 38.938, size = 73, normalized size = 0.82

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a+bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(-a + b*x**2)*elliptic_e(asin(sqrt
(d)*x/sqrt(c)), b*c/(a*d))/(sqrt(d)*sqrt(1 - b*x**2/a)*sqrt(c - d
*x**2))
```

Mathematica [A] time = 0.0718343, size = 89, normalized size = 1.

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*
x], (b*c)/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] time = 0.025, size = 165, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2-cx^2b+ac)\,d}\sqrt{bx^2-a}\sqrt{-dx^2+c}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}\left(aEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcEllipticF\left(x\sqrt{\frac{b}{a}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-c)
/c)^(1/2)*(a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*Ellip
ticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(
a*d/b/c)^(1/2)))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

3.261 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqr
t[-c + d*x^2])
```

Rubi [A] time = 0.158352, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqr
t[-c + d*x^2])
```

Rubi in Sympy [A] time = 40.6402, size = 73, normalized size = 0.82

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(a - b*x**2)*elliptic_e(asin(sqrt(
d)*x/sqrt(c)), b*c/(a*d))/(sqrt(d)*sqrt(1 - b*x**2/a)*sqrt(-c + d
*x**2))
```

Mathematica [A] time = 0.0721078, size = 89, normalized size = 1.

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x
], (b*c)/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] time = 0.02, size = 166, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2-cx^2b+ac)\,d}\left(-aEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcEllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b
/a)^(1/2),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^
(1/2)))*(-b*x^2+a)^(1/2)*(d*x^2-c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*(-(
d*x^2-c)/c)^(1/2)/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="fricas")

[Out] integral(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)

[Out] Integral(sqrt(a - b*x**2)/sqrt(-c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

3.262 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sq
rt[-c + d*x^2])
```

Rubi [A] time = 0.160863, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sq
rt[-c + d*x^2])
```

Rubi in Sympy [A] time = 40.1029, size = 73, normalized size = 0.81

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a+bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(1 - d*x**2/c)*sqrt(-a + b*x**2)*elliptic_e(asin(sqrt
(d)*x/sqrt(c)), b*c/(a*d))/(sqrt(d)*sqrt(1 - b*x**2/a)*sqrt(-c +
d*x**2))
```

Mathematica [A] time = 0.064657, size = 90, normalized size = 1.

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*
x], (b*c)/(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]
)
```

Maple [A] time = 0.014, size = 111, normalized size = 1.2

$$\frac{a}{-bdx^4+adx^2+cx^2b-ac}\sqrt{bx^2-a}\sqrt{dx^2-c}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}\mathit{EllipticE}\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x
^2-c)^(1/2)*a*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*EllipticE
(x*(d/c)^(1/2),(b*c/a/d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)

[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

3.263 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])
```

Rubi [A] time = 0.270919, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2])
```

Rubi in Sympy [A] time = 38.7842, size = 172, normalized size = 0.89

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) + b*x*sqrt(c + d*x**2)/(d*sqrt(a + b*x**2)) + sqrt
(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/
(a*d))/(sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*
x**2))
```

Mathematica [A] time = 0.0775645, size = 86, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*
x^2])
```

Maple [A] time = 0., size = 158, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,d}\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(aEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*Ellipti
cF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2)))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.264 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c
]*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])
```

Rubi [A] time = 0.291073, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c
]*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])
```

Rubi in Sympy [A] time = 42.3273, size = 177, normalized size = 0.87

$$-\frac{a^{\frac{3}{2}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}c\sqrt{-\frac{a(c+dx^2)}{c(-a-bx^2)}}\sqrt{-a-bx^2}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -a**(3/2)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(sqrt(b)*c*sqrt(-a*(c + d*x**2)/(c*(-a - b*x**2)))*
sqrt(-a - b*x**2)) - sqrt(c)*sqrt(-a - b*x**2)*elliptic_e(atan(sq
rt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(sqrt(d)*sqrt(c*(a + b*x**2)/(a*
(c + d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(-a - b*x**2)/sqrt(c + d
*x**2)
```

Mathematica [A] time = 0.0677775, size = 89, normalized size = 0.44

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d
*x^2])
```

Maple [A] time = 0.023, size = 104, normalized size = 0.5

$$\frac{a}{bdx^4+adx^2+cx^2b+ac}\sqrt{-bx^2-a}\sqrt{dx^2+c}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)
/a)^(1/2)*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="fricas")

[Out] integral(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(-a - b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.265 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))])
```

Rubi [A] time = 0.295679, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))])
```

Rubi in Sympy [A] time = 42.7143, size = 180, normalized size = 0.89

$$\frac{\sqrt{a}\sqrt{b}\sqrt{-c-dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}-\frac{bx\sqrt{-c-dx^2}}{d\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-\frac{c(a+bx^2)}{a(-c-dx^2)}}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(b)*sqrt(-c - d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(
a)), -a*d/(b*c) + 1)/(d*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqr
t(a + b*x**2)) - b*x*sqrt(-c - d*x**2)/(d*sqrt(a + b*x**2)) + sqr
t(c)*sqrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c
/(a*d))/(sqrt(d)*sqrt(-c*(a + b*x**2)/(a*(-c - d*x**2)))*sqrt(-c
- d*x**2))
```

Mathematica [A] time = 0.0804456, size = 89, normalized size = 0.44

$$\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d
*x^2])
```

Maple [A] time = 0.019, size = 108, normalized size = 0.5

$$\frac{a}{-bdx^4-adx^2-cx^2b-ac}\sqrt{bx^2+a}\sqrt{-dx^2-c}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)
/a)^(1/2)*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(-b*d*x^4-a*d
*x^2-b*c*x^2-a*c)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 + a}}{\sqrt{-dx^2 - c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2}}{\sqrt{-c - dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}}{\sqrt{-dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.266 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=212

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d
]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt
[c]*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))])
```

Rubi [A] time = 0.323565, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d
]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt
[c]*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))])
```

Rubi in Sympy [A] time = 55.1375, size = 189, normalized size = 0.89

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{-c-dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{d\sqrt{\frac{a(-c-dx^2)}{c(-a-bx^2)}}\sqrt{-a-bx^2}}+\frac{bx\sqrt{-c-dx^2}}{d\sqrt{-a-bx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(-a-bx^2)}{a(-c-dx^2)}}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(-c - d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt
(a)), -a*d/(b*c) + 1)/(d*sqrt(a*(-c - d*x**2)/(c*(-a - b*x**2)))*
sqrt(-a - b*x**2)) + b*x*sqrt(-c - d*x**2)/(d*sqrt(-a - b*x**2))
+ sqrt(c)*sqrt(-a - b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1
 - b*c/(a*d))/(sqrt(d)*sqrt(c*(-a - b*x**2)/(a*(-c - d*x**2)))*sq
rt(-c - d*x**2))
```

Mathematica [A] time = 0.0651233, size = 92, normalized size = 0.43

$$\frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c -
d*x^2])
```

Maple [A] time = 0.019, size = 165, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,d}\left(-aEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcEllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcEllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2)))*(-b*x^2-a)^(1/2)*(-d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.267 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*
Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d
)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.391468, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*
Sqrt[1 + (d*x^2)/c])) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d
)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 76.1481, size = 162, normalized size = 0.86

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{d\sqrt{1+\frac{dx^2}{c}}\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(1 - b*x**2/a)*sqrt(c + d*x**2)*elliptic_e(a
sin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(d*sqrt(1 + d*x**2/c)*sqrt(a
```

```
- b*x**2)) + sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 + d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*d*
sqrt(a - b*x**2)*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.0742127, size = 89, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], -((b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c +
 d*x^2])
```

Maple [A] time = 0.018, size = 164, normalized size = 0.9

$$\frac{1}{(bdx^4-adx^2+cx^2b-ac)\,d}\left(-aEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcEllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(
b/a)^(1/2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/
c)^(1/2)))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^2}}{\sqrt{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2 + a}}{\sqrt{dx^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

3.268 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*S
qrt[1 + (d*x^2)/c]) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.382336, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*S
qrt[1 + (d*x^2)/c]) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 75.6895, size = 162, normalized size = 0.85

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{d\sqrt{1+\frac{dx^2}{c}}\sqrt{-a+bx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(b)*sqrt(1 - b*x**2/a)*sqrt(c + d*x**2)*elliptic_e(as
in(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(d*sqrt(1 + d*x**2/c)*sqrt(-a
```

```
+ b*x**2)) - sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 + d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*d*
sqrt(-a + b*x**2)*sqrt(c + d*x**2))
```

Mathematica [A] time = 0.0756661, size = 90, normalized size = 0.47

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], -((b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.022, size = 109, normalized size = 0.6

$$\frac{a}{-bdx^4+adx^2-cx^2b+ac}\sqrt{bx^2-a}\sqrt{dx^2+c}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*
x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

3.269 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*S
qrt[1 + (d*x^2)/c]) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[-c - d*x^2])
```

Rubi [A] time = 0.394472, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*S
qrt[1 + (d*x^2)/c]) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/
(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[-c - d*x^2])
```

Rubi in Sympy [A] time = 99.8421, size = 165, normalized size = 0.85

$$\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{d\sqrt{1+\frac{dx^2}{c}}\sqrt{a-bx^2}}+\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(b)*sqrt(1 - b*x**2/a)*sqrt(-c - d*x**2)*elliptic_e(a
sin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(d*sqrt(1 + d*x**2/c)*sqrt(a
```

```
- b*x**2)) + sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 + d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*d*
sqrt(a - b*x**2)*sqrt(-c - d*x**2))
```

Mathematica [A] time = 0.0789673, size = 92, normalized size = 0.47

$$\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], -((b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c
- d*x^2])
```

Maple [A] time = 0.017, size = 111, normalized size = 0.6

$$\frac{a}{bdx^4-adx^2+cx^2b-ac}\sqrt{-bx^2+a}\sqrt{-dx^2-c}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*(-(b*x^2-
a)/a)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4-a
*d*x^2+b*c*x^2-a*c)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

3.270 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2
]*Sqrt[1 + (d*x^2)/c])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqrt[-c - d*x^2])
```

Rubi [A] time = 0.39615, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE
[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2
]*Sqrt[1 + (d*x^2)/c])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a
*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqrt[-c - d*x^2])
```

Rubi in Sympy [A] time = 93.126, size = 167, normalized size = 0.84

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{d\sqrt{1+\frac{dx^2}{c}}\sqrt{-a+bx^2}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(1 - b*x**2/a)*sqrt(-c - d*x**2)*elliptic_e(
asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(d*sqrt(1 + d*x**2/c)*sqrt(-
```

```
a + b*x**2)) - sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(1 + d*x**2/c)*(a*d
 + b*c)*elliptic_f(asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*
d*sqrt(-a + b*x**2)*sqrt(-c - d*x**2))
```

Mathematica [A] time = 0.0649757, size = 93, normalized size = 0.47

$$\frac{\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], -((b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c
 - d*x^2])
```

Maple [A] time = 0.017, size = 167, normalized size = 0.8

$$\frac{1}{(bdx^4-adx^2+cx^2b-ac)\,d}\sqrt{bx^2-a}\sqrt{-dx^2-c}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}\left(aEllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcEllipticF\left(x\sqrt{\frac{b}{a}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x)
```

```
[Out] (b*x^2-a)^(1/2)*(-d*x^2-c)^(1/2)*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*(a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*Ellip
ticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),
(-a*d/b/c)^(1/2)))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2 - a}}{\sqrt{-dx^2 - c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a + bx^2}}{\sqrt{-c - dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c - d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 - a}}{\sqrt{-dx^2 - c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

3.271 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt
[1 + (d*x^2)/c])
```

Rubi [A] time = 0.150989, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt
[1 + (d*x^2)/c])
```

Rubi in Sympy [A] time = 31.343, size = 75, normalized size = 0.86

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1+\frac{dx^2}{c}}\sqrt{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(c + d*x**2)*elliptic_e(asin(sqrt(
b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*sqrt(1 + d*x**2/c)*sqrt(a - b
*x**2))
```

Mathematica [A] time = 0.0847225, size = 87, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x
], -((a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c
])
```

Maple [A] time = 0.017, size = 106, normalized size = 1.2

$$\frac{c}{-bdx^4+adx^2-cx^2b+ac}\sqrt{dx^2+c}\sqrt{-bx^2+a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4+a*
d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.272 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqr
t[1 + (d*x^2)/c])
```

Rubi [A] time = 0.158979, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqr
t[1 + (d*x^2)/c])
```

Rubi in Sympy [A] time = 39.1347, size = 76, normalized size = 0.84

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1+\frac{dx^2}{c}}\sqrt{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(-c - d*x**2)*elliptic_e(asin(sqrt
(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*sqrt(1 + d*x**2/c)*sqrt(a -
b*x**2))
```

Mathematica [A] time = 0.0685608, size = 90, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*
x], -((a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/
c])
```

Maple [B] time = 0.025, size = 171, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2+cx^2b-ac)\,b}\left(-adEllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cEllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adEllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(-
d/c)^(1/2),(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(-b*c
/a/d)^(1/2)))*(-d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/
2)*(-(b*x^2-a)/a)^(1/2)/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(-d/c)^(1/2
)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.273 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqr
t[1 + (d*x^2)/c])
```

Rubi [A] time = 0.15032, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqr
t[1 + (d*x^2)/c])
```

Rubi in Sympy [A] time = 31.3644, size = 75, normalized size = 0.85

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1+\frac{dx^2}{c}}\sqrt{-a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(c + d*x**2)*elliptic_e(asin(sqrt(
b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*sqrt(1 + d*x**2/c)*sqrt(-a +
b*x**2))
```

Mathematica [A] time = 0.0695233, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x
], -((a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/
c])
```

Maple [B] time = 0.018, size = 168, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2+cx^2b-ac)\,b}\sqrt{dx^2+c}\sqrt{bx^2-a}\sqrt{\frac{dx^2+c}{c}}\sqrt{-\frac{bx^2-a}{a}}\left(adEllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cEllipticF\left(x\sqrt{-\frac{d}{c}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*(-(b*x^2-a)/a
)^(1/2)*(a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))+c*Ellipti
cF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2
),(-b*c/a/d)^(1/2)))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.274 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sq
rt[1 + (d*x^2)/c])
```

Rubi [A] time = 0.158053, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sq
rt[1 + (d*x^2)/c])
```

Rubi in Sympy [A] time = 37.5815, size = 76, normalized size = 0.84

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1+\frac{dx^2}{c}}\sqrt{-a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(-c - d*x**2)*elliptic_e(asin(sqrt
(b)*x/sqrt(a)), -a*d/(b*c))/(sqrt(b)*sqrt(1 + d*x**2/c)*sqrt(-a +
 b*x**2))
```

Mathematica [A] time = 0.0630408, size = 91, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*
x], -((a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)
/c])
```

Maple [A] time = 0.016, size = 110, normalized size = 1.2

$$\frac{c}{bdx^4-adx^2+cx^2b-ac}\sqrt{-dx^2-c}\sqrt{bx^2-a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4-a*d
*x^2+b*c*x^2-a*c)/(b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.275 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1
- (d*x^2)/c])
```

Rubi [A] time = 0.158541, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1
- (d*x^2)/c])
```

Rubi in Sympy [A] time = 42.2648, size = 73, normalized size = 0.83

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1-\frac{dx^2}{c}}\sqrt{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(c - d*x**2)*elliptic_e(asin(sqrt(
b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*sqrt(1 - d*x**2/c)*sqrt(a - b*
x**2))
```

Mathematica [A] time = 0.0834989, size = 88, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x
], (a*d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] time = 0.019, size = 164, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2-cx^2b+ac)\,b}\left(-adEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+cEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+adEllipticE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(d/c
)^(1/2),(b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^
(1/2)))*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*(-
(b*x^2-a)/a)^(1/2)/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(c - d*x**2)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

3.276 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1
 - (d*x^2)/c])
```

Rubi [A] time = 0.156421, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1
 - (d*x^2)/c])
```

Rubi in Sympy [A] time = 39.0801, size = 73, normalized size = 0.82

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1-\frac{dx^2}{c}}\sqrt{a-bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(-c + d*x**2)*elliptic_e(asin(sqrt
(b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*sqrt(1 - d*x**2/c)*sqrt(a - b
*x**2))
```

Mathematica [A] time = 0.0694258, size = 89, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*
x], (a*d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.024, size = 110, normalized size = 1.2

$$\frac{c}{bdx^4-adx^2-cx^2b+ac}\sqrt{dx^2-c}\sqrt{-bx^2+a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}EllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-
c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4-a*d
*x^2-b*c*x^2+a*c)/(b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

3.277 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1
 - (d*x^2)/c])
```

Rubi [A] time = 0.156891, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1
 - (d*x^2)/c])
```

Rubi in Sympy [A] time = 40.8068, size = 73, normalized size = 0.82

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(c - d*x**2)*elliptic_e(asin(sqrt(
b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*sqrt(1 - d*x**2/c)*sqrt(-a + b
*x**2))
```

Mathematica [A] time = 0.070451, size = 89, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x
], (a*d)/(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] time = 0.021, size = 110, normalized size = 1.2

$$\frac{c}{-bdx^4+adx^2+cx^2b-ac}\sqrt{-dx^2+c}\sqrt{bx^2-a}\sqrt{-\frac{bx^2-a}{a}}\sqrt{-\frac{dx^2-c}{c}}EllipticE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*c*(-(b*x^2-a)/a)^(1/2)*(-(d*x^2-
c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(-b*d*x^4+a*
d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c - dx^2}}{\sqrt{-a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}}{\sqrt{bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

3.278 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[
1 - (d*x^2)/c])
```

Rubi [A] time = 0.156304, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[
1 - (d*x^2)/c])
```

Rubi in Sympy [A] time = 39.3147, size = 73, normalized size = 0.81

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(1 - b*x**2/a)*sqrt(-c + d*x**2)*elliptic_e(asin(sqrt
(b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*sqrt(1 - d*x**2/c)*sqrt(-a +
b*x**2))
```

Mathematica [A] time = 0.0621381, size = 90, normalized size = 1.

$$\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{bx^2-a}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*
x], (a*d)/(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]
)
```

Maple [B] time = 0.016, size = 167, normalized size = 1.9

$$\frac{1}{(bdx^4-adx^2-cx^2b+ac)\,b}\sqrt{dx^2-c}\sqrt{bx^2-a}\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}\left(adEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/
a)^(1/2)*(a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))-c*Elliptic
F(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(b
*c/a/d)^(1/2)))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

3.279 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (
c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi [A] time = 0.276064, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```


```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt
[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (
c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 36.8311, size = 168, normalized size = 0.82

$$-\frac{\sqrt{a}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{c+dx^2}}{\sqrt{a+bx^2}}+\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a*
d/(b*c) + 1)/(sqrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(
a + b*x**2)) + x*sqrt(c + d*x**2)/sqrt(a + b*x**2) + c**(3/2)*sqr
t(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/
(a*sqrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2))))*sqrt(c + d*x**2)
)
```

Mathematica [A] time = 0.0818254, size = 86, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)
]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2
)/c])
```

Maple [A] time = 0., size = 101, normalized size = 0.5

$$\frac{c}{bdx^4+adx^2+cx^2b+ac}\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.280 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(b*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.298888, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*
Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(b*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
) - (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rubi in Sympy [A] time = 40.4029, size = 177, normalized size = 0.83

$$-\frac{\sqrt{a}\sqrt{-c-dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}}+\frac{x\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}-\frac{c^{\frac{3}{2}}\sqrt{a+bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-\frac{c(a+bx^2)}{a(-c-dx^2)}}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(-c - d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(sqrt(b)*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt
(a + b*x**2)) + x*sqrt(-c - d*x**2)/sqrt(a + b*x**2) - c**(3/2)*s
qrt(a + b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d)
)/(a*sqrt(d)*sqrt(-c*(a + b*x**2)/(a*(-c - d*x**2)))*sqrt(-c - d*
x**2))
```

Mathematica [A] time = 0.0682434, size = 89, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a
)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^
2)/c])
```

Maple [A] time = 0.024, size = 161, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,b}\sqrt{-dx^2-c}\sqrt{bx^2+a}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\left(adEllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-adEllipticF\left(x\sqrt{-\frac{d}{c}},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-d*x^2-c)^(1/2)*(b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a
)^(1/2)*(a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-a*d*Ellipt
icF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),(b
*c/a/d)^(1/2))*b)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2 - c}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c - dx^2}}{\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(-c - d*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="giac")

[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.281 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=214

$$-\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*
Sqrt[-a - b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
) - (c^(3/2)*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.304192, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*
Sqrt[-a - b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
) - (c^(3/2)*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 41.2471, size = 180, normalized size = 0.84

$$\frac{\sqrt{a}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}\sqrt{-\frac{a(c+dx^2)}{c(-a-bx^2)}}\sqrt{-a-bx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(b)*x/sqrt(a)), -a*d
/(b*c) + 1)/(sqrt(b)*sqrt(-a*(c + d*x**2)/(c*(-a - b*x**2)))*sqrt
(-a - b*x**2)) + sqrt(c)*sqrt(d)*sqrt(-a - b*x**2)*elliptic_e(ata
n(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(b*sqrt(c*(a + b*x**2)/(a*(c
 + d*x**2)))*sqrt(c + d*x**2)) - d*x*sqrt(-a - b*x**2)/(b*sqrt(c
+ d*x**2))
```

Mathematica [A] time = 0.0767726, size = 89, normalized size = 0.42

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)
]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^
2)/c])
```

Maple [A] time = 0.019, size = 162, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2+cx^2b+ac)\,b}\sqrt{dx^2+c}\sqrt{-bx^2-a}\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{bx^2+a}{a}}\left(adEllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cEllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a
)^(1/2)*(a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-c*Elliptic
F(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),
(b*c/a/d)^(1/2)))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2 + c}}{\sqrt{-bx^2 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="fricas")

[Out] integral(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)

[Out] Integral(sqrt(c + d*x**2)/sqrt(-a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{-bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.282 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=222

$$\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sq
rt[-a - b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(b*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
 + (c^(3/2)*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rubi [A] time = 0.326267, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sq
rt[-a - b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(b*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
 + (c^(3/2)*Sqrt[-a - b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rubi in Sympy [A] time = 52.4714, size = 185, normalized size = 0.83

$$-\frac{\sqrt{a}\sqrt{-c-dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}\sqrt{\frac{a(-c-dx^2)}{c(-a-bx^2)}}\sqrt{-a-bx^2}}+\frac{x\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}+\frac{c^{\frac{3}{2}}\sqrt{-a-bx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(-a-bx^2)}{a(-c-dx^2)}}\sqrt{-c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(-c - d*x**2)*elliptic_e(atan(sqrt(b)*x/sqrt(a)), -a
*d/(b*c) + 1)/(sqrt(b)*sqrt(a*(-c - d*x**2)/(c*(-a - b*x**2)))*sq
rt(-a - b*x**2)) + x*sqrt(-c - d*x**2)/sqrt(-a - b*x**2) + c**(3/
2)*sqrt(-a - b*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/
(a*d))/(a*sqrt(d)*sqrt(c*(-a - b*x**2)/(a*(-c - d*x**2)))*sqrt(-c
 - d*x**2))
```

Mathematica [A] time = 0.0625852, size = 92, normalized size = 0.41

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a
)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x
^2)/c])
```

Maple [A] time = 0.02, size = 111, normalized size = 0.5

$$\frac{c}{-bdx^4-adx^2-cx^2b-ac}\sqrt{-dx^2-c}\sqrt{-bx^2-a}\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)*(-d*x^2-c)^(1/2)*(-
b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2 - c}}{\sqrt{-bx^2 - a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c - dx^2}}{\sqrt{-a - bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 - c}}{\sqrt{-bx^2 - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.283 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)
/a]*Sqrt[c - d*x^2])) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c
)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.386785, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)
/a]*Sqrt[c - d*x^2])) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c
)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 74.5856, size = 162, normalized size = 0.86

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{a+bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(a + b*x**2)*elliptic_e(a
sin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(1 + b*x**2/a)*sqrt(c
```

```
- d*x**2)) + sqrt(c)*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(d)*
sqrt(a + b*x**2)*sqrt(c - d*x**2))
```

Mathematica [A] time = 0.0799097, size = 89, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)
]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*
x^2)/c])
```

Maple [A] time = 0.017, size = 164, normalized size = 0.9

$$\frac{1}{(bdx^4+adx^2-cx^2b-ac)\,b}\left(-adEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adEllipticE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(d/
c)^(1/2),(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/
d)^(1/2)))*(-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*
((b*x^2+a)/a)^(1/2)/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

3.284 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a
]*Sqrt[-c + d*x^2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/
(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.37521, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24

$$\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a
]*Sqrt[-c + d*x^2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/
(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[-c + d*x^2])
```

Rubi in Sympy [A] time = 75.2671, size = 162, normalized size = 0.85

$$\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{a+bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}-\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(a + b*x**2)*elliptic_e(as
in(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(1 + b*x**2/a)*sqrt(-c
```

```
+ d*x**2)) - sqrt(c)*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(d)*
sqrt(a + b*x**2)*sqrt(-c + d*x**2))
```

Mathematica [A] time = 0.0669913, size = 90, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d
*x^2)/c])
```

Maple [A] time = 0.022, size = 109, normalized size = 0.6

$$\frac{c}{-bdx^4-adx^2+cx^2b+ac}\sqrt{dx^2-c}\sqrt{bx^2+a}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)*(d*x^2-c)^(1/2)*(b*
x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*EllipticE
(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c + dx^2}}{\sqrt{a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 - c}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

3.285 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/
a]*Sqrt[c - d*x^2]) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/
(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.395104, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/
a]*Sqrt[c - d*x^2]) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/
(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 97.9565, size = 165, normalized size = 0.85

$$\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}+\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(c)*sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(-a - b*x**2)*elliptic_e(a
sin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(1 + b*x**2/a)*sqrt(c
```

```
- d*x**2)) + sqrt(c)*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*(a*d +
 b*c)*elliptic_f(asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(d)*
sqrt(-a - b*x**2)*sqrt(c - d*x**2))
```

Mathematica [A] time = 0.0707044, size = 92, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)
]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d
*x^2)/c])
```

Maple [A] time = 0.018, size = 111, normalized size = 0.6

$$\frac{c}{bdx^4+adx^2-cx^2b-ac}\sqrt{-dx^2+c}\sqrt{-bx^2-a}\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}EllipticE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (-d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*(-(d*x^2-
c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4+a
*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

3.286 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2
)/a]*Sqrt[-c + d*x^2])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b
*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[-c + d*x^2])
```

Rubi [A] time = 0.397058, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}}-\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2
)/a]*Sqrt[-c + d*x^2])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b
*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[-c + d*x^2])
```

Rubi in Sympy [A] time = 94.7698, size = 167, normalized size = 0.84

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{-a-bx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}-\frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,(ad+bc)\,F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{-c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(-a - b*x**2)*elliptic_e(
asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(1 + b*x**2/a)*sqrt(-
```

```
c + d*x**2)) - sqrt(c)*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*(a*d
 + b*c)*elliptic_f(asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(b*sqrt(d
)*sqrt(-a - b*x**2)*sqrt(-c + d*x**2))
```

Mathematica [A] time = 0.0637675, size = 93, normalized size = 0.47

$$\frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{dx^2-c}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a
)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c -
d*x^2)/c])
```

Maple [A] time = 0.018, size = 167, normalized size = 0.8

$$\frac{1}{(bdx^4+adx^2-cx^2b-ac)\,b}\sqrt{dx^2-c}\sqrt{-bx^2-a}\sqrt{-\frac{dx^2-c}{c}}\sqrt{\frac{bx^2+a}{a}}\left(adEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cEllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x)
```

```
[Out] (d*x^2-c)^(1/2)*(-b*x^2-a)^(1/2)*(-(d*x^2-c)/c)^(1/2)*((b*x^2+a)/
a)^(1/2)*(a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))+c*Ellipti
cF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),
(-b*c/a/d)^(1/2)))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a - b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

3.287 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{bx^2+2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)
/(2*d)])/(Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 +
d*x^2])
```

Rubi [A] time = 0.0536963, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{\sqrt{bx^2+2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)
/(2*d)])/(Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 +
d*x^2])
```

Rubi in Sympy [A] time = 8.78081, size = 75, normalized size = 0.96

$$\frac{\sqrt{3}\sqrt{bx^2+2}\,F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}\,x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{2\sqrt{d}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(b*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(d)*x/3), -3
*b/(2*d) + 1)/(2*sqrt(d)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sqrt
(d*x**2 + 3))
```

Mathematica [A] time = 0.0440885, size = 37, normalized size = 0.47

$$\frac{F\left(\sin^{-1}\left(\frac{\sqrt{-b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)}{\sqrt{3}\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[-b]*x)/Sqrt[2]], (2*d)/(3*b)]/(Sqrt[3]*Sqr
t[-b])
```

Maple [A] time = 0.032, size = 38, normalized size = 0.5

$$\frac{\sqrt{2}}{2}EllipticF\left(\frac{x\sqrt{3}}{3}\sqrt{-d},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2
)*(b/d)^(1/2))/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.288 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d
*x^2]
```

Rubi [A] time = 0.07319, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$\frac{\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d
*x^2]
```

Rubi in Sympy [A] time = 15.8944, size = 32, normalized size = 0.82

$$\frac{\sqrt{1+\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] sqrt(1 + d*x**2/c)*elliptic_f(asin(x/2), -4*d/c)/sqrt(c + d*x**2)
```

Mathematica [A] time = 0.0560501, size = 40, normalized size = 1.03

$$\frac{\sqrt{\frac{c+dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d
*x^2]
```

Maple [A] time = 0.032, size = 38, normalized size = 1.

$$1\sqrt{\frac{dx^2+c}{c}}EllipticF\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\frac{1}{\sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(1/2*x,2*(-d/c)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x-2\right)\left(x+2\right)}\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 2)*(x + 2))*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.289 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 +
 x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi [A] time = 0.0431241, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 +
 x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

Rubi in Sympy [A] time = 8.12714, size = 53, normalized size = 0.87

$$\frac{2\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{x}{2}\right)\middle|1-\frac{4d}{c}\right)}{c\sqrt{\frac{4c+4dx^2}{c(x^2+4)}}\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] 2*sqrt(c + d*x**2)*elliptic_f(atan(x/2), 1 - 4*d/c)/(c*sqrt((4*c
+ 4*d*x**2)/(c*(x**2 + 4)))*sqrt(x**2 + 4))
```

Mathematica [C] time = 0.0497062, size = 47, normalized size = 0.77

$$-\frac{i\sqrt{\frac{c+dx^2}{c}}F\left(i\sinh^{-1}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[(c + d*x^2)/c]*EllipticF[I*ArcSinh[x/2], (4*d)/c])/Sqr
t[c + d*x^2]
```

Maple [A] time = 0.028, size = 53, normalized size = 0.9

$$\frac{1}{2}\sqrt{\frac{dx^2+c}{c}}\,EllipticF\left(x\sqrt{-\frac{d}{c}},\frac{1}{2}\sqrt{\frac{c}{d}}\right)\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-d/c)^(1/2),
1/2*(c/d)^(1/2))/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{c + dx^2}\sqrt{x^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx^2 + c}\sqrt{x^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.290 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal. Leaf size=6

$$-F\left(\cos^{-1}(x)\middle|2\right)$$

```
[Out] -EllipticF[ArcCos[x], 2]
```

Rubi [A] time = 0.0246358, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-F\left(\cos^{-1}(x)\middle|2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] -EllipticF[ArcCos[x], 2]
```

Rubi in Sympy [A] time = 6.94774, size = 5, normalized size = 0.83

$$-F\left(\operatorname{acos}(x)\middle|2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)

[Out] -elliptic_f(acos(x), 2)
```

Mathematica [B] time = 0.0361786, size = 27, normalized size = 4.5

$$\frac{\sqrt{1-2x^2}F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]

[Out] (Sqrt[1 - 2*x^2]*EllipticF[ArcSin[x], 2])/Sqrt[-1 + 2*x^2]
```

Maple [A] time = 0.025, size = 25, normalized size = 4.2

$$\mathit{EllipticF}\left(x, \sqrt{2}\right)\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x)

[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="fricas")

[Out] integral(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x+1\right)}\sqrt{2x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)

[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)),x, algorithm="giac")

[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.291 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi [A] time = 0.0972371, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2F\left(\sin^{-1}(cx)\middle|-1\right)}{c}-\frac{E\left(\sin^{-1}(cx)\middle|-1\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/
c
```

Rubi in Sympy [A] time = 27.9475, size = 20, normalized size = 0.87

$$-\frac{E\left(\operatorname{asin}(cx)\middle|-1\right)}{c}+\frac{2F\left(\operatorname{asin}(cx)\middle|-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] -elliptic_e(asin(c*x), -1)/c + 2*elliptic_f(asin(c*x), -1)/c
```

Mathematica [A] time = 0.0391707, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] time = 0.028, size = 28, normalized size = 1.2

$$\frac{(2\,EllipticF\,(x\,csgn\,(c)\,c,i) - EllipticE\,(x\,csgn\,(c)\,c,i))\,csgn\,(c)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2 + 1}}{\sqrt{c^2x^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-c^2x^2 + 1}}{\sqrt{c^2x^2 + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

3.292 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*S
qrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/
(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi [A] time = 0.252982, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174

$$\frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}+\frac{\sqrt{2}\sqrt{bx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*S
qrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/
(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rubi in Sympy [A] time = 32.4892, size = 175, normalized size = 0.96

$$-\frac{\sqrt{2}\sqrt{b}\sqrt{dx^2+3}E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{b}x}{2}\right)\middle|1-\frac{2d}{3b}\right)}{d\sqrt{\frac{2dx^2+6}{3bx^2+6}}\sqrt{bx^2+2}}+\frac{bx\sqrt{dx^2+3}}{d\sqrt{bx^2+2}}+\frac{\sqrt{3}\sqrt{bx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{d}x}{3}\right)\middle|-\frac{3b}{2d}+1\right)}{\sqrt{d}\sqrt{\frac{3bx^2+6}{2dx^2+6}}\sqrt{dx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(b)*sqrt(d*x**2 + 3)*elliptic_e(atan(sqrt(2)*sqrt(b)
*x/2), 1 - 2*d/(3*b))/(d*sqrt((2*d*x**2 + 6)/(3*b*x**2 + 6))*sqrt
```

```
(b*x**2 + 2)) + b*x*sqrt(d*x**2 + 3)/(d*sqrt(b*x**2 + 2)) + sqrt(
3)*sqrt(b*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(d)*x/3), -3*b/(2
*d) + 1)/(sqrt(d)*sqrt((3*b*x**2 + 6)/(2*d*x**2 + 6))*sqrt(d*x**2
 + 3))
```

Mathematica [A] time = 0.0347604, size = 37, normalized size = 0.2

$$\frac{\sqrt{2}E\left(\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sq
rt[-d]
```

Maple [A] time = 0., size = 37, normalized size = 0.2

$$\sqrt{2}EllipticE\left(\frac{x\sqrt{3}}{3}\sqrt{-d}, \frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{b}{d}}\right)\frac{1}{\sqrt{-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x)
```

```
[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*3^(1/2)*2^(1/2)*(b/d)^(1/2
))*2^(1/2)/(-d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="fricas")

[Out] integral(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)

[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3),x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

3.293 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rubi [A] time = 0.0261532, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]

[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rubi in Sympy [A] time = 5.26547, size = 19, normalized size = 1.

$$-\frac{\sqrt{3}E\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{2}\right)\middle|2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)

[Out] -sqrt(3)*elliptic_e(acos(sqrt(6)*x/2), 2)/3
```

Mathematica [A] time = 0.0353428, size = 35, normalized size = 1.84

$$\frac{\sqrt{3x^2-1}E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3-9x^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (Sqrt[-1 + 3*x^2]*EllipticE[ArcSin[Sqrt[3/2]*x], 2])/Sqrt[3 - 9*x
^2]
```

Maple [A] time = 0.027, size = 37, normalized size = 2.

$$-\frac{\sqrt{3}}{3}EllipticE\left(\frac{x\sqrt{3}\sqrt{2}}{2},\sqrt{2}\right)\sqrt{-3x^2+1}\frac{1}{\sqrt{3x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x)
```

```
[Out] -1/3*EllipticE(1/2*x*3^(1/2)*2^(1/2),2^(1/2))*(-3*x^2+1)^(1/2)*3^
(1/2)/(3*x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2-1}}{\sqrt{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3x^2-1}}{\sqrt{-3x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3x^2 - 1}}{\sqrt{-3x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 - 1)/sqrt(-3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{3\,x^2 - 1}}{\sqrt{-3\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

3.294 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=95

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[c])
```

Rubi [A] time = 0.297252, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.017$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 -
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[c])
```

Rubi in Sympy [A] time = 23.6938, size = 87, normalized size = 0.92

$$\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\,E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\middle|-\frac{b+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(b + sqrt(-4*a*c + b**2))*elliptic_e(asin(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2))), -(b + sqrt(-4*a*c + b**2))
```

```
/(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c))
```

Mathematica [A] time = 0.413655, size = 92, normalized size = 0.97

$$\frac{E\left(\sin^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
```

```
[Out] EllipticE[ArcSin[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]/(Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int 1\sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1-2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
```

```
[Out] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2 -
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2
/(b + sqrt(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{b-\sqrt{b^2-4\,ac}}}}{\sqrt{-\frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{b+\sqrt{b^2-4\,ac}}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2 -
```

```
[Out] integral(sqrt((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(b - sqrt(b^2 - 4
*a*c)))/sqrt(-(2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(b + sqrt(b^2 - 4
*a*c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a
*c + b**2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqr
t(-4*a*c + b**2))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2 -
```

```
[Out] Timed out
```

3.295 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c])
```

Rubi [A] time = 0.258262, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 -
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c])
```

Rubi in Sympy [A] time = 38.0453, size = 85, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\,E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\middle|\frac{b+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(b + sqrt(-4*a*c + b**2))*elliptic_e(asin(sqrt(2)*sqr
t(c)*x/sqrt(b + sqrt(-4*a*c + b**2))), (b + sqrt(-4*a*c + b**2))/
```

```
(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c))
```

Mathematica [A] time = 0.358122, size = 92, normalized size = 0.98

$$\frac{E\left(\sin^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
```

```
[Out] EllipticE[ArcSin[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]/(Sqrt[2]*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.212, size = 0, normalized size = 0.

$$\int 1\sqrt{1-2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1-2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
```

```
[Out] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^
2/(b + sqrt(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-\frac{2\,cx^2-b+\sqrt{b^2-4\,ac}}{b-\sqrt{b^2-4\,ac}}}}{\sqrt{-\frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{b+\sqrt{b^2-4\,ac}}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2
```

```
[Out] integral(sqrt(-(2*c*x^2 - b + sqrt(b^2 - 4*a*c))/(b - sqrt(b^2 -
4*a*c)))/sqrt(-(2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(b + sqrt(b^2 -
4*a*c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4
*a*c + b**2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + s
qrt(-4*a*c + b**2))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt(b^2
```

```
[Out] Timed out
```

3.296 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=478

$$\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.10189, antiderivative size = 478, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.068

$$\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
```

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 159.989, size = 425, normalized size = 0.89

$$\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}}{\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}} - \frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\,E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\middle|-\frac{2\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}} + \frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\,F\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)\middle|-\frac{2\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] x*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)/sqrt(2*c*x**2/(b +
 sqrt(-4*a*c + b**2)) + 1) - sqrt(2)*sqrt(b + sqrt(-4*a*c + b**2)
)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*elliptic_e(atan(sq
rt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2))), -2*sqrt(-4*a*c +
b**2)/(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt((2*c*x**2/(b - s
qrt(-4*a*c + b**2)) + 1)/(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)
)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)) + sqrt(2)*sqrt(b
+ sqrt(-4*a*c + b**2))*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*elliptic_f(atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)
)), -2*sqrt(-4*a*c + b**2)/(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*
sqrt((2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)/(2*c*x**2/(b + sqrt
(-4*a*c + b**2)) + 1))*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) +
1))
```

Mathematica [A] time = 0.357726, size = 94, normalized size = 0.2

$$\frac{E\left(\sin^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
```

```
[Out] EllipticE[ArcSin[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]/(Sqrt[2]*Sqrt[-(c
/(b + Sqrt[b^2 - 4*a*c]))])
```

Maple [F] time = 0.203, size = 0, normalized size = 0.

$$\int 1\sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
```

```
[Out] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 - 
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/
(b + sqrt(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{b-\sqrt{b^2-4\,ac}}}}{\sqrt{\frac{2\,cx^2+b+\sqrt{b^2-4\,ac}}{b+\sqrt{b^2-4\,ac}}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 - 
```

```
[Out] integral(sqrt((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(b - sqrt(b^2 - 4
*a*c)))/sqrt((2*c*x^2 + b + sqrt(b^2 - 4*a*c))/(b + sqrt(b^2 - 4*
a*c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a
*c + b**2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(
-4*a*c + b**2))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 - 
```

```
[Out] Timed out
```

3.297 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt{2}bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[
b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) +
 (Sqrt[2]*b*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))]
)/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.661834, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051

$$\frac{\sqrt{2}bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
```

```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[
b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) +
 (Sqrt[2]*b*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))]
)/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 82.5352, size = 185, normalized size = 0.86

$$\frac{\sqrt{2}bF\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\middle|\frac{-b+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}\right)}{\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}}-\frac{\sqrt{2}\left(b+\sqrt{-4ac+b^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\middle|\frac{-b+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] sqrt(2)*b*elliptic_f(asin(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c
+ b**2))), (-b + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))/
(sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))) - sqrt(2)*(b + sqrt(-4*a*
c + b**2))*elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c
 + b**2))), (-b + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
/(2*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2)))
```

Mathematica [A] time = 0.313601, size = 94, normalized size = 0.44

$$\frac{E\left(\sin^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}-b}\right)}{\sqrt{2}\sqrt{-\frac{c}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
```

```
[Out] EllipticE[ArcSin[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (
b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]/(Sqrt[2]*Sqrt[-(
c/(b + Sqrt[b^2 - 4*a*c]))])
```

Maple [F] time = 0.137, size = 0, normalized size = 0.

$$\int 1\sqrt{1-2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))
```

```
[Out] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 -
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2
/(b + sqrt(b^2 - 4*a*c)) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-\frac{2\,cx^2-b+\sqrt{b^2-4\,ac}}{b-\sqrt{b^2-4\,ac}}}}{\sqrt{\frac{2\,cx^2+b+\sqrt{b^2-4\,ac}}{b+\sqrt{b^2-4\,ac}}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 -
```

```
[Out] integral(sqrt(-(2*c*x^2 - b + sqrt(b^2 - 4*a*c))/(b - sqrt(b^2 -
4*a*c)))/sqrt((2*c*x^2 + b + sqrt(b^2 - 4*a*c))/(b + sqrt(b^2 - 4
*a*c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4
*a*c + b**2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqr
t(-4*a*c + b**2))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(b^2 -

[Out] Timed out
```

3.298 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2^{-m-2}\sqrt{x^2}\left(2-4x^2\right)^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\left(1-2x^2\right)^2\right)}{(m+1)x}$$

```
[Out] -((2^(-2 - m)*Sqrt[x^2]*(2 - 4*x^2)^(1 + m)*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, (1 - 2*x^2)^2])/((1 + m)*x))
```

Rubi [C] time = 0.0345361, antiderivative size = 23, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$xF_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]

[Out] x*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2]
```

Rubi in Sympy [A] time = 6.13962, size = 15, normalized size = 0.24

$$x\,\text{appellf}_1\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},x^2,2x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)

[Out] x*appellf1(1/2, 1/2, -m, 3/2, x**2, 2*x**2)
```

Mathematica [C] time = 0.193788, size = 122, normalized size = 1.97

$$\frac{3x\left(1-2x^2\right)^m F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)}{\sqrt{1-x^2}\left(x^2\left(F_1\left(\frac{3}{2};-m,\frac{3}{2};\frac{5}{2};2x^2,x^2\right)-4mF_1\left(\frac{3}{2};1-m,\frac{1}{2};\frac{5}{2};2x^2,x^2\right)\right)+3F_1\left(\frac{1}{2};-m,\frac{1}{2};\frac{3}{2};2x^2,x^2\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] (3*x*(1 - 2*x^2)^m*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2])/(Sqrt
[1 - x^2]*(3*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2] + x^2*(-4*m*
AppellF1[3/2, 1 - m, 1/2, 5/2, 2*x^2, x^2] + AppellF1[3/2, -m, 3/
2, 5/2, 2*x^2, x^2])))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \left(-2 x^{2}+1\right)^{m} \frac{1}{\sqrt{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

```
[Out] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-2 x^{2}+1\right)^{m}}{\sqrt{-x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-2 x^{2}+1\right)^{m}}{\sqrt{-x^{2}+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1),x, algorithm="fricas")
```

```
[Out] integral((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-2x^{2}+1\right)^{m}}{\sqrt{-\left(x-1\right)\left(x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)

[Out] Integral((-2*x**2 + 1)**m/sqrt(-(x - 1)*(x + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-2\,x^{2}+1\right)^{m}}{\sqrt{-x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1),x, algorithm="giac")

[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

3.299 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*
Sqrt[3]]*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0994968, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*
Sqrt[3]]*Sqrt[-1 + x^2])
```

Rubi in Sympy [A] time = 15.7275, size = 41, normalized size = 0.89

$$\frac{\sqrt{-x^2+1}F\left(\operatorname{asin}(x)\middle|-7-4\sqrt{3}\right)}{\sqrt{-4\sqrt{3}+7}\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
```

```
[Out] sqrt(-x**2 + 1)*elliptic_f(asin(x), -7 - 4*sqrt(3))/(sqrt(-4*sqrt
(3) + 7)*sqrt(x**2 - 1))
```

Mathematica [A] time = 0.107458, size = 48, normalized size = 1.04

$$\frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)|\frac{1}{-7+4\sqrt{3}}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], (-7 + 4*Sqrt[3])^(-1)])/(Sqrt
[7 - 4*Sqrt[3]]*Sqrt[-1 + x^2])
```

Maple [B] time = 0.245, size = 117, normalized size = 2.5

$$\frac{-i\left(-2+\sqrt{3}\right)}{\left(4\sqrt{3}-7\right)\left(-x^4+4\sqrt{3}x^2-6x^2-4\sqrt{3}+7\right)}EllipticF\left(\frac{ix}{-2+\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{-\left(-x^2+4\sqrt{3}-7\right)\left(-4\sqrt{3}+7\right)}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x)
```

```
[Out] -I*EllipticF(I*x/(-2+3^(1/2)),2*I-I*3^(1/2))*(-x^2+1)^(1/2)*(-(-x
^2+4*3^(1/2)-7)*(-4*3^(1/2)+7))^(1/2)/(4*3^(1/2)-7)*(-2+3^(1/2))*
(x^2-1)^(1/2)*(7+x^2-4*3^(1/2))^(1/2)/(-x^4+4*3^(1/2)*x^2-6*x^2-4
*3^(1/2)+7)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2-4\sqrt{3}+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 - 4*sqrt(3) + 7)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

3.300 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

```
[Out] -(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1
 + Sqrt[3])/2])/6
```

Rubi [A] time = 0.134338, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024

$$-\frac{1}{6}\sqrt{3+\sqrt{3}}F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x]
```

```
[Out] -(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1
 + Sqrt[3])/2])/6
```

Rubi in Sympy [A] time = 15.1505, size = 66, normalized size = 1.4

$$-\frac{F\left(\operatorname{acos}\left(\frac{\sqrt{3}x\sqrt{-\sqrt{3}+3}}{3}\right)\middle|\frac{1}{2}+\frac{\sqrt{3}}{2}\right)}{\sqrt{-\sqrt{3}+3}\sqrt{-3\sqrt{3}+3-\frac{6\sqrt{3}}{-3+\sqrt{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*3**(1/2)*x**2)*
```

```
[Out] -elliptic_f(acos(sqrt(3)*x*sqrt(-sqrt(3) + 3)/3), 1/2 + sqrt(3)/2
)/(sqrt(-sqrt(3) + 3)*sqrt(-3*sqrt(3) + 3 - 6*sqrt(3)/(-3 + sqrt(
3))))
```

Mathematica [A] time = 0.209666, size = 81, normalized size = 1.72

$$\frac{\sqrt{-2\sqrt{3}x^2+3\sqrt{3}-3}F\left(\sin^{-1}\left(\frac{\sqrt{1+\sqrt{3}}x}{\sqrt[4]{3}}\right)|2-\sqrt{3}\right)}{3^{3/4}\sqrt{4\sqrt{3}x^2-6\sqrt{3}+6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x]
```

```
[Out] (Sqrt[-3 + 3*Sqrt[3] - 2*Sqrt[3]*x^2]*EllipticF[ArcSin[(Sqrt[1 +
Sqrt[3]]*x)/3^(1/4)], 2 - Sqrt[3]])/(3^(3/4)*Sqrt[6 - 6*Sqrt[3] +
 4*Sqrt[3]*x^2])
```

Maple [B] time = 0.601, size = 207, normalized size = 4.4

$$\frac{\sqrt{2}\left(-3+\sqrt{3}\right)}{18\left(\sqrt{3}-1\right)^2\left(2x^4\sqrt{3}-2x^4-6\sqrt{3}x^2+6x^2+3\sqrt{3}-3\right)\sqrt{\left(2\sqrt{3}-3\right)\left(\sqrt{3}-1\right)}}\sqrt{\sqrt{3}x^2-3x^2+3}\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2),x)
```

```
[Out] 1/18*(3^(1/2)*x^2-3*x^2+3)^(1/2)*(3-3*3^(1/2)+2*3^(1/2)*x^2)^(1/2
)*2^(1/2)/(3^(1/2)-1)^2*(-(4*3^(1/2)*x^2-6*x^2-3*3^(1/2)+3)*(3^(1
/2)-1))^(1/2)*(-(3-3*3^(1/2)+2*3^(1/2)*x^2)*(3^(1/2)-1))^(1/2)*El
lipticF(1/3*x*3^(1/2)*2^(1/2)/(3^(1/2)-1)*((2*3^(1/2)-3)*(3^(1/2)
-1))^(1/2),1/(3^(1/2)-1)*((3^(1/2)-1)*(1+3^(1/2)))^(1/2))*(-3+3^(
1/2))/(2*x^4*3^(1/2)-2*x^4-6*3^(1/2)*x^2+6*x^2+3*3^(1/2)-3)/((2*3
^(1/2)-3)*(3^(1/2)-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3)),x,
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*s
qrt(3) + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sqrt{3}x^2 - 3\,x^2 + 3}\sqrt{\sqrt{3}(2\,x^2 - 3) + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3)),x,
```

```
[Out] integral(1/(sqrt(sqrt(3)*x^2 - 3*x^2 + 3)*sqrt(sqrt(3)*(2*x^2 - 3
) + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + \sqrt{3}x^2 + 3}\sqrt{2\sqrt{3}x^2 - 3\sqrt{3} + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*3**(1/2)*x**2)**(1/2
```

```
[Out] Integral(1/(sqrt(-3*x**2 + sqrt(3)*x**2 + 3)*sqrt(2*sqrt(3)*x**2
- 3*sqrt(3) + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(\sqrt{3} - 3\right) + 3}\sqrt{2\,\sqrt{3}x^2 - 3\,\sqrt{3} + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3)),x,
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*s
qrt(3) + 3)), x)
```

3.301 $\int \frac{1}{\sqrt[4]{2+3x^2}(4+3x^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 +
 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1
/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*
Sqrt[3])
```

Rubi [A] time = 0.0599293, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{3x^2+2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{3x^2+2}}{\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] -ArcTan[(2^(3/4) + 2^(1/4)*Sqrt[2 + 3*x^2])/(Sqrt[3]*x*(2 + 3*x^2
)^(1/4))]/(2*2^(3/4)*Sqrt[3]) - ArcTanh[(2^(3/4) - 2^(1/4)*Sqrt[2
 + 3*x^2])/(Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 73.9714, size = 92, normalized size = 0.71

$$\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{-x^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3x^2+2}}{2}\right)\middle|-1\right)}{6x}-\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{-x^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3x^2+2}}{2}\right)\middle|-1\right)}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
```

```
[Out] 2**(1/4)*sqrt(3)*I*sqrt(-x**2)*elliptic_pi(-I, asin(2**(3/4)*(3*x
**2 + 2)**(1/4)/2), -1)/(6*x) - 2**(1/4)*sqrt(3)*I*sqrt(-x**2)*el
liptic_pi(I, asin(2**(3/4)*(3*x**2 + 2)**(1/4)/2), -1)/(6*x)
```

Mathematica [C] time = 0.173179, size = 135, normalized size = 1.05

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)}{\sqrt[4]{3x^2+2}\,(3x^2+4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)-4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{2},-\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2, (-3*x^2)/4])/((2 + 3
*x^2)^(1/4)*(4 + 3*x^2)*(-4*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/2
, (-3*x^2)/4] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (-3*x^2)/2, (-3
*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/2, (-3*x^2)/4])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2+4}\,\frac{1}{\sqrt[4]{3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x)
```

```
[Out] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{3x^2+2}\,(3x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
```

```
[Out] Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

3.302 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*
x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi [A] time = 0.054775, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\,2^{3/4}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rubi in Sympy [A] time = 73.619, size = 88, normalized size = 0.73

$$-\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{x^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{6x}+\frac{\sqrt[4]{2}\sqrt{3}i\sqrt{x^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-3x^2+2}}{2}\right)\middle|-1\right)}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -2**(1/4)*sqrt(3)*I*sqrt(x**2)*elliptic_pi(-I, asin(2**(3/4)*(-3*
x**2 + 2)**(1/4)/2), -1)/(6*x) + 2**(1/4)*sqrt(3)*I*sqrt(x**2)*el
liptic_pi(I, asin(2**(3/4)*(-3*x**2 + 2)**(1/4)/2), -1)/(6*x)
```

Mathematica [C] time = 0.196143, size = 135, normalized size = 1.12

$$-\frac{4xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)}{\sqrt[4]{2-3x^2}\,(3x^2-4)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)+4F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] (-4*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((2 - 3*x
^2)^(1/4)*(-4 + 3*x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (
3*x^2)/4] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/
4] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])))
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2+4}\frac{1}{\sqrt[4]{-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

```
[Out] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{3x^2\sqrt[4]{-3x^2+2}-4\sqrt[4]{-3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 + 2)**(1/4) - 4*(-3*x**2 + 2)**(1/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.303 $\int \frac{1}{\sqrt[4]{2+bx^2}(4+bx^2)}\,dx$

Optimal. Leaf size=129

$$-\frac{\tan^{-1}\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\,2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2\,2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 +
 b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1
/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*
Sqrt[b])
```

Rubi [A] time = 0.0689154, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt{bx^2+2}+2^{3/4}}{\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{bx^2+2}}{\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] -ArcTan[(2^(3/4) + 2^(1/4)*Sqrt[2 + b*x^2])/(Sqrt[b]*x*(2 + b*x^2
)^(1/4))]/(2*2^(3/4)*Sqrt[b]) - ArcTanh[(2^(3/4) - 2^(1/4)*Sqrt[2
 + b*x^2])/(Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 73.7195, size = 88, normalized size = 0.68

$$\frac{\sqrt[4]{2}i\sqrt{-bx^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{bx^2+2}}{2}\right)\middle|-1\right)}{2bx}-\frac{\sqrt[4]{2}i\sqrt{-bx^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{bx^2+2}}{2}\right)\middle|-1\right)}{2bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

```
[Out] 2**(1/4)*I*sqrt(-b*x**2)*elliptic_pi(-I, asin(2**(3/4)*(b*x**2 +
2)**(1/4)/2), -1)/(2*b*x) - 2**(1/4)*I*sqrt(-b*x**2)*elliptic_pi(
I, asin(2**(3/4)*(b*x**2 + 2)**(1/4)/2), -1)/(2*b*x)
```

Mathematica [C] time = 0.23179, size = 144, normalized size = 1.12

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)}{\sqrt[4]{bx^2+2}\,(bx^2+4)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)-12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{2},-\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2)/2, -(b*x^2)/4])/((2 +
b*x^2)^(1/4)*(4 + b*x^2)*(-12*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2)
/2, -(b*x^2)/4] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -(b*x^2)/2,
 -(b*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, -(b*x^2)/2, -(b*x^2)/4]
)))
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+4}\frac{1}{\sqrt[4]{bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

```
[Out] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx^2+2}\,(bx^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

```
[Out] Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

3.304 $\int \frac{1}{\sqrt[4]{2-bx^2}(4-bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^
2)^(1/4))]/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*
x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0715037, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\tan^{-1}\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-bx^2}}{\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt{2-bx^2}+2^{3/4}}{\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - b*x^2])/(Sqrt[b]*x*(2 - b*x^2)
^(1/4))]/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2
- b*x^2])/(Sqrt[b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 76.1992, size = 85, normalized size = 0.69

$$-\frac{\sqrt[4]{2}i\sqrt{bx^2}\left(-i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-bx^2+2}}{2}\right)\middle|-1\right)}{2bx}+\frac{\sqrt[4]{2}i\sqrt{bx^2}\left(i;\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{-bx^2+2}}{2}\right)\middle|-1\right)}{2bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
```

```
[Out] -2**(1/4)*I*sqrt(b*x**2)*elliptic_pi(-I, asin(2**(3/4)*(-b*x**2 +
 2)**(1/4)/2), -1)/(2*b*x) + 2**(1/4)*I*sqrt(b*x**2)*elliptic_pi(
I, asin(2**(3/4)*(-b*x**2 + 2)**(1/4)/2), -1)/(2*b*x)
```

Mathematica [C] time = 0.218044, size = 145, normalized size = 1.17

$$-\frac{12xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)}{\sqrt[4]{2-bx^2}\,(bx^2-4)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)+12F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{2},\frac{bx^2}{4}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] (-12*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2, (b*x^2)/4])/((2 - b*
x^2)^(1/4)*(-4 + b*x^2)*(12*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/2,
 (b*x^2)/4] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^2)/2, (b*x
^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, (b*x^2)/2, (b*x^2)/4])))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2+4}\frac{1}{\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

```
[Out] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{bx^2\sqrt[4]{-bx^2+2}-4\sqrt[4]{-bx^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

3.305 $\int \frac{1}{\sqrt[4]{a+3x^2}(2a+3x^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3
*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3
/4))
```

Rubi [A] time = 0.0649082, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3
*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3
/4))
```

Rubi in Sympy [A] time = 46.645, size = 122, normalized size = 1.02

$$\frac{\sqrt{3}\sqrt[4]{a}\sqrt{-\frac{x^2}{a}}\left(-\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a+3x^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{3x\sqrt{-a}}-\frac{\sqrt{3}\sqrt[4]{a}\sqrt{-\frac{x^2}{a}}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a+3x^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{3x\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] sqrt(3)*a**(1/4)*sqrt(-x**2/a)*elliptic_pi(-sqrt(a)/sqrt(-a), asi
n((a + 3*x**2)**(1/4)/a**(1/4)), -1)/(3*x*sqrt(-a)) - sqrt(3)*a**
```

```
(1/4)*sqrt(-x**2/a)*elliptic_pi(sqrt(a)/sqrt(-a), asin((a + 3*x**
2)**(1/4)/a**(1/4)), -1)/(3*x*sqrt(-a))
```

Mathematica [C] time = 0.229877, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{a+3x^2}\,(2a+3x^2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/(
(a + 3*x^2)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2,
(-3*x^2)/a, (-3*x^2)/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (
-3*x^2)/a, (-3*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/
a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.067, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2+2\,a}\frac{1}{\sqrt[4]{3\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

```
[Out] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a + 3x^2}\,(2a + 3x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2 + 2\,a)(3\,x^2 + a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

3.306 $\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)}\,dx$

Optimal. Leaf size=120

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*
x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/
4))
```

Rubi [A] time = 0.0614019, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*
x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/
4))
```

Rubi in Sympy [A] time = 45.9442, size = 119, normalized size = 0.99

$$-\frac{\sqrt{3}\sqrt[4]{a}\sqrt{\frac{x^2}{a}}\left(-\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a-3x^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{3x\sqrt{-a}}+\frac{\sqrt{3}\sqrt[4]{a}\sqrt{\frac{x^2}{a}}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a-3x^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{3x\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

```
[Out] -sqrt(3)*a**(1/4)*sqrt(x**2/a)*elliptic_pi(-sqrt(a)/sqrt(-a), asi
n((a - 3*x**2)**(1/4)/a**(1/4)), -1)/(3*x*sqrt(-a)) + sqrt(3)*a**
```

```
(1/4)*sqrt(x**2/a)*elliptic_pi(sqrt(a)/sqrt(-a), asin((a - 3*x**2
)**(1/4)/a**(1/4)), -1)/(3*x*sqrt(-a))
```

Mathematica [C] time = 0.249007, size = 155, normalized size = 1.29

$$-\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{\sqrt[4]{a-3x^2}\,(3x^2-2a)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((a
 - 3*x^2)^(1/4)*(-2*a + 3*x^2)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3
*x^2)/a, (3*x^2)/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^
2)/a, (3*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x
^2)/(2*a)])))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2+2\,a}\,\frac{1}{\sqrt[4]{-3\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

```
[Out] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

3.307 $\int \frac{1}{\sqrt[4]{a+bx^2}(2a+bx^2)}\,dx$

Optimal. Leaf size=120

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b
*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt
[b])
```

Rubi [A] time = 0.0723392, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{\tan^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b
*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a
+ b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt
[b])
```

Rubi in Sympy [A] time = 52.7652, size = 116, normalized size = 0.97

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{bx\sqrt{-a}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{bx\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)/sqrt(-a), asin((a +
 b*x**2)**(1/4)/a**(1/4)), -1)/(b*x*sqrt(-a)) - a**(1/4)*sqrt(-b*
```

```
x**2/a)*elliptic_pi(sqrt(a)/sqrt(-a), asin((a + b*x**2)**(1/4)/a*
*(1/4)), -1)/(b*x*sqrt(-a))
```

Mathematica [C] time = 0.236927, size = 165, normalized size = 1.38

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{a+bx^2}\,(2a+bx^2)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(2*a)])/
((a + b*x^2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2,
-((b*x^2)/a), -(b*x^2)/(2*a)] - b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/
2, -((b*x^2)/a), -(b*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, -((
b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2+2\,a}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)

[Out] Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

3.308 $\int \frac{1}{\sqrt[4]{a-bx^2}(2a-bx^2)}\,dx$

Optimal. Leaf size=124

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*
x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[
b])
```

Rubi [A] time = 0.0725318, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*
x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a -
 b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[
b])
```

Rubi in Sympy [A] time = 54.2092, size = 112, normalized size = 0.9

$$-\frac{\sqrt[4]{a}\sqrt{\frac{bx^2}{a}}\left(-\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a-bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{bx\sqrt{-a}}+\frac{\sqrt[4]{a}\sqrt{\frac{bx^2}{a}}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{a-bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{bx\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -a**(1/4)*sqrt(b*x**2/a)*elliptic_pi(-sqrt(a)/sqrt(-a), asin((a -
 b*x**2)**(1/4)/a**(1/4)), -1)/(b*x*sqrt(-a)) + a**(1/4)*sqrt(b*x
```

```
**2/a)*elliptic_pi(sqrt(a)/sqrt(-a), asin((a - b*x**2)**(1/4)/a**
(1/4)), -1)/(b*x*sqrt(-a))
```

Mathematica [C] time = 0.246662, size = 162, normalized size = 1.31

$$\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{\sqrt[4]{a-bx^2}\,(2a-bx^2)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] (6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((a
- b*x^2)^(1/4)*(2*a - b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b*x
^2)/a, (b*x^2)/(2*a)] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*x^
2)/a, (b*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b*x
^2)/(2*a)])))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2+2\,a}\frac{1}{\sqrt[4]{-bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4
)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

3.309 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0340955, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi in Sympy [A] time = 50.2988, size = 168, normalized size = 2.75

$$\frac{\sqrt{2}x(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{3x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{3x^2-1}+1}\sqrt{i\sqrt{3x^2-1}+1}}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)\middle|\frac{1}{2}\right)}{12x}-\frac{\sqrt{6}\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{6}\sqrt[4]{3x^2-1}}{3\sqrt{x^2}}\right)}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(3*x**2 - 1
)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(3*x**2 - 1) + 1)*sqrt(I*sqrt(3*x
**2 - 1) + 1)) - sqrt(3)*sqrt(x**2/(sqrt(3*x**2 - 1) + 1)**2)*(sq
```

```
rt(3*x**2 - 1) + 1)*elliptic_f(2*atan((3*x**2 - 1)**(1/4)), 1/2)/
(12*x) - sqrt(6)*sqrt(x**2)*atanh(sqrt(6)*(3*x**2 - 1)**(1/4)/(3*
sqrt(x**2)))/(12*x)
```

Mathematica [C] time = 0.210795, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)}{(3x^2-2)\sqrt[4]{3x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};3x^2,\frac{3x^2}{2}\right)\right)+2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};3x^2,\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*
(-1 + 3*x^2)^(1/4)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2
] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF
1[3/2, 5/4, 1, 5/2, 3*x^2, (3*x^2)/2])))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2-2}\frac{1}{\sqrt[4]{3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Fricas [A] time = 2.70102, size = 139, normalized size = 2.28

$$\frac{1}{24}\sqrt{6}\left(2\arctan\left(\frac{\sqrt{6}\left(3x^2-1\right)^{\frac{1}{4}}}{3x}\right)+\log\left(\frac{36\left(3x^2-1\right)^{\frac{1}{4}}x^3-12\sqrt{6}\sqrt{3x^2-1}x^2+24\left(3x^2-1\right)^{\frac{3}{4}}x-\sqrt{6}\left(9x^4+12x^2-4\right)}{9x^4-12x^2+4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] 1/24*sqrt(6)*(2*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + log((36
*(3*x^2 - 1)^(1/4)*x^3 - 12*sqrt(6)*sqrt(3*x^2 - 1)*x^2 + 24*(3*x
^2 - 1)^(3/4)*x - sqrt(6)*(9*x^4 + 12*x^2 - 4))/(9*x^4 - 12*x^2 +
 4)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3x^2-1)^{\frac{1}{4}}(3x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.310 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi [A] time = 0.0333361, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6]) - ArcTanh[(
Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rubi in Sympy [A] time = 51.1358, size = 185, normalized size = 3.03

$$\frac{\sqrt{2}x\,(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{-3x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{-3x^2-1}+1}\sqrt{i\sqrt{-3x^2-1}+1}}+\frac{\sqrt{6}\sqrt{-x^2}\operatorname{atanh}\left(\frac{\sqrt{6}\sqrt[4]{-3x^2-1}}{3\sqrt{-x^2}}\right)}{12x}+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-1}+1\right)^2}}\left(\sqrt{-3x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{-3x^2-1}\right)\middle|\frac{1}{2}\right)}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(-3*x**2 -
1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(-3*x**2 - 1) + 1)*sqrt(I*sqrt(-
3*x**2 - 1) + 1)) + sqrt(6)*sqrt(-x**2)*atanh(sqrt(6)*(-3*x**2 -
```

```
1)**(1/4)/(3*sqrt(-x**2)))/(12*x) + sqrt(3)*sqrt(-x**2/(sqrt(-3*x
**2 - 1) + 1)**2)*(sqrt(-3*x**2 - 1) + 1)*elliptic_f(2*atan((-3*x
**2 - 1)**(1/4)), 1/2)/(12*x)
```

Mathematica [C] time = 0.221579, size = 127, normalized size = 2.08

$$\frac{2xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)}{\sqrt[4]{-3x^2-1}\,(3x^2+2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-3x^2,-\frac{3x^2}{2}\right)\right)-2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-3x^2,-\frac{3x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] (2*x*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^2)/2])/((-1 - 3*x^2
)^(1/4)*(2 + 3*x^2)*(-2*AppellF1[1/2, 1/4, 1, 3/2, -3*x^2, (-3*x^
2)/2] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -3*x^2, (-3*x^2)/2] + A
ppellF1[3/2, 5/4, 1, 5/2, -3*x^2, (-3*x^2)/2])))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2-2}\,\frac{1}{\sqrt[4]{-3\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2+2)(-3\,x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Fricas [A] time = 2.76999, size = 332, normalized size = 5.44

$$-\frac{1}{24}\sqrt{6}\left(\log\left(\frac{\sqrt{6}\left(3\sqrt{-3x^2-1}x+\sqrt{6}(-3x^2-1)^{\frac{3}{4}}-3x-\sqrt{6}(-3x^2-1)^{\frac{1}{4}}\right)}{9(3x^2+2)}\right)-\log\left(-\frac{\sqrt{6}\left(3\sqrt{-3x^2-1}x-\sqrt{6}(-3x^2-1\right.}{9(3x^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*(log(1/9*sqrt(6)*(3*sqrt(-3*x^2 - 1)*x + sqrt(6)*(-
3*x^2 - 1)^(3/4) - 3*x - sqrt(6)*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
 - log(-1/9*sqrt(6)*(3*sqrt(-3*x^2 - 1)*x - sqrt(6)*(-3*x^2 - 1)^
(3/4) - 3*x + sqrt(6)*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) - I*log(1/
18*sqrt(6)*(6*I*sqrt(-3*x^2 - 1)*x + 2*sqrt(6)*(-3*x^2 - 1)^(3/4)
 + 6*I*x + 2*sqrt(6)*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + I*log(1/1
8*sqrt(6)*(-6*I*sqrt(-3*x^2 - 1)*x + 2*sqrt(6)*(-3*x^2 - 1)^(3/4)
 - 6*I*x + 2*sqrt(6)*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{(3x^2+2)(-3x^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

3.311 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt
[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[
2]*Sqrt[b])
```

Rubi [A] time = 0.046275, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt
[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[
2]*Sqrt[b])
```

Rubi in Sympy [A] time = 57.1789, size = 170, normalized size = 2.21

$$\frac{\sqrt{2}x(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{bx^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{bx^2-1}+1}\sqrt{i\sqrt{bx^2-1}+1}}-\frac{\sqrt{2}\sqrt{bx^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{bx^2-1}}{\sqrt{bx^2}}\right)}{4bx}-\frac{\sqrt{\frac{bx^2}{\left(\sqrt{bx^2-1}+1\right)^2}}\left(\sqrt{bx^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{bx^2-1}\right)\middle|\frac{1}{2}\right)}{4bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

```
[Out] sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(b*x**2 - 1
)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(b*x**2 - 1) + 1)*sqrt(I*sqrt(b*x
```

```
**2 - 1) + 1)) - sqrt(2)*sqrt(b*x**2)*atanh(sqrt(2)*(b*x**2 - 1)*
*(1/4)/sqrt(b*x**2))/(4*b*x) - sqrt(b*x**2/(sqrt(b*x**2 - 1) + 1)
**2)*(sqrt(b*x**2 - 1) + 1)*elliptic_f(2*atan((b*x**2 - 1)**(1/4)
), 1/2)/(4*b*x)
```

Mathematica [C] time = 0.250127, size = 132, normalized size = 1.71

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)}{(bx^2-2)\sqrt[4]{bx^2-1}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};bx^2,\frac{bx^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};bx^2,\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2])/((-2 + b*x^2)*
(-1 + b*x^2)^(1/4)*(6*AppellF1[1/2, 1/4, 1, 3/2, b*x^2, (b*x^2)/2
] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, b*x^2, (b*x^2)/2] + Appel
lF1[3/2, 5/4, 1, 5/2, b*x^2, (b*x^2)/2])))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2-2}\frac{1}{\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Fricas [A] time = 9.33184, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\log\left(\frac{4(bx^2-1)^{\frac{1}{4}}b^2x^3-4\sqrt{2}\sqrt{bx^2-1}b^{\frac{3}{2}}x^2+8(bx^2-1)^{\frac{3}{4}}bx-\sqrt{2}(b^2x^4+4bx^2-4)\sqrt{b}}{b^2x^4-4bx^2+4}\right)\right)}{8\sqrt{b}}, -\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\log\left(\frac{4(bx^2-1)^{\frac{1}{4}}b^2x^3+4\sqrt{2}\sqrt{bx^2-1}\sqrt{-b}bx^2-8(bx^2-1)^{\frac{3}{4}}bx-\sqrt{2}(b^2x^4+4bx^2-4)\sqrt{-b}}{b^2x^4-4bx^2+4}\right)\right)}{8\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*(2*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + l
og((4*(b*x^2 - 1)^(1/4)*b^2*x^3 - 4*sqrt(2)*sqrt(b*x^2 - 1)*b^(3/
2)*x^2 + 8*(b*x^2 - 1)^(3/4)*b*x - sqrt(2)*(b^2*x^4 + 4*b*x^2 - 4
)*sqrt(b))/(b^2*x^4 - 4*b*x^2 + 4)))/sqrt(b), -1/8*sqrt(2)*(2*arc
tan(sqrt(2)*(b*x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - log((4*(b*x^2 - 1
)^(1/4)*b^2*x^3 + 4*sqrt(2)*sqrt(b*x^2 - 1)*sqrt(-b)*b*x^2 - 8*(b
*x^2 - 1)^(3/4)*b*x - sqrt(2)*(b^2*x^4 + 4*b*x^2 - 4)*sqrt(-b))/(
b^2*x^4 - 4*b*x^2 + 4)))/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.312 $\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt
[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[
2]*Sqrt[b])
```

Rubi [A] time = 0.0500597, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt
[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[
2]*Sqrt[b])
```

Rubi in Sympy [A] time = 65.5565, size = 187, normalized size = 2.37

$$\frac{\sqrt{2}x\,(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{-bx^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{-bx^2-1}+1}\sqrt{i\sqrt{-bx^2-1}+1}}+\frac{\sqrt{2}\sqrt{-bx^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{-bx^2-1}}{\sqrt{-bx^2}}\right)}{4bx}+\frac{\sqrt{-\frac{bx^2}{\left(\sqrt{-bx^2-1}+1\right)^2}}\left(\sqrt{-bx^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{-bx^2-1}\right)\middle|\frac{1}{2}\right)}{4bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
```

```
[Out] sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(-b*x**2 -
1)**(1/4)/2), -1)/(2*sqrt(-I*sqrt(-b*x**2 - 1) + 1)*sqrt(I*sqrt(-
```

```
b*x**2 - 1) + 1)) + sqrt(2)*sqrt(-b*x**2)*atanh(sqrt(2)*(-b*x**2
- 1)**(1/4)/sqrt(-b*x**2))/(4*b*x) + sqrt(-b*x**2/(sqrt(-b*x**2 -
 1) + 1)**2)*(sqrt(-b*x**2 - 1) + 1)*elliptic_f(2*atan((-b*x**2 -
 1)**(1/4)), 1/2)/(4*b*x)
```

Mathematica [C] time = 0.238794, size = 137, normalized size = 1.73

$$\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)}{\sqrt[4]{-bx^2-1}\,(bx^2+2)\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-bx^2,-\frac{bx^2}{2}\right)\right)-6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-bx^2,-\frac{bx^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] (6*x*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -(b*x^2)/2])/((-1 - b*x
^2)^(1/4)*(2 + b*x^2)*(-6*AppellF1[1/2, 1/4, 1, 3/2, -(b*x^2), -(
b*x^2)/2] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, -(b*x^2), -(b*x^2
)/2] + AppellF1[3/2, 5/4, 1, 5/2, -(b*x^2), -(b*x^2)/2])))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-2}\frac{1}{\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Fricas [A] time = 9.12209, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)+\log\left(\frac{4\,(-bx^2-1)^{\frac{1}{4}}b^2x^3+8\,(-bx^2-1)^{\frac{3}{4}}bx-\sqrt{2}\left(b^2x^4+4\,\sqrt{-bx^2-1}bx^2-4\,bx^2-4\right)\sqrt{b}}{b^2x^4+4\,bx^2+4}\right)\right)}{8\,\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}\sqrt{-b}}{bx}\right)-\log\left(\frac{4\,(-bx^2-1)^{\frac{1}{4}}b^2x^3-8\,(-bx^2-1)^{\frac{3}{4}}bx-\sqrt{2}\left(b^2x^4-4\,\sqrt{-bx^2-1}bx^2-4\,bx^2-4\right)\sqrt{-b}}{b^2x^4+4\,bx^2+4}\right)\right)}{8\,\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(2)*(2*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) +
log((4*(-b*x^2 - 1)^(1/4)*b^2*x^3 + 8*(-b*x^2 - 1)^(3/4)*b*x - sq
rt(2)*(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 4)*sqrt(b))
/(b^2*x^4 + 4*b*x^2 + 4)))/sqrt(b), -1/8*sqrt(2)*(2*arctan(sqrt(2
)*(-b*x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - log((4*(-b*x^2 - 1)^(1/4)*
b^2*x^3 - 8*(-b*x^2 - 1)^(3/4)*b*x - sqrt(2)*(b^2*x^4 - 4*sqrt(-b
*x^2 - 1)*b*x^2 - 4*b*x^2 - 4)*sqrt(-b))/(b^2*x^4 + 4*b*x^2 + 4))
)/sqrt(-b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)),x, algorithm="giac")

[Out] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

3.313 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^
(3/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*S
qrt[6]*a^(3/4))
```

Rubi [A] time = 0.0538701, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^
(3/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*S
qrt[6]*a^(3/4))
```

Rubi in Sympy [A] time = 90.765, size = 231, normalized size = 2.72

$$\frac{\sqrt{6}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt{\frac{x^2}{a}}\,\mathrm{atan}\left(\frac{\sqrt{6}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt[4]{-a+3x^2}}{3\sqrt{\frac{x^2}{a}}}\right)}{12x}+\frac{x\sqrt[4]{-a}\left(\frac{\sqrt{a}}{\sqrt{-a}};\mathrm{asin}\left(\frac{\sqrt[4]{-a+3x^2}}{\sqrt[4]{-a}}\right)\middle|-1\right)}{a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{-a+3x^2}}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{-a+3x^2}}{\sqrt{-a}}}}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{a}+\sqrt{-a+3x^2}\right)^2}}\left(\sqrt{a}+\sqrt{-a+3x^2}\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{-a+3x^2}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{12a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

```
[Out] sqrt(6)*sqrt(-1/sqrt(a))*sqrt(x**2/a)*atan(sqrt(6)*sqrt(-1/sqrt(a
))*(-a + 3*x**2)**(1/4)/(3*sqrt(x**2/a)))/(12*x) + x*(-a)**(1/4)*
elliptic_pi(sqrt(a)/sqrt(-a), asin((-a + 3*x**2)**(1/4)/(-a)**(1/
4)), -1)/(a**(3/2)*sqrt(1 - sqrt(-a + 3*x**2)/sqrt(-a))*sqrt(1 +
sqrt(-a + 3*x**2)/sqrt(-a))) - sqrt(3)*sqrt(x**2/(sqrt(a) + sqrt(
-a + 3*x**2))**2)*(sqrt(a) + sqrt(-a + 3*x**2))*elliptic_f(2*atan
((-a + 3*x**2)**(1/4)/a**(1/4)), 1/2)/(12*a**(3/4)*x)
```

Mathematica [C] time = 0.269638, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)}{(3x^2-2a)\sqrt[4]{3x^2-a}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)+2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{3x^2}{a},\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/a, (3*x^2)/(2*a)])/((-2
*a + 3*x^2)*(-a + 3*x^2)^(1/4)*(2*a*AppellF1[1/2, 1/4, 1, 3/2, (3
*x^2)/a, (3*x^2)/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (3*x^
2)/a, (3*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (3*x^2)/a, (3*x
^2)/(2*a)])))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^2-2\,a}\frac{1}{\sqrt[4]{3\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)),x, algorithm="giac")
```

```
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

3.314 $\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^
(3/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*S
qrt[6]*a^(3/4))
```

Rubi [A] time = 0.0488076, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^
(3/4)) - ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*S
qrt[6]*a^(3/4))
```

Rubi in Sympy [A] time = 96.1355, size = 248, normalized size = 2.92

$$-\frac{\sqrt{6}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt{-\frac{x^2}{a}}\operatorname{atan}\left(\frac{\sqrt{6}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt[4]{-a-3x^2}}{3\sqrt{-\frac{x^2}{a}}}\right)}{12x}+\frac{x\sqrt[4]{-a}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{-a-3x^2}}{\sqrt[4]{-a}}\right)\middle|-1\right)}{a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{-a-3x^2}}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{-a-3x^2}}{\sqrt{-a}}}}$$
$$+\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{a}+\sqrt{-a-3x^2}\right)^2}}\left(\sqrt{a}+\sqrt{-a-3x^2}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{-a-3x^2}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{12a^{\frac{3}{4}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

```
[Out] -sqrt(6)*sqrt(-1/sqrt(a))*sqrt(-x**2/a)*atan(sqrt(6)*sqrt(-1/sqrt
(a))*(-a - 3*x**2)**(1/4)/(3*sqrt(-x**2/a)))/(12*x) + x*(-a)**(1/
4)*elliptic_pi(sqrt(a)/sqrt(-a), asin((-a - 3*x**2)**(1/4)/(-a)**
(1/4)), -1)/(a**(3/2)*sqrt(1 - sqrt(-a - 3*x**2)/sqrt(-a))*sqrt(1
 + sqrt(-a - 3*x**2)/sqrt(-a))) + sqrt(3)*sqrt(-x**2/(sqrt(a) + s
qrt(-a - 3*x**2))**2)*(sqrt(a) + sqrt(-a - 3*x**2))*elliptic_f(2*
atan((-a - 3*x**2)**(1/4)/a**(1/4)), 1/2)/(12*a**(3/4)*x)
```

Mathematica [C] time = 0.234987, size = 157, normalized size = 1.85

$$\frac{2axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)}{\sqrt[4]{-a-3x^2}\,(2a+3x^2)\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)-2aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{3x^2}{a},-\frac{3x^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] (2*a*x*AppellF1[1/2, 1/4, 1, 3/2, (-3*x^2)/a, (-3*x^2)/(2*a)])/((
-a - 3*x^2)^(1/4)*(2*a + 3*x^2)*(-2*a*AppellF1[1/2, 1/4, 1, 3/2,
(-3*x^2)/a, (-3*x^2)/(2*a)] + x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (
-3*x^2)/a, (-3*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (-3*x^2)/
a, (-3*x^2)/(2*a)])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{-3\,x^2-2\,a}\frac{1}{\sqrt[4]{-3\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/
4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)
```

3.315 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$

Optimal. Leaf size=101

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*Sqrt
[2]*a^(3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a +
 b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0682901, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2*Sqrt
[2]*a^(3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a +
 b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 93.729, size = 233, normalized size = 2.31

$$\frac{\sqrt{2}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt{\frac{bx^2}{a}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt[4]{-a+bx^2}}{\sqrt{\frac{bx^2}{a}}}\right)}{4bx}+\frac{x\sqrt[4]{-a}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{-a}}\right)\middle|-1\right)}{a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{-a+bx^2}}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{-a+bx^2}}{\sqrt{-a}}}}$$
$$-\frac{\sqrt{\frac{bx^2}{\left(\sqrt{a}+\sqrt{-a+bx^2}\right)^2}}\left(\sqrt{a}+\sqrt{-a+bx^2}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
```

```
[Out] sqrt(2)*sqrt(-1/sqrt(a))*sqrt(b*x**2/a)*atan(sqrt(2)*sqrt(-1/sqrt
(a))*(-a + b*x**2)**(1/4)/sqrt(b*x**2/a))/(4*b*x) + x*(-a)**(1/4)
*elliptic_pi(sqrt(a)/sqrt(-a), asin((-a + b*x**2)**(1/4)/(-a)**(1
/4)), -1)/(a**(3/2)*sqrt(1 - sqrt(-a + b*x**2)/sqrt(-a))*sqrt(1 +
 sqrt(-a + b*x**2)/sqrt(-a))) - sqrt(b*x**2/(sqrt(a) + sqrt(-a +
b*x**2))**2)*(sqrt(a) + sqrt(-a + b*x**2))*elliptic_f(2*atan((-a
+ b*x**2)**(1/4)/a**(1/4)), 1/2)/(4*a**(3/4)*b*x)
```

Mathematica [C] time = 0.251291, size = 163, normalized size = 1.61

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)}{(2a-bx^2)\sqrt[4]{bx^2-a}\left(bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)+6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};\frac{bx^2}{a},\frac{bx^2}{2a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, (b*x^2)/a, (b*x^2)/(2*a)])/((2
*a - b*x^2)*(-a + b*x^2)^(1/4)*(6*a*AppellF1[1/2, 1/4, 1, 3/2, (b
*x^2)/a, (b*x^2)/(2*a)] + b*x^2*(2*AppellF1[3/2, 1/4, 2, 5/2, (b*
x^2)/a, (b*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, (b*x^2)/a, (b
*x^2)/(2*a)])))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{bx^2-2\,a}\frac{1}{\sqrt[4]{bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2-a)^{\frac{1}{4}}(bx^2-2\,a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-2a + bx^2)\sqrt[4]{-a + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 - a)^{\frac{1}{4}}(bx^2 - 2\,a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

3.316 $\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*Sqrt
[2]*a^(3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a -
 b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.0666019, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2*Sqrt
[2]*a^(3/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a -
 b*x^2)^(1/4))]/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 107.999, size = 250, normalized size = 2.43

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt{-\frac{bx^2}{a}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{-\frac{1}{\sqrt{a}}}\sqrt[4]{-a-bx^2}}{\sqrt{-\frac{bx^2}{a}}}\right)}{4bx}+\frac{x\sqrt[4]{-a}\left(\frac{\sqrt{a}}{\sqrt{-a}};\operatorname{asin}\left(\frac{\sqrt[4]{-a-bx^2}}{\sqrt[4]{-a}}\right)\middle|-1\right)}{a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{-a-bx^2}}{\sqrt{-a}}}\sqrt{1+\frac{\sqrt{-a-bx^2}}{\sqrt{-a}}}}+\frac{\sqrt{-\frac{bx^2}{\left(\sqrt{a}+\sqrt{-a-bx^2}\right)^2}}\left(\sqrt{a}+\sqrt{-a-bx^2}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{-a-bx^2}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{4a^{\frac{3}{4}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
```

```
[Out] -sqrt(2)*sqrt(-1/sqrt(a))*sqrt(-b*x**2/a)*atan(sqrt(2)*sqrt(-1/sq
rt(a))*(-a - b*x**2)**(1/4)/sqrt(-b*x**2/a))/(4*b*x) + x*(-a)**(1
/4)*elliptic_pi(sqrt(a)/sqrt(-a), asin((-a - b*x**2)**(1/4)/(-a)*
*(1/4)), -1)/(a**(3/2)*sqrt(1 - sqrt(-a - b*x**2)/sqrt(-a))*sqrt(
1 + sqrt(-a - b*x**2)/sqrt(-a))) + sqrt(-b*x**2/(sqrt(a) + sqrt(-
a - b*x**2))**2)*(sqrt(a) + sqrt(-a - b*x**2))*elliptic_f(2*atan(
(-a - b*x**2)**(1/4)/a**(1/4)), 1/2)/(4*a**(3/4)*b*x)
```

Mathematica [C] time = 0.252381, size = 168, normalized size = 1.63

$$-\frac{6axF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)}{\sqrt[4]{-a-bx^2}\,(2a+bx^2)\left(6aF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)-bx^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{bx^2}{2a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] (-6*a*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -(b*x^2)/(2*a)])
/((-a - b*x^2)^(1/4)*(2*a + b*x^2)*(6*a*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -(b*x^2)/(2*a)] - b*x^2*(2*AppellF1[3/2, 1/4, 2,
5/2, -((b*x^2)/a), -(b*x^2)/(2*a)] + AppellF1[3/2, 5/4, 1, 5/2, -
((b*x^2)/a), -(b*x^2)/(2*a)])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^2-2\,a}\frac{1}{\sqrt[4]{-bx^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

```
[Out] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^2+2\,a)(-bx^2-a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{2a\sqrt[4]{-a-bx^2}+bx^2\sqrt[4]{-a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/
4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(bx^2+2\,a)(-bx^2-a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

3.317 $\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqr
t[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2])
```

Rubi [A] time = 0.0282404, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqr
t[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2])
```

Rubi in Sympy [A] time = 45.8595, size = 150, normalized size = 2.83

$$-\frac{\sqrt{2}x(1-i)\left(i;\operatorname{asin}\left(\frac{\sqrt{2}(1+i)\sqrt[4]{x^2-1}}{2}\right)\middle|-1\right)}{2\sqrt{-i\sqrt{x^2-1}+1}\sqrt{i\sqrt{x^2-1}+1}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{x^2-1}+1\right)^2}}\left(\sqrt{x^2-1}+1\right)F\left(2\operatorname{atan}\left(\sqrt[4]{x^2-1}\right)\middle|\frac{1}{2}\right)}{4x}+\frac{\sqrt{2}\sqrt{x^2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt[4]{x^2-1}}{\sqrt{x^2}}\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -sqrt(2)*x*(1 - I)*elliptic_pi(I, asin(sqrt(2)*(1 + I)*(x**2 - 1)
**(1/4)/2), -1)/(2*sqrt(-I*sqrt(x**2 - 1) + 1)*sqrt(I*sqrt(x**2 -
 1) + 1)) + sqrt(x**2/(sqrt(x**2 - 1) + 1)**2)*(sqrt(x**2 - 1) +
1)*elliptic_f(2*atan((x**2 - 1)**(1/4)), 1/2)/(4*x) + sqrt(2)*sqr
```

```
t(x**2)*atanh(sqrt(2)*(x**2 - 1)**(1/4)/sqrt(x**2))/(4*x)
```

Mathematica [C] time = 0.22797, size = 115, normalized size = 2.17

$$-\frac{6xF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)}{(x^2-2)\sqrt[4]{x^2-1}\left(x^2\left(2F_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};x^2,\frac{x^2}{2}\right)+F_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};x^2,\frac{x^2}{2}\right)\right)+6F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};x^2,\frac{x^2}{2}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] (-6*x*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2])/((-2 + x^2)*(-1 + x
^2)^(1/4)*(6*AppellF1[1/2, 1/4, 1, 3/2, x^2, x^2/2] + x^2*(2*Appe
llF1[3/2, 1/4, 2, 5/2, x^2, x^2/2] + AppellF1[3/2, 5/4, 1, 5/2, x
^2, x^2/2])))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{1}{-x^2+2}\frac{1}{\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+2)/(x^2-1)^(1/4),x)
```

```
[Out] int(1/(-x^2+2)/(x^2-1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Fricas [A] time = 2.4003, size = 124, normalized size = 2.34

$$-\frac{1}{8}\sqrt{2}\left(2\arctan\left(\frac{\sqrt{2}(x^2-1)^{\frac{1}{4}}}{x}\right)-\log\left(-\frac{4\,(x^2-1)^{\frac{1}{4}}x^3+4\sqrt{2}\sqrt{x^2-1}x^2+8\,(x^2-1)^{\frac{3}{4}}x+\sqrt{2}(x^4+4\,x^2-4)}{x^4-4\,x^2+4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(2*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) - log(-(4*(x^2
- 1)^(1/4)*x^3 + 4*sqrt(2)*sqrt(x^2 - 1)*x^2 + 8*(x^2 - 1)^(3/4)*
x + sqrt(2)*(x^4 + 4*x^2 - 4))/(x^4 - 4*x^2 + 4)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.318 $\int \frac{(a+bx^2)^{7/4}}{c+dx^2} dx$

Optimal. Leaf size=362

$$-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}-\frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}}+\frac{6abx}{5d\sqrt[4]{a+bx^2}}+\frac{2bx\left(a+bx^2\right)^{3/4}}{5d}$$

```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a +
 b*x^2)^(1/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqr
t[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(5*d*(a + b*x^2)^(1/4)) + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(
1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)^(3/2)*Sqrt[-((
b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), Ar
cSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b
*c) + a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5
/2)*x)
```

Rubi [A] time = 0.683797, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$-\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}-\frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}}+\frac{6abx}{5d\sqrt[4]{a+bx^2}}+\frac{2bx\left(a+bx^2\right)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a +
 b*x^2)^(1/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqr
t[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(5*d*(a + b*x^2)^(1/4)) + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(
1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)^(3/2)*Sqrt[-((
b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), Ar
cSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b
*c) + a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5
/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(ad-bc)^{\frac{3}{2}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{\frac{5}{2}}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(ad-bc)^{\frac{3}{2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{\frac{5}{2}}x} - \frac{3a^2b\int\frac{1}{(a+bx^2)^{\frac{5}{4}}}\,dx}{5d} + \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{ab\,(ad-bc)\int\frac{1}{(a+bx^2)^{\frac{5}{4}}}\,dx}{d^2} + \frac{2bx\left(a+bx^2\right)^{\frac{3}{4}}}{5d} + \frac{2bx\,(ad-bc)}{d^2\sqrt[4]{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c)**(3/2)*elliptic_pi(-sqrt(a)*
sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/
(d**(5/2)*x) - a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c)**(3/2)*ellipt
ic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a
**(1/4)), -1)/(d**(5/2)*x) - 3*a**2*b*Integral((a + b*x**2)**(-5/
4), x)/(5*d) + 6*a*b*x/(5*d*(a + b*x**2)**(1/4)) - a*b*(a*d - b*c
)*Integral((a + b*x**2)**(-5/4), x)/d**2 + 2*b*x*(a + b*x**2)**(3
/4)/(5*d) + 2*b*x*(a*d - b*c)/(d**2*(a + b*x**2)**(1/4))
```

Mathematica [C] time = 1.10477, size = 431, normalized size = 1.19

$$\frac{2x\left(\frac{b\left(3x^2\left(a+bx^2\right)\left(c+dx^2\right)\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-5ac\left(6ac+14adx^2+bcx^2+6bdx^4\right)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{x^2\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)} - \frac{\cdots}{x\cdots}\right.}{15d\sqrt[4]{a+bx^2}\,(c+dx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (2*x*((-9*a^2*c*(-2*b*c + 5*a*d)*AppellF1[1/2, 1/4, 1, 3/2, -((b*
x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((
b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)])) + (b*(-5*a*c*(6*a*c + b*c*x^2 + 14*a*d*x^2
 + 6*b*d*x^4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/
c)] + 3*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[5/2, 1/4, 2,
7/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2,
 -((b*x^2)/a), -((d*x^2)/c)])))/(-10*a*c*AppellF1[3/2, 1/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 1/4, 2,
 7/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2
, -((b*x^2)/a), -((d*x^2)/c)]))))/(15*d*(a + b*x^2)^(1/4)*(c + d*
x^2))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{7}{4}}}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.319 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2} \, dx$

Optimal. Leaf size=302

$$\frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{2bx\sqrt[4]{a+bx^2}}{3d}$$

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)
/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a +
b*x^2)^(3/4)) - (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)
^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((S
qrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(
1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(d^2*x)
```

Rubi [A] time = 0.580659, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{2bx\sqrt[4]{a+bx^2}}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)
/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a +
b*x^2)^(3/4)) - (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)
^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((S
qrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(
```

```
1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(d^2*x)
```

Rubi in Sympy [A] time = 98.3325, size = 270, normalized size = 0.89

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(ad-bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}$$

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(ad-bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}+\frac{2a^{\frac{3}{2}}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{3d\,(a+bx^2)^{\frac{3}{4}}}$$

$$+\frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}(ad-bc)\,F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{d^2\,(a+bx^2)^{\frac{3}{4}}}+\frac{2bx\sqrt[4]{a+bx^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c)*elliptic_pi(-sqrt(a)*sqrt(d
)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(d**2*
x) - a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c)*elliptic_pi(sqrt(a)*sqr
t(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(d*
*2*x) + 2*a**(3/2)*sqrt(b)*(1 + b*x**2/a)**(3/4)*elliptic_f(atan(
sqrt(b)*x/sqrt(a))/2, 2)/(3*d*(a + b*x**2)**(3/4)) + 2*sqrt(a)*sq
rt(b)*(1 + b*x**2/a)**(3/4)*(a*d - b*c)*elliptic_f(atan(sqrt(b)*x
/sqrt(a))/2, 2)/(d**2*(a + b*x**2)**(3/4)) + 2*b*x*(a + b*x**2)**
(1/4)/(3*d)
```

Mathematica [C] time = 1.11216, size = 435, normalized size = 1.44

$$\frac{2x\left(\frac{b\left(3x^2\left(a+bx^2\right)\left(c+dx^2\right)\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-5ac\left(6ac+10adx^2+3bcx^2+6bdx^4\right)F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}-\right.}{9d\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (2*x*((-9*a^2*c*(-2*b*c + 3*a*d)*AppellF1[1/2, 3/4, 1, 3/2, -((b*
x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x
^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((
```

```
b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*
x^2)/a), -((d*x^2)/c)])) + (b*(-5*a*c*(6*a*c + 3*b*c*x^2 + 10*a*d
*x^2 + 6*b*d*x^4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + 3*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[5/2, 3/4,
 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1
, 7/2, -((b*x^2)/a), -((d*x^2)/c)])))/(-10*a*c*AppellF1[3/2, 3/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 3
/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4
, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(9*d*(a + b*x^2)^(3/4)*
(c + d*x^2))
```

Maple [F] time = 0.085, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c} \left(bx^2+a\right)^{\frac{5}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/4)/(d*x^2+c),x)

[Out] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{4}}}{dx^2+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^{\frac{5}{4}}}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)

[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{5}{4}}}{dx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c),x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

3.320 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2} \, dx$

Optimal. Leaf size=244

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}+\frac{2bx}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}}$$

```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a
)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^
2)^(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*Sq
rt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d],
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```

Rubi [A] time = 0.455438, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}+\frac{2bx}{d\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a
)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^
2)^(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^
(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*Sq
rt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d],
```

```
ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{\frac{3}{2}}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{\frac{3}{2}}x} + \frac{b\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*sqrt(a*d - b*c)*elliptic_pi(-sqrt(a)*sqr
t(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(d*
*(3/2)*x) - a**(1/4)*sqrt(-b*x**2/a)*sqrt(a*d - b*c)*elliptic_pi(
sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4
)), -1)/(d**(3/2)*x) + b*Integral((a + b*x**2)**(-1/4), x)/d
```

Mathematica [C] time = 0.249817, size = 161, normalized size = 0.66

$$\frac{6acx\left(a+bx^2\right)^{3/4}F_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(x^2\left(3bcF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-4adF_1\left(\frac{3}{2};-\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+6acF_1\left(\frac{1}{2};-\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(3/4)*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -3/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -3/4, 2
, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 1/4, 1,
5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int\frac{1}{dx^2+c}\left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/4)/(d*x^2+c),x)

[Out] int((b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^{\frac{3}{4}}}{dx^2 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^{\frac{3}{4}}}{c + dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)

[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.321 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}}$$

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x
^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSi
n[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2
)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rubi [A] time = 0.445002, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b
]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x
^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSi
n[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2
)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rubi in Sympy [A] time = 82.7589, size = 172, normalized size = 0.86

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}+\frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{d\left(a+bx^2\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqrt(a*d -
 b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(d*x) - a**(1/4)*s
qrt(-b*x**2/a)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin(
(a + b*x**2)**(1/4)/a**(1/4)), -1)/(d*x) + 2*sqrt(a)*sqrt(b)*(1 +
 b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(d*(a
+ b*x**2)**(3/4))
```

Mathematica [C] time = 0.250046, size = 160, normalized size = 0.8

$$\frac{6acx\sqrt[4]{a+bx^2}F_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(x^2\left(bcF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-4adF_1\left(\frac{3}{2};-\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+6acF_1\left(\frac{1}{2};-\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(1/4)*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -1/4, 2
, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 3/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\sqrt[4]{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/4)/(d*x^2+c),x)

[Out] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)

[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

3.322 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx$

Optimal. Leaf size=167

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*S
qrt[-(b*c) + a*d]*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rubi [A] time = 0.346882, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*S
qrt[-(b*c) + a*d]*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rubi in Sympy [A] time = 58.2096, size = 146, normalized size = 0.87

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqrt(a*d -
b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(sqrt(d)*x*sqrt(a*d
```

```
 - b*c)) - a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(sqrt(a)*sqrt(d)/s
qrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(sqrt(d)*
x*sqrt(a*d - b*c))
```

Mathematica [C] time = 0.0879217, size = 160, normalized size = 0.96

$$-\frac{6acxF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt[4]{a+bx^2}\,(c+dx^2)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])
/((a + b*x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\frac{1}{\sqrt[4]{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a + bx^2}\,(c + dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

3.323 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx$

Optimal. Leaf size=152

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}$$

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)*x) + (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c
 - a*d)*x)
```

Rubi [A] time = 0.365383, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)*x) + (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c
 - a*d)*x)
```

Rubi in Sympy [A] time = 66.3259, size = 131, normalized size = 0.86

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\,(ad-bc)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqrt(a*d -
 b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b*c)) -
```

```
a**(1/4)*sqrt(-b*x**2/a)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b
*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b*c))
```

Mathematica [C] time = 0.0898739, size = 161, normalized size = 1.06

$$-\frac{6acxF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(a+bx^2)^{3/4}(c+dx^2)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])
/((a + b*x^2)^(3/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + bx^{2}\right)^{\frac{3}{4}} \left(c + dx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2} + a\right)^{\frac{3}{4}} \left(dx^{2} + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

3.324 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx$

Optimal. Leaf size=233

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}$$

```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*S
qrt[d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b
*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a
*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rubi [A] time = 0.485656, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```


```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*S
qrt[d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b
*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a
*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{\frac{3}{2}}} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{\frac{3}{2}}} - \frac{b\int\frac{1}{(a+bx^2)^{\frac{5}{4}}}\,dx}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] a**(1/4)*sqrt(d)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqr
t(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b
*c)**(3/2)) - a**(1/4)*sqrt(d)*sqrt(-b*x**2/a)*elliptic_pi(sqrt(a
)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1
)/(x*(a*d - b*c)**(3/2)) - b*Integral((a + b*x**2)**(-5/4), x)/(a
*d - b*c)
```

Mathematica [C] time = 0.547943, size = 339, normalized size = 1.45

$$\frac{2x\left(-\frac{5bcdx^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(x^2\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)} - \frac{9c(a\cdots}{(c+dx^2)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx\cdots}{c}\right.\right.\right.}\right.}{3\sqrt[4]{a+bx^2}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (2*x*((-3*b)/a - (9*c*(b*c + a*d)*AppellF1[1/2, 1/4, 1, 3/2, -((b
*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4,
1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/
4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1
, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))) - (5*b*c*d*x^2*AppellF1[3/2
, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(-10*a*c
*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*
a*d*AppellF1[5/2, 1/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*
AppellF1[5/2, 5/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(3*(-
(b*c) + a*d)*(a + b*x^2)^(1/4))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int\frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{5}{4}}(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2)^{\frac{5}{4}}\,(c + dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

3.325 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

Optimal. Leaf size=254

$$\frac{2bx}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}$$

```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqr
t[a]*(b*c - a*d)*(a + b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a
)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x) - (a^(1/4)*d*Sqrt
[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```

Rubi [A] time = 0.475576, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{2bx}{3a\left(a+bx^2\right)^{3/4}(bc-ad)}+\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3\sqrt{a}\left(a+bx^2\right)^{3/4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqr
t[a]*(b*c - a*d)*(a + b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a
)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x) - (a^(1/4)*d*Sqrt
[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```

Rubi in Sympy [A] time = 88.9241, size = 223, normalized size = 0.88

$$-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\left(ad-bc\right)^2}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\left(ad-bc\right)^2}$$
$$-\frac{2bx}{3a\left(a+bx^2\right)^{\frac{3}{4}}\left(ad-bc\right)}-\frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{3\sqrt{a}\left(a+bx^2\right)^{\frac{3}{4}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] -a**(1/4)*d*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqrt(a*d
 - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b*c)**
2) - a**(1/4)*d*sqrt(-b*x**2/a)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(
a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b*c
)**2) - 2*b*x/(3*a*(a + b*x**2)**(3/4)*(a*d - b*c)) - 2*sqrt(b)*(
1 + b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(3*
sqrt(a)*(a + b*x**2)**(3/4)*(a*d - b*c))
```

Mathematica [C] time = 0.511052, size = 342, normalized size = 1.35

$$\frac{2x\left(\frac{5bcdx^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}+\frac{9c(bc-}{\left(c+dx^2\right)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right.\right.\right.}\right.}{9\left(a+bx^2\right)^{3/4}\left(ad-bc\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (2*x*((-3*b)/a + (9*c*(b*c - 3*a*d)*AppellF1[1/2, 3/4, 1, 3/2, -(
(b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4
, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2,
3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/
4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))) + (5*b*c*d*x^2*AppellF1
[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(-10
*a*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2
*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] +
3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))))
/(9*(-(b*c) + a*d)*(a + b*x^2)^(3/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{d x^{2}+c} \left(b x^{2}+a\right)^{-\frac{7}{4}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b x^{2}+a)^{\frac{7}{4}}(d x^{2}+c)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b x^{2}\right)^{\frac{7}{4}} \left(c+d x^{2}\right)} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}}(dx^2 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

3.326 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$

Optimal. Leaf size=274

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}(bc-ad)^2}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c -
 8*a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(5*a^(3/2)*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*
d^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((-(b*c) +
a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(S
qrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1
/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```

Rubi [A] time = 0.964766, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}(bc-ad)^2}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{5/2}}+\frac{2bx}{5a\left(a+bx^2\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```



```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c -
 8*a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(5*a^(3/2)*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*
d^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((-(b*c) +
```

```
a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(S
qrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1
/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}d^{\frac{3}{2}}\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{\frac{5}{2}}}-\frac{\sqrt[4]{a}d^{\frac{3}{2}}\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(ad-bc)^{\frac{5}{2}}}$$
$$-\frac{2bx}{5a(a+bx^2)^{\frac{5}{4}}(ad-bc)}-\frac{2bx(8ad-3bc)}{5a^2\sqrt[4]{a+bx^2}(ad-bc)^2}+\frac{b(8ad-3bc)\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5a^2(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
```

```
[Out] a**(1/4)*d**(3/2)*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sq
rt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d -
b*c)**(5/2)) - a**(1/4)*d**(3/2)*sqrt(-b*x**2/a)*elliptic_pi(sqrt
(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)),
-1)/(x*(a*d - b*c)**(5/2)) - 2*b*x/(5*a*(a + b*x**2)**(5/4)*(a*d
- b*c)) - 2*b*x*(8*a*d - 3*b*c)/(5*a**2*(a + b*x**2)**(1/4)*(a*d
- b*c)**2) + b*(8*a*d - 3*b*c)*Integral((a + b*x**2)**(-1/4), x)/
(5*a**2*(a*d - b*c)**2)
```

Mathematica [C] time = 1.62462, size = 404, normalized size = 1.47

$$\frac{2x\left(\frac{9ac(5a^2d^2+8abcd-3b^2c^2)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}+\frac{3b(-9a^2d+4ab(c-2dx^2)+3b^2cx^2)}{a+bx^2}-\frac{\cdots}{(c+\cdots}\right.}{15a^2\sqrt[4]{a+bx^2}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (2*x*((3*b*(-9*a^2*d + 3*b^2*c*x^2 + 4*a*b*(c - 2*d*x^2)))/(a + b
*x^2) + (9*a*c*(-3*b^2*c^2 + 8*a*b*c*d + 5*a^2*d^2)*AppellF1[1/2,
 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(6*a*c*Ap
pellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d
*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*App
ellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))) - (5*a*b*c
*d*(-3*b*c + 8*a*d)*x^2*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
-((d*x^2)/c)])/((c + d*x^2)*(-10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2
```

```
, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -(
(b*x^2)/a), -((d*x^2)/c)])))))/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^
(1/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)^{\frac{9}{4}}\,(c+dx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

3.327 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx$

Optimal. Leaf size=304

$$\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{2bx(5bc-12ad)}{21a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*
d)*x)/(21*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*
c - 12*a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(21*a^(3/2)*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (a^(
1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqr
t[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])
/((b*c - a*d)^3*x)
```

Rubi [A] time = 0.874059, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{2bx(5bc-12ad)}{21a^2\left(a+bx^2\right)^{3/4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x(bc-ad)^3}+\frac{2bx}{7a\left(a+bx^2\right)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*
d)*x)/(21*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*
c - 12*a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/2, 2])/(21*a^(3/2)*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (a^(
1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
```

```
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqr
t[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])
/((b*c - a*d)^3*x)
```

Rubi in Sympy [A] time = 169.644, size = 277, normalized size = 0.91

$$-\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\left(ad-bc\right)^3} - \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{x\left(ad-bc\right)^3} - \frac{2bx}{7a\left(a+bx^2\right)^{\frac{7}{4}}\left(ad-bc\right)} - \frac{2bx\left(12ad-5bc\right)}{21a^2\left(a+bx^2\right)^{\frac{3}{4}}\left(ad-bc\right)^2} - \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}\left(12ad-5bc\right)F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{21a^{\frac{3}{2}}\left(a+bx^2\right)^{\frac{3}{4}}\left(ad-bc\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)
```

```
[Out] -a**(1/4)*d**2*sqrt(-b*x**2/a)*elliptic_pi(-sqrt(a)*sqrt(d)/sqrt(
a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d - b*c
)**3) - a**(1/4)*d**2*sqrt(-b*x**2/a)*elliptic_pi(sqrt(a)*sqrt(d)
/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(x*(a*d
 - b*c)**3) - 2*b*x/(7*a*(a + b*x**2)**(7/4)*(a*d - b*c)) - 2*b*x
*(12*a*d - 5*b*c)/(21*a**2*(a + b*x**2)**(3/4)*(a*d - b*c)**2) -
2*sqrt(b)*(1 + b*x**2/a)**(3/4)*(12*a*d - 5*b*c)*elliptic_f(atan(
sqrt(b)*x/sqrt(a))/2, 2)/(21*a**(3/2)*(a + b*x**2)**(3/4)*(a*d -
b*c)**2)
```

Mathematica [C] time = 1.66161, size = 408, normalized size = 1.34

$$\frac{2x\left(\frac{9ac\left(21a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}+\frac{3b\left(-15a^2d+4ab\left(2c-3dx^2\right)+5b^2cx^2\right)}{a+bx^2}+\cdots\right.}{63a^2\left(a+bx^2\right)^{3/4}\left(bc-ad\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] (2*x*((3*b*(-15*a^2*d + 5*b^2*c*x^2 + 4*a*b*(2*c - 3*d*x^2)))/(a
+ b*x^2) + (9*a*c*(5*b^2*c^2 - 12*a*b*c*d + 21*a^2*d^2)*AppellF1[
```

```
1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2)*(6*a*
c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b
*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))) + (5
*a*b*c*d*(-5*b*c + 12*a*d)*x^2*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)])/((c + d*x^2)*(-10*a*c*AppellF1[3/2, 3/4, 1,
 5/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 3/4,
 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1
, 7/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(63*a^2*(b*c - a*d)^2*(a
+ b*x^2)^(3/4))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{dx^2+c}\left(bx^2+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

3.328 $\int \frac{\left(a+bx^2\right)^{7/4}}{\left(c+dx^2\right)^2}\,dx$

Optimal. Leaf size=340

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}+\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*
(a + b*x^2)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c
- a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/2, 2])/(2*c*d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d
]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d
])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(
4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt
[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rubi [A] time = 0.771873, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}+\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{x\left(a+bx^2\right)^{3/4}(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```


```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*
(a + b*x^2)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c
- a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]
]/2, 2])/(2*c*d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d
```

```
]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d
])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(
4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt
[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,(2ad+5bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{\frac{5}{2}}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,(2ad+5bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{\frac{5}{2}}x} + \frac{ab\,(ad-5bc)\int\frac{1}{(a+bx^2)^{\frac{5}{4}}}\,dx}{4cd^2} - \frac{bx\,(ad-5bc)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{x\left(a+bx^2\right)^{\frac{3}{4}}(ad-bc)}{2cd\,(c+dx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*sqrt(a*d - b*c)*(2*a*d + 5*b*c)*elliptic
_pi(-sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a*
*(1/4)), -1)/(4*c*d**(5/2)*x) - a**(1/4)*sqrt(-b*x**2/a)*sqrt(a*d
 - b*c)*(2*a*d + 5*b*c)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*
c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(4*c*d**(5/2)*x) + a*
b*(a*d - 5*b*c)*Integral((a + b*x**2)**(-5/4), x)/(4*c*d**2) - b*
x*(a*d - 5*b*c)/(2*c*d**2*(a + b*x**2)**(1/4)) + x*(a + b*x**2)**
(3/4)*(a*d - b*c)/(2*c*d*(c + d*x**2))
```

Mathematica [C] time = 0.639957, size = 436, normalized size = 1.28

$$\frac{x\left(\frac{5ac\left(6a^2d+ab\left(5dx^2-6c\right)-b^2cx^2\right)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3x^2\left(a+bx^2\right)(bc-ad)\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{c\left(10acF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)} - \frac{\cdots}{x^2\left(4a\cdots\right.}\right.}{6d\sqrt[4]{a+bx^2}\,(c+dx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((-18*a^2*(b*c + a*d)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a),
 -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a
```

```
), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])) + (5*a*c*(6*a^2*d - b^2*c*x^2 + a*b*(-6*c + 5*d*x^
2))*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*(b
*c - a*d)*x^2*(a + b*x^2)*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((b*
x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^2)
/a), -((d*x^2)/c)]))/(c*(10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((
b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^
2)/a), -((d*x^2)/c)])))))/(6*d*(a + b*x^2)^(1/4)*(c + d*x^2))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{7}{4}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.329 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=279

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(1/4))/(2*c*d*(c + d*x^2)) + (Sqrt[a]
*Sqrt[b]*(3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(
3*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c
*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^
(1/4)], -1])/(4*c*d^2*x)
```

Rubi [A] time = 0.706925, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^2)^(1/4))/(2*c*d*(c + d*x^2)) + (Sqrt[a]
*Sqrt[b]*(3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sq
rt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(
3*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c
*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
```

```
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^
(1/4)], -1])/(4*c*d^2*x)
```

Rubi in Sympy [A] time = 107.621, size = 245, normalized size = 0.88

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(2ad+3bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,(2ad+3bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}(ad+3bc)\,F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{2cd^2\left(a+bx^2\right)^{\frac{3}{4}}}+\frac{x\sqrt[4]{a+bx^2}\,(ad-bc)}{2cd\left(c+dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*(2*a*d + 3*b*c)*elliptic_pi(-sqrt(a)*sq
rt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(4
*c*d**2*x) - a**(1/4)*sqrt(-b*x**2/a)*(2*a*d + 3*b*c)*elliptic_pi
(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/
4)), -1)/(4*c*d**2*x) + sqrt(a)*sqrt(b)*(1 + b*x**2/a)**(3/4)*(a*
d + 3*b*c)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(2*c*d**2*(a
+ b*x**2)**(3/4)) + x*(a + b*x**2)**(1/4)*(a*d - b*c)/(2*c*d*(c +
 d*x**2))
```

Mathematica [C] time = 0.632185, size = 439, normalized size = 1.57

$$\frac{x\left(\frac{5ac\left(6a^2d+ab\left(7dx^2-6c\right)-3b^2cx^2\right)F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3x^2\left(a+bx^2\right)(bc-ad)\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}{c\left(10acF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}-\frac{\cdots}{x^2\left(4\cdots\right.}\right.}{6d\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((-18*a^2*(b*c + a*d)*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
 -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a
), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])) + (5*a*c*(6*a^2*d - 3*b^2*c*x^2 + a*b*(-6*c + 7*
d*x^2))*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] +
```

```
3*(b*c - a*d)*x^2*(a + b*x^2)*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -
((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((
b*x^2)/a), -((d*x^2)/c)]))/(c*(10*a*c*AppellF1[3/2, 3/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[5/2, 3/4, 2, 7/
2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2,
 -((b*x^2)/a), -((d*x^2)/c)])))))/(6*d*(a + b*x^2)^(3/4)*(c + d*x
^2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^2+c)^2}\left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{4}}}{(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{5}{4}}}{\left(c+dx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{5}{4}}}{\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

3.330 $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}$$

```
[Out] -(b*x)/(2*c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c
+ d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/
4)*(b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4
*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c + 2*a*d)*Sqrt[-(
(b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcS
in[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*
d]*x)
```

Rubi [A] time = 0.618487, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}-\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] -(b*x)/(2*c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c
+ d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/
```

```
4)*(b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4
*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c + 2*a*d)*Sqrt[-(
(b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcS
in[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*
d]*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad+\frac{bc}{2}\right)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cd^{\frac{3}{2}}x\sqrt{ad-bc}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad+\frac{bc}{2}\right)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cd^{\frac{3}{2}}x\sqrt{ad-bc}} + \frac{ab\int\frac{1}{(a+bx^2)^{\frac{5}{4}}}\,dx}{4cd} - \frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x\left(a+bx^2\right)^{\frac{3}{4}}}{2c\left(c+dx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*(a*d + b*c/2)*elliptic_pi(-sqrt(a)*sqrt(
d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(2*c*
d**(3/2)*x*sqrt(a*d - b*c)) - a**(1/4)*sqrt(-b*x**2/a)*(a*d + b*c
/2)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2
)**(1/4)/a**(1/4)), -1)/(2*c*d**(3/2)*x*sqrt(a*d - b*c)) + a*b*In
tegral((a + b*x**2)**(-5/4), x)/(4*c*d) - b*x/(2*c*d*(a + b*x**2)
**(1/4)) + x*(a + b*x**2)**(3/4)/(2*c*(c + d*x**2))
```

Mathematica [C] time = 0.293699, size = 320, normalized size = 1.04

$$\frac{x\left(-\frac{18a^2F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}+\frac{5abx^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\ldots\right.}{x^2\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\ldots\right.\right.}\right.}{6\sqrt[4]{a+bx^2}\left(c+dx^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((3*(a + b*x^2))/c - (18*a^2*AppellF1[1/2, 1/4, 1, 3/2, -((b*x
^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^
2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b
```

```
*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2
)/a), -((d*x^2)/c)])) + (5*a*b*x^2*AppellF1[3/2, 1/4, 1, 5/2, -((
b*x^2)/a), -((d*x^2)/c)])/(-10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -((
b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2,
-((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b
*x^2)/a), -((d*x^2)/c)]))))/(6*(a + b*x^2)^(1/4)*(c + d*x^2))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^{2}+c\right)^{2}} \left(bx^{2}+a\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)^{\frac{3}{4}}}{\left(dx^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{3}{4}}}{(c+dx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2)**2, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.331 $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2} dx$

Optimal. Leaf size=278

$$\frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)}+\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}$$

```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (
b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c
*d*(a + b*x^2)^(3/4)) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]
*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a +
b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c - a*d)*x) - (a^(1/4)*(b*c
 - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x)
```

Rubi [A] time = 0.604801, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)}+\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cdx(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (
b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c
*d*(a + b*x^2)^(3/4)) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]
*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a +
b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c - a*d)*x) - (a^(1/4)*(b*c
```

```
 - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x)
```

Rubi in Sympy [A] time = 106.984, size = 233, normalized size = 0.84

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{bc}{2}\right)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cdx\,(ad-bc)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{bc}{2}\right)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cdx\,(ad-bc)}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{2cd\,(a+bx^2)^{\frac{3}{4}}}+\frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c/2)*elliptic_pi(-sqrt(a)*sqrt
(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(2*c
*d*x*(a*d - b*c)) - a**(1/4)*sqrt(-b*x**2/a)*(a*d - b*c/2)*ellipt
ic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a
**(1/4)), -1)/(2*c*d*x*(a*d - b*c)) + sqrt(a)*sqrt(b)*(1 + b*x**2
/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(2*c*d*(a + b
*x**2)**(3/4)) + x*(a + b*x**2)**(1/4)/(2*c*(c + d*x**2))
```

Mathematica [C] time = 0.281755, size = 322, normalized size = 1.16

$$\frac{x\left(-\frac{18a^2F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}-\frac{5abx^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\right.}{x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\right.\right.}\right.}{6\,(a+bx^2)^{3/4}\,(c+dx^2)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((3*(a + b*x^2))/c - (18*a^2*AppellF1[1/2, 3/4, 1, 3/2, -((b*x
^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^
2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b
*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x
^2)/a), -((d*x^2)/c)])) - (5*a*b*x^2*AppellF1[3/2, 3/4, 1, 5/2, -
```

```
((b*x^2)/a), -((d*x^2)/c)])/(-10*a*c*AppellF1[3/2, 3/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2
, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2,
-((b*x^2)/a), -((d*x^2)/c)])))/(6*(a + b*x^2)^(3/4)*(c + d*x^2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{2}+c\right)^{2}} \sqrt[4]{b x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)

[Out] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{2}+a\right)^{\frac{1}{4}}}{\left(d x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+b x^{2}}}{\left(c+d x^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{1}{4}}}{(dx^2 + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

3.332 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}\,dx$

Optimal. Leaf size=336

$$\frac{bx}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}$$

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4
))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c -
 a*d)*(a + b*x^2)^(1/4)) - (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[
(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2
)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rubi [A] time = 0.718315, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$\frac{bx}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{dx\left(a+bx^2\right)^{3/4}}{2c\left(c+dx^2\right)(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4
))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c -
```

```
 a*d)*(a + b*x^2)^(1/4)) - (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[
(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2
)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sq
rt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/
4)], -1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{3bc}{2}\right)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c\sqrt{d}x\left(ad-bc\right)^{\frac{3}{2}}}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{3bc}{2}\right)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2c\sqrt{d}x\left(ad-bc\right)^{\frac{3}{2}}}+\frac{ab\int\frac{1}{\left(a+bx^2\right)^{\frac{5}{4}}}\,dx}{4c\left(ad-bc\right)}-\frac{bx}{2c\sqrt[4]{a+bx^2}\left(ad-bc\right)}+\frac{dx\left(a+bx^2\right)^{\frac{3}{4}}}{2c\left(c+dx^2\right)\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] a**(1/4)*sqrt(-b*x**2/a)*(a*d - 3*b*c/2)*elliptic_pi(-sqrt(a)*sqr
t(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(2*
c*sqrt(d)*x*(a*d - b*c)**(3/2)) - a**(1/4)*sqrt(-b*x**2/a)*(a*d -
 3*b*c/2)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a +
b*x**2)**(1/4)/a**(1/4)), -1)/(2*c*sqrt(d)*x*(a*d - b*c)**(3/2))
+ a*b*Integral((a + b*x**2)**(-5/4), x)/(4*c*(a*d - b*c)) - b*x/(
2*c*(a + b*x**2)**(1/4)*(a*d - b*c)) + d*x*(a + b*x**2)**(3/4)/(2
*c*(c + d*x**2)*(a*d - b*c))
```

Mathematica [C] time = 0.98374, size = 358, normalized size = 1.07

$$\frac{x\left(\frac{5abdx^2F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ad-bc)\left(x^2\left(4adF_1\left(\frac{5}{2};\frac{1}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{5}{2};\frac{5}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}+\frac{18a(ad-2}{(bc-ad)\left(x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right.\right.\right.}\right.}{6\sqrt[4]{a+bx^2}\left(c+dx^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((-3*d*(a + b*x^2))/(c*(b*c - a*d)) + (18*a*(-2*b*c + a*d)*App
ellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((b*c - a*d)
```

```
*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)
] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
 + (5*a*b*d*x^2*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2
)/c)])/((-(b*c) + a*d)*(-10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((
b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^
2)/a), -((d*x^2)/c)])))))/(6*(a + b*x^2)^(1/4)*(c + d*x^2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{2}+c\right)^{2}} \frac{1}{\sqrt[4]{b x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+a\right)^{\frac{1}{4}} \left(d x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{1}{4}}(dx^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

3.333 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=292

$$-\frac{dx\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\,(a+bx^2)^{3/4}\,(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}$$

```
[Out] -(d*x*(a + b*x^2)^(1/4))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]
*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c
- 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
- a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*Ellipti
cPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rubi [A] time = 0.646343, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381

$$-\frac{dx\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\,(a+bx^2)^{3/4}\,(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```



```
[Out] -(d*x*(a + b*x^2)^(1/4))/(2*c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]
*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c
- 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
- a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*Ellipti
```

```
cPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rubi in Sympy [A] time = 113.161, size = 250, normalized size = 0.86

$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{5bc}{2}\right)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cx\left(ad-bc\right)^2}-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(ad-\frac{5bc}{2}\right)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{2cx\left(ad-bc\right)^2}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{2c\left(a+bx^2\right)^{\frac{3}{4}}\left(ad-bc\right)}+\frac{dx\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] -a**(1/4)*sqrt(-b*x**2/a)*(a*d - 5*b*c/2)*elliptic_pi(-sqrt(a)*sq
rt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/(2
*c*x*(a*d - b*c)**2) - a**(1/4)*sqrt(-b*x**2/a)*(a*d - 5*b*c/2)*e
lliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1
/4)/a**(1/4)), -1)/(2*c*x*(a*d - b*c)**2) + sqrt(a)*sqrt(b)*(1 +
b*x**2/a)**(3/4)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(2*c*(a
 + b*x**2)**(3/4)*(a*d - b*c)) + d*x*(a + b*x**2)**(1/4)/(2*c*(c
+ d*x**2)*(a*d - b*c))
```

Mathematica [C] time = 0.811733, size = 340, normalized size = 1.16

$$\frac{x\left(\frac{5abdx^2F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-10acF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}+\frac{18a(ad-2bc)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2}\ldots\right.}{x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\ldots\right.\right.}\right.}{6\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)\left(bc-ad\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((-3*d*(a + b*x^2))/c + (18*a*(-2*b*c + a*d)*AppellF1[1/2, 3/4
, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4,
 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3
/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4
, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + (5*a*b*d*x^2*AppellF1[3
```

```
/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-10*a*c*AppellF1[3
/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF
1[5/2, 3/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[
5/2, 7/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(6*(b*c - a*d)*
(a + b*x^2)^(3/4)*(c + d*x^2))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)

[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

3.334 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=314

$$-\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c\sqrt[4]{a+bx^2}(bc-ad)^2}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]
*(4*b*c + a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(2*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) -
(a^(1/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-(
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^(1/4)*Sqrt[d]*(7*
b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b
*c) + a*d)^(5/2)*x)
```

Rubi [A] time = 1.04017, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$-\frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c\sqrt[4]{a+bx^2}(bc-ad)^2}-\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}+\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```


```
[Out] -(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]
*(4*b*c + a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(2*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) -
(a^(1/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-(
```

```
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^(1/4)*Sqrt[d]*(7*
b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b
*c) + a*d)^(5/2)*x)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,(2ad-7bc)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx\,(ad-bc)^{\frac{5}{2}}} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,(2ad-7bc)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx\,(ad-bc)^{\frac{5}{2}}} + \frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(ad-bc)} + \frac{bx\,(ad+4bc)}{2ac\sqrt[4]{a+bx^2}\,(ad-bc)^2} - \frac{b\,(ad+4bc)\int\frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4ac\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] a**(1/4)*sqrt(d)*sqrt(-b*x**2/a)*(2*a*d - 7*b*c)*elliptic_pi(-sqr
t(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)),
 -1)/(4*c*x*(a*d - b*c)**(5/2)) - a**(1/4)*sqrt(d)*sqrt(-b*x**2/a
)*(2*a*d - 7*b*c)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), as
in((a + b*x**2)**(1/4)/a**(1/4)), -1)/(4*c*x*(a*d - b*c)**(5/2))
+ d*x/(2*c*(a + b*x**2)**(1/4)*(c + d*x**2)*(a*d - b*c)) + b*x*(a
*d + 4*b*c)/(2*a*c*(a + b*x**2)**(1/4)*(a*d - b*c)**2) - b*(a*d +
 4*b*c)*Integral((a + b*x**2)**(-1/4), x)/(4*a*c*(a*d - b*c)**2)
```

Mathematica [C] time = 0.972185, size = 480, normalized size = 1.53

$$\frac{x\left(\frac{18\left(-a^2d^2+4abcd+2b^2c^2\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)} + \frac{5ac\left(6a^2d^2+5abd^2x^2+4b^2c\left(6c+5dx^2\right)\right)F_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a} \cdots\right.}{ac\left(10acF_1\left(\frac{3}{2};\frac{1}{4},1;\frac{5}{2};-\frac{bx^2}{a} \cdots\right.\right.}\right.}{6\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((18*(2*b^2*c^2 + 4*a*b*c*d - a^2*d^2)*AppellF1[1/2, 1/4, 1, 3
/2, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/
2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2,
```

```
 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])) + (5*a*c*(6*a^2*d^2 + 5*a*b*d^2*x
^2 + 4*b^2*c*(6*c + 5*d*x^2))*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2
)/a), -((d*x^2)/c)] - 3*x^2*(a^2*d^2 + a*b*d^2*x^2 + 4*b^2*c*(c +
 d*x^2))*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((b*x^2)/a), -((d*x^2
)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)
]))/(a*c*(10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^
2)/c)] - x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((b*x^2)/a), -((d
*x^2)/c)] + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^2)/a), -((d*x^2
)/c)])))))/(6*(b*c - a*d)^2*(a + b*x^2)^(1/4)*(c + d*x^2))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{2}+c\right)^{2}} \left(b x^{2}+a\right)^{-\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+a\right)^{\frac{5}{4}} \left(d x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

3.335 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2} dx$

Optimal. Leaf size=345

$$\frac{bx(3ad+4bc)}{6ac\,(a+bx^2)^{3/4}\,(bc-ad)^2}-\frac{dx}{2c\,(a+bx^2)^{3/4}\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c\,(a+bx^2)^{3/4}\,(bc-ad)^2}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}$$

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(
4*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) -
(a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[
a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)
], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-
((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], Arc
Sin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rubi [A] time = 0.972986, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429

$$\frac{bx(3ad+4bc)}{6ac\,(a+bx^2)^{3/4}\,(bc-ad)^2}-\frac{dx}{2c\,(a+bx^2)^{3/4}\,(c+dx^2)\,(bc-ad)}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c\,(a+bx^2)^{3/4}\,(bc-ad)^2}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(
4*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)
/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) -
(a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[
a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)
], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-
((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], Arc
Sin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rubi in Sympy [A] time = 173.819, size = 303, normalized size = 0.88

$$-\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,(2ad-9bc)\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx\,(ad-bc)^3} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,(2ad-9bc)\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\operatorname{asin}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx\,(ad-bc)^3} + \frac{dx}{2c\,(a+bx^2)^{\frac{3}{4}}\,(c+dx^2)\,(ad-bc)} + \frac{bx\,(3ad+4bc)}{6ac\,(a+bx^2)^{\frac{3}{4}}\,(ad-bc)^2} + \frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}(3ad+4bc)\,F\left(\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}\middle|2\right)}{6\sqrt{a}c\,(a+bx^2)^{\frac{3}{4}}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] -a**(1/4)*d*sqrt(-b*x**2/a)*(2*a*d - 9*b*c)*elliptic_pi(-sqrt(a)*
sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)**(1/4)/a**(1/4)), -1)/
(4*c*x*(a*d - b*c)**3) - a**(1/4)*d*sqrt(-b*x**2/a)*(2*a*d - 9*b*
c)*elliptic_pi(sqrt(a)*sqrt(d)/sqrt(a*d - b*c), asin((a + b*x**2)
**(1/4)/a**(1/4)), -1)/(4*c*x*(a*d - b*c)**3) + d*x/(2*c*(a + b*x
**2)**(3/4)*(c + d*x**2)*(a*d - b*c)) + b*x*(3*a*d + 4*b*c)/(6*a*
c*(a + b*x**2)**(3/4)*(a*d - b*c)**2) + sqrt(b)*(1 + b*x**2/a)**(
3/4)*(3*a*d + 4*b*c)*elliptic_f(atan(sqrt(b)*x/sqrt(a))/2, 2)/(6*
sqrt(a)*c*(a + b*x**2)**(3/4)*(a*d - b*c)**2)
```

Mathematica [C] time = 1.03105, size = 485, normalized size = 1.41

$$\frac{x\left(\frac{5ac\left(18a^2d^2+21abd^2x^2+4b^2c\left(6c+7dx^2\right)\right)F_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-3x^2\left(3a^2d^2+3abd^2x^2+4b^2c\left(c+dx^2\right)\right)\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\right.\right.}{ac\left(10acF_1\left(\frac{3}{2};\frac{3}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-x^2\left(4adF_1\left(\frac{5}{2};\frac{3}{4},2;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{5}{2};\frac{7}{4},1;\frac{7}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)}\right.}{18\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)\left(bc-ad\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((-18*(2*b^2*c^2 - 12*a*b*c*d + 3*a^2*d^2)*AppellF1[1/2, 3/4,
1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1
, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4
, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4,
1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + (5*a*c*(18*a^2*d^2 + 21*a
*b*d^2*x^2 + 4*b^2*c*(6*c + 7*d*x^2))*AppellF1[3/2, 3/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)] - 3*x^2*(3*a^2*d^2 + 3*a*b*d^2*x^2 +
4*b^2*c*(c + d*x^2))*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -((b*x^2)/
a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((b*x^2)/a)
, -((d*x^2)/c)]))/(a*c*(10*a*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -((b
*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((b*x
^2)/a), -((d*x^2)/c)])))))/(18*(b*c - a*d)^2*(a + b*x^2)^(3/4)*(c
 + d*x^2))
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{2}+c\right)^{2}} \left(b x^{2}+a\right)^{-\frac{7}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{2}+a\right)^{\frac{7}{4}} \left(d x^{2}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^{\frac{7}{4}}(dx^2 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

3.336 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=371

$$\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(5ad+4bc)}{10ac\left(a+bx^2\right)^{5/4}(bc-ad)^2}$$

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) -
(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*
(12*b^2*c^2 - 52*a*b*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*Ellip
ticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)
^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-(
(b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(7/2)*
x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*Ellipti
cPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rubi [A] time = 1.43189, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}\left(-5a^2d^2-52abcd+12b^2c^2\right)E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(ad-bc)^{7/2}}-\frac{dx}{2c\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(5ad+4bc)}{10ac\left(a+bx^2\right)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) -
(d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*
(12*b^2*c^2 - 52*a*b*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*Ellip
ticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)
^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-(
(b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), A
rcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(7/2)*
x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*Ellipti
cPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.30074, size = 634, normalized size = 1.71

$$x\left(\frac{18a\left(5a^3d^3-30a^2bcd^2-26ab^2c^2d+6b^3c^3\right)F_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{1}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bcF_1\left(\frac{3}{2};\frac{5}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{1}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}+\frac{3x^2\left(5a^4d^3+10a^3bd^3x^2+a^2b^2d\left(56c^2+56cdx^2+5d^2x^4\right)+\cdots\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((18*a*(6*b^3*c^3 - 26*a*b^2*c^2*d - 30*a^2*b*c*d^2 + 5*a^3*d^
3)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*
c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4
*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c
*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + (-5*a
*c*(30*a^4*d^3 + 55*a^3*b*d^3*x^2 - 12*b^4*c^2*x^2*(6*c + 5*d*x^2
) + a^2*b^2*d*(336*c^2 + 284*c*d*x^2 + 25*d^2*x^4) + 4*a*b^3*c*(-
24*c^2 + 57*c*d*x^2 + 65*d^2*x^4))*AppellF1[3/2, 1/4, 1, 5/2, -((
b*x^2)/a), -((d*x^2)/c)] + 3*x^2*(5*a^4*d^3 + 10*a^3*b*d^3*x^2 -
12*b^4*c^2*x^2*(c + d*x^2) + a^2*b^2*d*(56*c^2 + 56*c*d*x^2 + 5*d
^2*x^4) + 4*a*b^3*c*(-4*c^2 + 9*c*d*x^2 + 13*d^2*x^4))*(4*a*d*App
ellF1[5/2, 1/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF
```

```
1[5/2, 5/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))/(c*(a + b*x^2)*
(10*a*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] -
x^2*(4*a*d*AppellF1[5/2, 1/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)]
 + b*c*AppellF1[5/2, 5/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))))
)/(30*a^2*(b*c - a*d)^3*(a + b*x^2)^(1/4)*(c + d*x^2))
```

Maple [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^2+c\right)^2}\left(bx^2+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^{\frac{9}{4}}\left(dx^2+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

3.337 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx$

Optimal. Leaf size=419

$$\frac{bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{42a^2c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{42a^{3/2}c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(7ad+4bc)}{14ac\left(a+bx^2\right)^{7/4}(bc-ad)^2}$$

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) +
(b*(20*b^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d
)^3*(a + b*x^2)^(3/4)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)
*(c + d*x^2)) + (Sqrt[b]*(20*b^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*(1
3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c
*(b*c - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a +
b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^4*x)
```

Rubi [A] time = 1.24462, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\frac{bx\left(-21a^2d^2-76abcd+20b^2c^2\right)}{42a^2c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\left(-21a^2d^2-76abcd+20b^2c^2\right)F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{42a^{3/2}c\left(a+bx^2\right)^{3/4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}+\frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}};\sin^{-1}\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cx(bc-ad)^4}-\frac{dx}{2c\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)(bc-ad)}+\frac{bx(7ad+4bc)}{14ac\left(a+bx^2\right)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) +
(b*(20*b^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d
)^3*(a + b*x^2)^(3/4)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)
*(c + d*x^2)) + (Sqrt[b]*(20*b^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*(1
3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c
*(b*c - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/
a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a +
b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^4*x)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.38838, size = 637, normalized size = 1.52

$$x\left(\frac{18a\left(21a^3d^3-126a^2bcd^2+38ab^2c^2d-10b^3c^3\right)F_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x^2\left(4adF_1\left(\frac{3}{2};\frac{3}{4},2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bcF_1\left(\frac{3}{2};\frac{7}{4},1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-6acF_1\left(\frac{1}{2};\frac{3}{4},1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}+\frac{3x^2\left(21a^4d^3+42a^3bd^3x^2+a^2b^2d\left(88c^2+88cdx^2+21d^2x\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*((18*a*(-10*b^3*c^3 + 38*a*b^2*c^2*d - 126*a^2*b*c*d^2 + 21*a^
3*d^3)*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(-
6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^
2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] +
 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) +
 (-5*a*c*(126*a^4*d^3 + 273*a^3*b*d^3*x^2 - 20*b^4*c^2*x^2*(6*c +
 7*d*x^2) + 4*a*b^3*c*(-48*c^2 + 61*c*d*x^2 + 133*d^2*x^4) + a^2*
b^2*d*(528*c^2 + 604*c*d*x^2 + 147*d^2*x^4))*AppellF1[3/2, 3/4, 1
, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*x^2*(21*a^4*d^3 + 42*a^3*b
```

```
*d^3*x^2 - 20*b^4*c^2*x^2*(c + d*x^2) + 4*a*b^3*c*(-8*c^2 + 11*c*
d*x^2 + 19*d^2*x^4) + a^2*b^2*d*(88*c^2 + 88*c*d*x^2 + 21*d^2*x^4
))*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -((b*x^2)/a), -((d*x^2)/c)]
+ 3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))/
(c*(a + b*x^2)*(10*a*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)] - x^2*(4*a*d*AppellF1[5/2, 3/4, 2, 7/2, -((b*x^2)/a)
, -((d*x^2)/c)] + 3*b*c*AppellF1[5/2, 7/4, 1, 7/2, -((b*x^2)/a),
-((d*x^2)/c)])))))/(126*a^2*(b*c - a*d)^3*(a + b*x^2)^(3/4)*(c +
d*x^2))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^{2}+c\right)^{2}}\left(bx^{2}+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+a\right)^{\frac{11}{4}}\left(dx^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

3.338 $\int (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^
2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] time = 0.106855, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^
2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi in Sympy [A] time = 29.4338, size = 61, normalized size = 0.77

$$x\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(a+bx^2\right)^p\left(c+dx^2\right)^q \operatorname{appellf}_1\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(1 + d*x**2/c)**(-q)*(a + b*x**2)**p*(c +
d*x**2)**q*appellf1(1/2, -p, -q, 3/2, -b*x**2/a, -d*x**2/c)
```

Mathematica [B] time = 0.349211, size = 172, normalized size = 2.18

$$\frac{3acx\left(a+bx^2\right)^p\left(c+dx^2\right)^qF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{2x^2\left(bcpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)+3acF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -
((b*x^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((
b*x^2)/a), -((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q,
 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2+a\right)^p \left(dx^2+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p*(d*x^2 + c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.339 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx$

Optimal. Leaf size=296

$$\frac{dx\left(a+bx^2\right)^{p+1}\left(15a^2d^2-8abcd(p+6)+b^2c^2\left(4p^2+28p+57\right)\right)}{b^3(2p+3)(2p+5)(2p+7)}$$
$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15a^3d^3-9a^2bcd^2(2p+7)+3ab^2c^2d\left(4p^2+24p+35\right)-b^3c^3\left(8p^3+60p^2+142p+105\right)\right)\,{}_2F_1\left(\right.}{b^3(2p+3)(2p+5)(2p+7)}$$
$$-\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b^2(2p+5)(2p+7)}+\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))
*x*(a + b*x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*
(5*a*d - b*c*(11 + 2*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*
(5 + 2*p)*(7 + 2*p)) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b
*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d^2*(7 + 2*p) + 3*a*b^2*c^
2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 60*p^2 + 8*p^3))
*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.638722, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$\frac{dx\left(a+bx^2\right)^{p+1}\left(15a^2d^2-8abcd(p+6)+b^2c^2\left(4p^2+28p+57\right)\right)}{b^3(2p+3)(2p+5)(2p+7)}$$
$$-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15a^3d^3-9a^2bcd^2(2p+7)+3ab^2c^2d\left(4p^2+24p+35\right)-b^3c^3\left(8p^3+60p^2+142p+105\right)\right)\,{}_2F_1\left(\right.}{b^3(2p+3)(2p+5)(2p+7)}$$
$$-\frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}(5ad-bc(2p+11))}{b^2(2p+5)(2p+7)}+\frac{dx\left(c+dx^2\right)^2\left(a+bx^2\right)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))
*x*(a + b*x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*
(5*a*d - b*c*(11 + 2*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*
(5 + 2*p)*(7 + 2*p)) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b
*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d^2*(7 + 2*p) + 3*a*b^2*c^
2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 60*p^2 + 8*p^3))
*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**3,x)

[Out] Timed out
```

Mathematica [A] time = 0.0975513, size = 136, normalized size = 0.46

$$\frac{1}{35}x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(35c^3\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\left(35c^2\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+dx^2\left(21c\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)+5dx^2\,{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};-\frac{bx^2}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^p*(c + d*x^2)^3,x]

[Out] (x*(a + b*x^2)^p*(35*c^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2
)/a)] + d*x^2*(35*c^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a
)] + d*x^2*(21*c*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)] +
5*d*x^2*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)]))))/(35*(1
+ (b*x^2)/a)^p)
```

Maple [F] time = 0.072, size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^p\left(dx^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^3,x)

[Out] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2+c\right)^3\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*(b*x^2 + a)^p,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3\right)\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*(b*x^2 + a)^p,x, algorithm="fricas")

[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^2 + a)^
p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)^3 \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^3*(b*x^2 + a)^p,x, algorithm="giac")

[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

3.340 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx$

Optimal. Leaf size=176

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2d^2-2abcd(2p+5)+b^2c^2\left(4p^2+16p+15\right)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)} - \frac{dx\left(a+bx^2\right)^{p+1}(3ad-bc(2p+7))}{b^2(2p+3)(2p+5)} + \frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p
)*(5 + 2*p))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p
)) + ((3*a^2*d^2 - 2*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p
^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)
])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.26312, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3a^2d^2-2abcd(2p+5)+b^2c^2\left(4p^2+16p+15\right)\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)} - \frac{dx\left(a+bx^2\right)^{p+1}(3ad-bc(2p+7))}{b^2(2p+3)(2p+5)} + \frac{dx\left(c+dx^2\right)\left(a+bx^2\right)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p
)*(5 + 2*p))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p
)) + ((3*a^2*d^2 - 2*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p
^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)
])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi in Sympy [A] time = 37.0355, size = 160, normalized size = 0.91

$$\frac{dx\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)}{b\left(2p+5\right)} - \frac{dx\left(a+bx^2\right)^{p+1}\left(3ad-2bcp-7bc\right)}{b^2\left(2p+3\right)\left(2p+5\right)} + \frac{x\left(1+\frac{bx^2}{a}\right)^{-p}\left(a+bx^2\right)^p\left(ad\left(3ad-2bcp-7bc\right)-bc\left(2p+3\right)\left(ad-bc\left(2p+5\right)\right)\right){}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{b^2\left(2p+3\right)\left(2p+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c)**2,x)
```

```
[Out] d*x*(a + b*x**2)**(p + 1)*(c + d*x**2)/(b*(2*p + 5)) - d*x*(a + b
*x**2)**(p + 1)*(3*a*d - 2*b*c*p - 7*b*c)/(b**2*(2*p + 3)*(2*p +
5)) + x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*(a*d*(3*a*d - 2*b*c*
p - 7*b*c) - b*c*(2*p + 3)*(a*d - b*c*(2*p + 5)))*hyper((-p, 1/2)
, (3/2,), -b*x**2/a)/(b**2*(2*p + 3)*(2*p + 5))
```

Mathematica [A] time = 0.0637352, size = 106, normalized size = 0.6

$$\frac{1}{15}x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\left(10c\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)+3dx^2\,{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};-\frac{bx^2}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*(15*c^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2
)/a)] + d*x^2*(10*c*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]
 + 3*d*x^2*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])))/(15*(
1 + (b*x^2)/a)^p)
```

Maple [F] time = 0.061, size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(dx^2+c\right)^2\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*(b*x^2 + a)^p,x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^4 + 2\,cdx^2 + c^2\right)\left(bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*(b*x^2 + a)^p,x, algorithm="fricas")

[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 88.4894, size = 88, normalized size = 0.5

$$a^p c^2 x\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2 e^{i\pi}}{a}\right) + \frac{2a^p cdx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2}, -p \\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p d^2 x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c)**2,x)

[Out] a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) +
2*a**p*c*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a
)/3 + a**p*d**2*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*
pi)/a)/5
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)^2 \left(bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^2*(b*x^2 + a)^p,x, algorithm="giac")

[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

3.341 $\int \left(a+bx^2\right)^p \left(c+dx^2\right)\, dx$

Optimal. Leaf size=93

$$\frac{dx\left(a+bx^2\right)^{p+1}}{b(2p+3)}-\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ad-bc(2p+3))\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b(2p+3)}$$

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - ((a*d - b*c*(3 + 2*p))*
x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b
*(3 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0961799, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c-\frac{ad}{2bp+3b}\right)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+\frac{dx\left(a+bx^2\right)^{p+1}}{b(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((c - (a*d)/(3*b + 2*b*
p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]
)/(1 + (b*x^2)/a)^p
```

Rubi in Sympy [A] time = 12.7575, size = 73, normalized size = 0.78

$$\frac{dx\left(a+bx^2\right)^{p+1}}{b\,(2p+3)}-\frac{x\left(1+\frac{bx^2}{a}\right)^{-p}\left(a+bx^2\right)^p(ad-bc\,(2p+3))\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{b\,(2p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p*(d*x**2+c),x)
```

```
[Out] d*x*(a + b*x**2)**(p + 1)/(b*(2*p + 3)) - x*(1 + b*x**2/a)**(-p)*
(a + b*x**2)**p*(a*d - b*c*(2*p + 3))*hyper((-p, 1/2), (3/2,), -b
*x**2/a)/(b*(2*p + 3))
```

Mathematica [A] time = 0.030472, size = 75, normalized size = 0.81

$$\frac{1}{3}x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(3c\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)+dx^2\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{bx^2}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*(3*c*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)] + d*x^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)]))/(3*(1
+ (b*x^2)/a)^p)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int\left(bx^2+a\right)^p\left(dx^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p*(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(dx^2+c\right)\left(bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(b*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^2+c\right)\left(bx^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(b*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Sympy [A] time = 39.9354, size = 53, normalized size = 0.57

$$a^p c x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p d x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p*(d*x**2+c),x)
```

```
[Out] a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**
p*d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^2 + c\right)\left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)*(b*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

3.342 $\int \left(a+bx^2\right)^p\, dx$

Optimal. Leaf size=44

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p
```

Rubi [A] time = 0.0253148, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p
```

Rubi in Sympy [A] time = 4.91128, size = 34, normalized size = 0.77

$$x\left(1+\frac{bx^2}{a}\right)^{-p}\left(a+bx^2\right)^p{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p,x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 1/2), (3/2,), -
b*x**2/a)
```

Mathematica [A] time = 0.00944366, size = 44, normalized size = 1.

$$x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^p
```

Maple [F] time = 0.001, size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p,x)
```

```
[Out] int((b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b x^{2}+a\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [A] time = 9.09413, size = 22, normalized size = 0.5

$$a^p x_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p,x)
```

```
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p, x)
```

3.343 $\int \frac{(a+bx^2)^p}{c+dx^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0711738, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c*(1 + (b*x^2)/a)^p)
```

Rubi in Sympy [A] time = 29.9045, size = 42, normalized size = 0.74

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{-p}\left(a+bx^2\right)^p\operatorname{appellf}_1\left(\frac{1}{2},1,-p,\frac{3}{2},-\frac{dx^2}{c},-\frac{bx^2}{a}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p/(d*x**2+c),x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*appellf1(1/2, 1, -p, 3/2,
-d*x**2/c, -b*x**2/a)/c
```

Mathematica [B] time = 0.307129, size = 162, normalized size = 2.84

$$-\frac{3acx\left(a+bx^2\right)^p F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(2x^2\left(adF_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-bcpF_1\left(\frac{3}{2};1-p,1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -
((d*x^2)/c)])/((c + d*x^2)*(-3*a*c*AppellF1[1/2, -p, 1, 3/2, -((b
*x^2)/a), -((d*x^2)/c)] + 2*x^2*(-(b*c*p*AppellF1[3/2, 1 - p, 1,
5/2, -((b*x^2)/a), -((d*x^2)/c)]) + a*d*AppellF1[3/2, -p, 2, 5/2,
 -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d*x^2 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p/(d*x**2+c),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

3.344 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c^2*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0697793, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x\,(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c^2*(1 + (b*x^2)/a)^p)
```

Rubi in Sympy [A] time = 27.6563, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p\,\operatorname{appellf}_1\left(\frac{1}{2},2,-p,\frac{3}{2},-\frac{dx^2}{c},-\frac{bx^2}{a}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*appellf1(1/2, 2, -p, 3/2,
-d*x**2/c, -b*x**2/a)/c**2
```

Mathematica [B] time = 0.318779, size = 162, normalized size = 2.84

$$-\frac{3acx\,(a+bx^2)^p\,F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^2\left(-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,2;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2adF_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -
((d*x^2)/c)])/((c + d*x^2)^2*(-3*a*c*AppellF1[1/2, -p, 2, 3/2, -(
(b*x^2)/a), -((d*x^2)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] - 2*a*d*AppellF1[3/2, -p, 3, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(dx^2 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(dx^2 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^p}{\left(dx^2 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

3.345 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c^3*(1 + (b*x^2)/a)^p)
```

Rubi [A] time = 0.0687538, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^p/(c + d*x^2)^3,x]
```


```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2
)/c)])/(c^3*(1 + (b*x^2)/a)^p)
```

Rubi in Sympy [A] time = 27.7148, size = 44, normalized size = 0.77

$$\frac{x\left(1+\frac{bx^2}{a}\right)^{-p}\left(a+bx^2\right)^p\operatorname{appellf}_1\left(\frac{1}{2},3,-p,\frac{3}{2},-\frac{dx^2}{c},-\frac{bx^2}{a}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
```

```
[Out] x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*appellf1(1/2, 3, -p, 3/2,
-d*x**2/c, -b*x**2/a)/c**3
```

Mathematica [B] time = 0.383989, size = 162, normalized size = 2.84

$$-\frac{3acx\left(a+bx^2\right)^pF_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)^3\left(-2x^2\left(bcpF_1\left(\frac{3}{2};1-p,3;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-3adF_1\left(\frac{3}{2};-p,4;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)-3acF_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -
((d*x^2)/c)])/((c + d*x^2)^3*(-3*a*c*AppellF1[1/2, -p, 3, 3/2, -(
(b*x^2)/a), -((d*x^2)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 3,
5/2, -((b*x^2)/a), -((d*x^2)/c)] - 3*a*d*AppellF1[3/2, -p, 4, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^p}{\left(dx^2+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^p}{d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)^p/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(dx^2 + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^p/(d*x^2 + c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

3.346 $\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(
2*b*c - 2*a*d)))
```

Rubi [A] time = 0.0575358, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02

$$\frac{x\left(a+bx^2\right)^{-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-
1 + (a*d)/(2*b*c - 2*a*d)),x]
```

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(
2*b*c - 2*a*d)))
```

Rubi in Sympy [A] time = 13.1508, size = 44, normalized size = 0.83

$$\frac{x\left(a+bx^2\right)^{\frac{bc}{2(ad-bc)}}\left(c+dx^2\right)^{-\frac{ad}{2(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-
1+a*d/(-2*a*d+2*b*c)),x)
```

```
[Out] x*(a + b*x**2)**(b*c/(2*(a*d - b*c)))*(c + d*x**2)**(-a*d/(2*(a*d
 - b*c)))/(a*c)
```

Mathematica [C] time = 2.40049, size = 594, normalized size = 11.21

$$3acx\left(a+bx^2\right)^{\frac{bc}{2ad-2bc}}\left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}\left(\frac{bF_1\left(\frac{1}{2};\frac{bc}{2bc-2ad}+1,\frac{ad}{2ad-2bc};\frac{3}{2};-\frac{bx^2}{a},\right.}{\left(a+bx^2\right)\left(x^2\left(a^2d^2F_1\left(\frac{3}{2};\frac{bc}{2bc-2ad}+1,\frac{ad}{2ad-2bc}+1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bc(2ad-3bc)F_1\left(\frac{3}{2};\frac{bc}{2bc-2ad}+2,\frac{ad}{2ad-2bc}\right.\right.\right.}\right.$$
$$+\frac{dF_1\left(\frac{1}{2};\frac{bc}{2bc-2ad},\frac{ad}{2ad-2bc}+1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(x^2\left(b^2c^2F_1\left(\frac{3}{2};\frac{bc}{2bc-2ad}+1,\frac{ad}{2ad-2bc}+1;\frac{5}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+ad(2bc-3ad)F_1\left(\frac{3}{2};\frac{bc}{2bc-2ad},\frac{ad}{2ad-2bc}+2;\frac{5}{2};-\frac{bx^2}{a},-\frac{d}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-
1 + (a*d)/(2*b*c - 2*a*d)),x]
```

```
[Out] 3*a*c*x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(
2*b*c - 2*a*d))*((d*AppellF1[1/2, (b*c)/(2*b*c - 2*a*d), 1 + (a*d
)/(-2*b*c + 2*a*d), 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((c + d*x^2
)*(3*a*c*(-(b*c) + a*d)*AppellF1[1/2, (b*c)/(2*b*c - 2*a*d), 1 +
(a*d)/(-2*b*c + 2*a*d), 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(a
*d*(2*b*c - 3*a*d)*AppellF1[3/2, (b*c)/(2*b*c - 2*a*d), 2 + (a*d)
/(-2*b*c + 2*a*d), 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b^2*c^2*App
ellF1[3/2, 1 + (b*c)/(2*b*c - 2*a*d), 1 + (a*d)/(-2*b*c + 2*a*d),
 5/2, -((b*x^2)/a), -((d*x^2)/c)]))) + (b*AppellF1[1/2, 1 + (b*c)
/(2*b*c - 2*a*d), (a*d)/(-2*b*c + 2*a*d), 3/2, -((b*x^2)/a), -((d
*x^2)/c)])/((a + b*x^2)*(3*a*c*(b*c - a*d)*AppellF1[1/2, 1 + (b*c
)/(2*b*c - 2*a*d), (a*d)/(-2*b*c + 2*a*d), 3/2, -((b*x^2)/a), -((
d*x^2)/c)] + x^2*(a^2*d^2*AppellF1[3/2, 1 + (b*c)/(2*b*c - 2*a*d)
, 1 + (a*d)/(-2*b*c + 2*a*d), 5/2, -((b*x^2)/a), -((d*x^2)/c)] +
b*c*(-3*b*c + 2*a*d)*AppellF1[3/2, 2 + (b*c)/(2*b*c - 2*a*d), (a*
d)/(-2*b*c + 2*a*d), 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))
```

Maple [A] time = 0.005, size = 71, normalized size = 1.3

$$\frac{x}{ac}\left(bx^2+a\right)^{1-\frac{2\,ad-3\,bc}{2\,ad-2\,bc}}\left(dx^2+c\right)^{1-\frac{3\,ad-2\,bc}{2\,ad-2\,bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x)
```

```
[Out] (b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d
-2*b*c)/(a*d-b*c))/a/c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{bc}{2(bc-ad)}-1}\left(dx^2+c\right)^{\frac{ad}{2(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c - a*d)
```

```
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2
*a*d/(b*c - a*d) - 1), x)
```

Fricas [A] time = 0.277354, size = 123, normalized size = 2.32

$$\frac{bdx^5+(bc+ad)x^3+acx}{\left(bx^2+a\right)^{\frac{3\,bc-2\,ad}{2\,(bc-ad)}}\left(dx^2+c\right)^{\frac{2\,bc-3\,ad}{2\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c - a*d)
```

```
[Out] (b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*
a*d)/(b*c - a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a
*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-
2*a*d+2*b*c)),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)^{-\frac{bc}{2(bc-ad)}-1}\left(dx^2+c\right)^{\frac{ad}{2(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c - a*d)

[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2
*a*d/(b*c - a*d) - 1), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```